PHY 2048
December 8, 2018

Final Exam Solutions

1. This is based on problem 5 from Exam 1. We need to relate Jack's velocity with respect to the shore to his velocity with respect to the water using the relation $\vec{v}_{JS} = \vec{v}_{JW} + \vec{v}_{WS}$. We know the direction $\vec{v}_{JS} = -v_{JS}\hat{\imath}$ and we know $\vec{v}_{WS} = -0.6$ m/s$\hat{\jmath}$. If Jack's velocity with respect to the water points at an angle α North of West then $\vec{v}_{JW} = 1$ m/s$(-\cos(\alpha)\hat{\imath} + \sin(\alpha)\hat{\jmath})$. The y-component tells us $\sin(\alpha) = 0.6$ so $\cos(\alpha) = 0.8$, hence $v_{JS} = 0.8$ m/s. The travel time is $\Delta t = (250\text{ m})/(0.8\text{ m/s}) = 312.5\text{ s} \simeq 5.2$ min.

2. This is based on problem 7 from Exam 1. Take left as positive and call the force of M on m ΔF. Then Newton's 2nd law for each block reads:

$$ma = \Delta F - \mu_k mg \ , \tag{1}$$
$$Ma = F - \Delta F - \mu_k Mg \ . \tag{2}$$

Solving the first equation for a and substituting into the second equation gives $\Delta F = \frac{mF}{m+M} = 12$ N.

3. This is based on problem 16 from Exam 1.i Because the block is not moving the totla force on it must be zero. Take the right as the positive x direction and the top of the page as the positive y direction, then the x and y components of the total force are,

$$x \implies F\cos(37^\circ) - \mu_s N = 0 \ , \tag{3}$$
$$y \implies -F\sin(37^\circ) - mg + N = 0 \ . \tag{4}$$

The y equation gives $N = F\sin(37^\circ) + mg$. Substituting this into the x equation gives

$$\mu_s = \frac{F\cos(37^\circ)}{F\sin(37^\circ) + mg} \simeq 0.145 \ . \tag{5}$$

4. This is based on problem 19 from Exam 1. We must first find the velocity of the ball in the frame of reference of the ground: $v_{BG} = v_{BB} + v_{BG} = +1$ m/s. Because the ball has a constant acceleration of

$-g$ its velocity at any time is $v(t) = v_{BG} - gt$. Plugging in $t = 0.5$ s gives $v = +1$ m/s $-$ 4.9 m/s $= -3.9$ m/s. The ball's speed is the magnitude of this.

5. This is based on problem 3 from Exam 2. The work-energy theorem implies that the final kinetic energy must be $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + W =$ 160 J. Solving for the final speed gives $v_f = \sqrt{\frac{320 \text{ J}}{5 \text{ kg}}} = 8$ m/s.

6. This is based on problem 9 from Exam 2. Having the same momenta implies $m_1 v_1 = m_2 v_2$, and hence $v_1/v_2 = m_2/m_1$. This means the ratio of the kinetic energies is

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}m_1 v_1^2}{\frac{1}{2}m_2 v_2^2} = \frac{m_2}{m_1} \ . \tag{6}$$

7. This is based on problem 14 on Exam 2. First compute the balls total kinetic energy $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. A solid ball has $I = \frac{1}{5}mR^2$, and $v = R\omega$, so $K = \frac{7}{10}mv^2$. Because total energy is conserved we must have 75 J $= \frac{7}{10}mv^2 + U(x)$. The ball turns around when its velocity drops to zero, at which point the potential must be $U = 75$ J. Now note that, in the range $1 < x < 5$ the potential energy is a straight line: $U(x) = 125 \text{ J} - 25x$ J/m. This becomes equal to 75 J at $x = 2$ m.

8. This is based on problem 19 from Exam 2. First use the eleastic force equation to relate the force on A to the force on B: $F_A = A_A \times E_A \times \Delta L/L = 2A_B \times 2E_B \times \Delta L/L = 4F_B$. Hence the torques (about the COM) due to A and B are $\tau_A = -d_A F_A = -4 d_A F_B$ and $\tau_B = +d_B F_B$. Because the brick is not rotating the total torque on it must vanish, which implies $d_A/d_B = 1/4$.

9. This was problem 2 on HW 11, which is problem 13.13 in the text. Recall that the volume of a sphere of radius R is $V = \frac{4}{3}\pi R^3$. The bit cut out of the large sphere has radius $R/2$, so its volume (and mass) is 1/8 that of the larger sphere. we can think of the sphere with the cutout as being a complete sphere minus the cutout sphere. Hence the force is,

$$+\frac{GMm}{d^2} - \frac{G\frac{1}{8}Mm}{(d - \frac{1}{2}R)^2} \simeq 8.31 \times 10^{-9} \text{ N} \ . \tag{7}$$

10. This was based on problem 5 in HW 11, which is problem 13.33 in the text. Recall that the total energy in gravity is $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$. Just escaping to infinity wiuth zero velocity implies $E = 0$, and hence $\frac{1}{2}mv^2 = \frac{GMm}{r}$. So the ratio is

$$\frac{K_P}{K_E} = \frac{\frac{GM_P m}{R_P}}{\frac{GM_E m}{R_E}} = \frac{M_P}{M_E} \times \frac{R_E}{R_P} = \frac{3}{2} \, . \tag{8}$$

11. This was problem 9 in HW 11, which is problem 13.65 in the text. A circular orbit of radius r with veolity v about a mass M requires $v^2/r = GM/r^2$. Hence the orbital velocity must be $v\sqrt{GM/r}$. The angular momentum is $L = mrv = m\sqrt{GMr}$.

12. This was problem 2 on HW 12, which is problem 14.19 in the text. Recall the pressure versus depth relation, $P = P_0 + \rho g d$. The air outside the tank pushes inward, which cancels the P_0 term so the net force on a small rectangle of area $\Delta A = w\Delta d$ is $\Delta F = (P - P_0) \times \Delta A = \rho g w d \Delta d$. Hence the total force on a segmet of wall from depth d_2 to d_1 is $F = \frac{1}{2}\rho g w(d_2^2 - d_1^2)$. The initial force is $F_{\text{before}} = 1.568 \times 10^5$ N, and the force after it is filled a further depth of 4 m is $F_{\text{after}} = 6.272 \times 10^5$ N, so the force increases by $F_{\text{after}} - F_{\text{before}} = 4.704 \times 10^5$ N.

13. This was problem 10 on HW 12, which is problem 14.71 in the text. First find the speed v using Bernoulli's equation. The pressure at the top is P_0, as is the pressure at the opening. Because the area of the tank is huge with respect to the hole we can neglect the speed of the water at the top. Hence we have $0 + \rho g h + P_0 = \frac{1}{2}\rho v^2 + 0 + P_0$, which implies $v = \sqrt{2gh}$. Now use kinematics of a bit of water which exits the hole at $t = 0$,

$$x(t) = vt \qquad , \qquad y(t) = H - h - \frac{1}{2}gt^2 \, . \tag{9}$$

The water hit ground ($y = 0$) at $t = \sqrt{2(H-h)/g}$, at which time its position is $x = 2\sqrt{h(H-h)} \simeq 34.6$ cm.

14. This was problem 1 on HW 13, which is problem 15.18 in the text. Note that the amplitude of oscillation is $\frac{1}{2}d$. So going from maximum displacement of $\frac{1}{2}d$ to a displacement of $\frac{1}{4}d$ means that $\cos(2\pi t/T) = \frac{1}{2}$, and hence $t = T/6 = 2$ hrs.

15. This is a simplified version of problem 5 on HW 13, which is problem 15.63 in the text. The period of oscillation is the time it takes to go from one bump to another, $T = \Delta x/v = 0.8$ s. The angular frequency is $\omega = 2\pi/T = \sqrt{k/m}$, hence $k = 4\pi^2 m/T^2 \simeq 61.7 \times 10^3$ kg/s^2.

16. This is a simplified version of problem 8 on HW 13, which is problem 16.29 in the text. The wave speed is $v = \omega/k = 4/20$ m/s $= 0.2$ m/s.

17. This was problem 10 on HW 13, which is problem 16.58 in the text. The 4th harmonic has $\lambda = \frac{1}{2}L = v/f = \sqrt{\tau/\mu f^2} = \sqrt{mg/\mu f^2}$. Solving for the mass gives $m = \mu L^2 f^2/4g \simeq 0.85$ kg.

18. This is problem 17.13 in the text and was covered on Nov. 30. Because $2 = \frac{1}{3} \times 6$ we have $s_B = \frac{1}{3}s_m = s_m \cos(k \times 0.07\ \text{m})$. Hence the wave length is $\lambda = 2\pi \times 0.07\ \text{m}/\cos^{-1}(\frac{1}{3}) \simeq 0.357$ m. Finally, we find the frequency $f = v/\lambda \simeq 960$ Hz.

19. This is problem 17.34 in the text, and was covered on Dec. 3. The distance between horizontal tick marks is 5 dB, hence $\beta_A - \beta_B = 5\ \text{dB} = (10\ \text{dB}) \log_{10}(I_A/I_B)$. Because the surface area is the same for both sources we have $I_A/I_B = P_A/P_B$. Dividing by 10 dB implies,

$$\log_{10}\Big(\frac{P_A}{P_B}\Big) = \frac{1}{2} \qquad \Longrightarrow \qquad \frac{P_A}{P_B} = 10^{0.5} \simeq 3.16\ . \tag{10}$$

20. This is problem 17.59 in the text, and was covered on Dec. 5. Use the Doppler shift formula to compute the frequency received at the American sub,

$$f_{US} = (1000\ \text{Hz}) \times \Big(\frac{5470+70}{5470-50}\Big) \simeq 1022\ \text{Hz}\ . \tag{11}$$