Name (print, last first): $\qquad$ Signature: $\qquad$
On my honor, I have neither given nor received unauthorized aid on this examination.

## YOUR TEST NUMBER IS THE 5-DIGIT NUMBER AT THE TOP OF EACH PAGE.

(1) Code your test number on your answer sheet (use lines 76-80 on the answer sheet for the 5-digit number). Code your name on your answer sheet. DARKEN CIRCLES COMPLETELY. Code your UFID number on your answer sheet.
(2) Print your name on this sheet and sign it also.
(3) Do all scratch work anywhere on this exam that you like. Circle your answers on the test form. At the end of the test, this exam printout is to be turned in. No credit will be given without both answer sheet and printout.
(4) Blacken the circle of your intended answer completely, using a $\# 2$ pencil or blue or black ink. Do not make any stray marks or some answers may be counted as incorrect.
(5) The answers are rounded off. Choose the closest to exact. There is no penalty for guessing.
(6) Hand in the answer sheet (scantron) separately. Only the scantron is graded.

$$
\text { Use } g=9.80 \mathrm{~m} / \mathrm{s}^{2}
$$

Axis

## PHY2048 Exam 1 Formula Sheet

## Vectors

$\vec{a}=a_{x} \hat{i}+a_{y} \hat{j}+a_{z} \hat{k} \quad \vec{b}=b_{x} \hat{i}+b_{y} \hat{j}+b_{z} \hat{k} \quad$ Magnitudes: $\quad|\vec{a}|=\sqrt{a_{x}^{2}+a_{y}^{2}+a_{z}^{2}} \quad|\vec{b}|=\sqrt{b_{x}^{2}+b_{y}^{2}+b_{z}^{2}}$
Scalar Product: $\vec{a} \cdot \vec{b}=a_{x} b_{x}+a_{y} b_{y}+a_{z} b_{z} \quad$ Magnitude: $\vec{a} \cdot \vec{b}=|\vec{a}||\vec{b}| \cos \theta(\theta=$ angle between $\vec{a}$ and $\vec{b})$
Vector Product: $\vec{a} \times \vec{b}=\left(a_{y} b_{z}-a_{z} b_{y}\right) \hat{i}+\left(a_{z} b_{x}-a_{x} b_{z}\right) \hat{j}+\left(a_{x} b_{y}-a_{y} b_{x}\right) \hat{k}$
Magnitude: $|\vec{a} \times \vec{b}|=|\vec{a}||\vec{b}| \sin \theta(\theta=$ angle between $\vec{a}$ and $\vec{b})$
Motion
Displacement: $\Delta \vec{r}=\vec{r}\left(t_{2}\right)-\vec{r}\left(t_{1}\right)$
Average Velocity: $\vec{v}_{\text {ave }}=\frac{\Delta \vec{r}}{\Delta t}=\frac{\vec{r}\left(t_{2}\right)-\vec{r}\left(t_{1}\right)}{t_{2}-t_{1}} \quad \quad$ Average Speed: $s_{\text {ave }}=($ total distance $) / \Delta t$
Instantaneous Velocity: $\vec{v}=\frac{d \vec{r}(t)}{d t} \quad$ Relative Velocity: $\vec{v}_{A C}=\vec{v}_{A B}+\vec{v}_{B C}$
Average Acceleration: $\vec{a}_{\text {ave }}=\frac{\Delta \vec{v}}{\Delta t}=\frac{\vec{v}\left(t_{2}\right)-\vec{v}\left(t_{1}\right)}{t_{2}-t_{1}} \quad$ Instantaneous Acceleration: $\vec{a}=\frac{d \vec{v}}{d t}=\frac{d^{2} \vec{r}}{d t^{2}}$

## Equations of Motion for Constant Acceleration

$\vec{v}=\vec{v}_{0}+\vec{a} t$
$\vec{r}-\vec{r}_{0}=\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2}$
$v_{x}^{2}=v_{x 0}^{2}+2 a_{x}\left(x-x_{0}\right)($ in each of $3 \operatorname{dim})$

## Newton's Laws

$\vec{F}_{n e t}=0 \Leftrightarrow \vec{v}$ is a constant (Newton's First Law)
$\vec{F}_{n e t}=m \vec{a}$ (Newton's Second Law)
"Action = Reaction" (Newton's Third Law)

> Force due to Gravity

Weight (near the surface of the Earth) $=\mathrm{mg}\left(\right.$ use $\left.\mathbf{g}=\mathbf{9 . 8} \mathrm{m} / \mathrm{s}^{2}\right)$
Magnitude of the Frictional Force
Static: $f_{s} \leq \mu_{s} F_{N} \quad$ Kinetic: $f_{k}=\mu_{k} F_{N}$
Uniform Circular Motion (Radius R, Tangential Speed $v=R \omega$, Angular Velocity $\omega$ )
Centripetal Acceleration: $a=\frac{v^{2}}{R}=R \omega^{2} \quad$ Period: $T=\frac{2 \pi R}{v}=\frac{2 \pi}{\omega}$

## Projectile Motion

Range: $R=\frac{v_{0}^{2} \sin \left(2 \theta_{0}\right)}{g}$

## Quadratic Formula

If: $a x^{2}+b x+c=0 \quad$ Then: $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
Work $(W)$, Mechanical Energy ( $E$, Kinetic Energy $(K)$ ), Potential Energy $(U)$
Kinetic Energy: $K=\frac{1}{2} m v^{2} \quad$ Work: $W=\int_{\vec{r}_{1}}^{\vec{r}_{2}} \vec{F} \cdot d \vec{r} \quad$ When force is constant $W=\vec{F} \cdot \vec{d}$
Power: $P=\frac{d W}{d t}=\vec{F} \cdot \vec{v} \quad$ Work-Energy Theorem: $K_{f}=K_{i}+W$

## PHY2048 Exam 2 Formula Sheet

$\Delta U=-W=-\int_{\vec{r}_{1}}^{\vec{r}_{2}} \vec{F} \cdot d \vec{r} \quad F_{x}=-\frac{d U}{d x} \quad$ Mechanical Energy: $E_{\text {mec }}=K+U$
Work-Energy: $W($ external $)=\Delta K+\Delta U+\Delta E($ thermal $)$
Springs
Hooke's Law: $F_{x}=-k x$
Elastic Potential energy ( $x$ from spring equilibrium): $U(x)=\frac{1}{2} k x^{2}$

## Center of Mass and Momentum

Center of Mass: $\quad \vec{r}_{\text {com }}=\frac{1}{M_{\text {tot }}} \sum_{i=1}^{N} m_{i} \vec{r}_{i}$
Linear Momentum: $\vec{p}=m \vec{v} \quad$ Impulse: $\vec{J}=\Delta \vec{p}=\int_{t_{i}}^{t_{f}} \vec{F}(t) d t \quad \vec{F}=\frac{d \vec{p}}{d t}$
$\vec{P}_{\mathrm{tot}}=M_{\mathrm{tot}} \vec{v}_{\mathrm{com}} \quad \vec{F}_{\mathrm{net}}=\frac{d \overrightarrow{\mathrm{P}}_{\mathrm{tot}}}{d t}=M_{\mathrm{tot}} \vec{a}_{\mathrm{com}}$
Rockets: Thrust $=M a=v_{\text {rel }} \frac{d M}{d t}$

$$
\Delta v=v_{\mathrm{rel}} \ln \left(\frac{M_{i}}{M_{f}}\right)
$$

Elastic Collisions of Two Bodies, 1D
$v_{1 f}=\frac{m_{1}-m_{2}}{m_{1}+m_{2}} v_{1 i}+\frac{2 m_{2}}{m_{1}+m_{2}} v_{2 i} \quad v_{2 f}=\frac{2 m_{1}}{m_{1}+m_{2}} v_{1 i}+\frac{m_{2}-m_{1}}{m_{1}+m_{2}} v_{2 i}$

## Rotational Variables

angular position: $\theta(t) \quad$ angular velocity: $\omega(t)=\frac{d \theta(t)}{d t} \quad$ angular acceleration: $\alpha(t)=\frac{d \omega(t)}{d t}=\frac{d^{2} \theta(t)}{d t^{2}}$
arc length: $s=r \theta \quad$ velocity: $v=r \omega$ tangential acceleration: $a_{\mathrm{T}}=r \alpha \quad$ centripetal acceleration: $a_{\mathrm{c}}=r \omega^{2}$
For constant angular acceleration $\alpha$ :

$$
\omega=\omega_{0}+\alpha t \quad \omega^{2}=\omega_{0}^{2}+2 \alpha\left(\theta-\theta_{0}\right) \quad \theta=\theta_{0}+\omega_{0} t+\frac{1}{2} \alpha t^{2}
$$

Rotational (Moment of) Inertia and Rolling
$I=\sum_{i=1}^{N} m_{i} r_{i}^{2}$ (discrete) $\quad I=\int r^{2} d m$ (continuous)
Parallel Axis: $I=I_{\text {com }}+M_{\mathrm{tot}} d^{2}$ ( $d$ is displacement from c.o.m.)
Kinetic Energy: $K_{\text {rot }}=\frac{1}{2} I \omega^{2} \quad K_{\text {roll }}=\frac{1}{2} M v_{\text {com }}^{2}+\frac{1}{2} I_{\text {com }} \omega^{2}$
Torque etc.
$\vec{\tau}=\vec{r} \times \vec{F} \quad \tau=r F \sin \theta \quad$ Angular Momentum: $\vec{L}=\vec{r} \times \vec{p} \quad L=I \omega \quad \vec{\tau}=\frac{d \vec{L}}{d t}$
Work done by a constant torque: $W=\tau \Delta \theta=\Delta K_{\text {rot }}$
Power done by a constant torque: $P=\tau \omega \quad$ For torque acting on a body with rotational inertia $I: \vec{\tau}=I \vec{\alpha}$
Precession frequency: $\Omega=\frac{m g r}{I \omega}$ ( $r$ is moment arm)

$$
\underline{\text { Stress and Strain }}(Y=\text { Young's modulus, } B=\text { bulk modulus })
$$

Linear: $\frac{F}{A}=Y \frac{\Delta L}{L} \quad$ Volume: $P=\frac{F}{A}=-B \frac{\Delta V}{V}$

$$
\underline{\text { Law of Gravitation }}
$$

Magnitude of Force: $F_{\text {grav }}=G \frac{m_{1} m_{2}}{r^{2}}$
Potential Energy: $U_{\text {grav }}=-G \frac{m_{1} m_{2}}{r}$
Law of Periods: $T^{2}=\left(\frac{4 \pi^{2}}{G M}\right) r^{3}$

$$
G=6.67 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}
$$

Total Mechanical Energy for circular orbit: $E=-\frac{G M m}{2 r}$
Escape Speed: $v_{\text {escape }}=\sqrt{\frac{2 G M}{R}}$

## Correct answer marked with

1. A horizontal spring with an object attached to its end is stretched along a horizontal frictionless surface from its initial (equilibrium) position $(x=0)$ to a position $x=d$ and released. What is the magnitude of its velocity at a moment the body moves through the point $x=d / 2$ in its subsequent motion? (Give in terms of the spring constant $(k)$, the distance $(d)$, and the mass of the object $(m))$.
$\boldsymbol{\&}(1) d \sqrt{\frac{3 k}{4 m}}$
(2) $d \sqrt{\frac{k}{2 m}}$
(3) $d \sqrt{\frac{k}{m}}$
(4) $d \sqrt{\frac{m}{k}}$
(5) $d \sqrt{\frac{2 m}{k}}$
2. You drag a box above over level ground by means of pulling a rope $37^{\circ}$ above the horizontal with a force of 20 N , moving the box at a constant velocity of $2 \mathrm{~m} / \mathrm{s}$ for a distance of 3 meters. If the box is of mass 10 kg how much work does the force of friction do, in Joules?
\&(1) -48
(2) -36
(3) -96
(4) There is insufficient information
(5) -60
3. A 2.0 kg cart is moving in a northerly direction at $5.0 \mathrm{~m} / \mathrm{s}$. Twenty seconds later it was observed to be moving in a southerly direction at $5.0 \mathrm{~m} / \mathrm{s}$. What was the net work done on the cart, in Joules?
\&(1) 0
(2) 50
(3) 100
(4) 200
(5) 400
4. A 0.5 kg ball is dropped from century tower ( 50 meters high). During the fall the total thermal energy of the ball and air increases by 100 J . Just before it hits the ground, what is the speed of the ball in $\mathrm{m} / \mathrm{s}$ ?
$\boldsymbol{\%}(1) 24$
(2) 17
(3) 31
(4) 12
(5) 10
5. A bowling ball is on the end of a rope of length 5 meters, the other end of which is attached to a hook in the ceiling. The ball is displaced to one side so that the rope is horizontal, then released so that it swings down through the vertical to the other side. What is the magnitude of its acceleration as it moves through its lowest point?
\&(1) $2 g$
(2) 0
(3) g
(4) 3 g
(5) 5 g
6. The figure shows a plot of potential energy $U$ versus position $x$ of a 0.5 kg particle that can travel only along the $x$ axis under the influence of a conservative force. The graph has these values: $U_{A}=32 J, U_{B}=48 J, U_{C}=60 \mathrm{~J}$.The particle is released at a point just with x a tiny bit more than 2 m . What is the maximum speed (in $\mathrm{m} / \mathrm{s}$ ) of the particle in its subsequent motion?
\&(1) 8
(2) 16
(3) 4
(4) 32
(5) 64

7. A 2 kg ball drops vertically onto the floor with a speed at point of impact of $10 \mathrm{~m} / \mathrm{s}$. It rebounds with an initial speed of $5 \mathrm{~m} / \mathrm{s}$. The ball is in contact with the floor for 0.4 s . What is the magnitude of the average force on the ball from the floor, in Newtons?
\&(1) 75
(2) 25
(3) 30
(4) 10
(5) 40
8. A stationary hand grenade of mass 5 kg explodes into 3 pieces. Two pieces, each of 1 kg , fly off at right angles to one another each with $10 \mathrm{~m} / \mathrm{s}$. How fast does the third ( 3 kg ) piece travel in $\mathrm{m} / \mathrm{s}$ ?
\&(1) 4.7
(2) 6.7
(3) 28
(4) 14
(5) 9.4
9. A man of mass $m$ stands on the end of a barge in a canal (with stationary water). The barge has a length $L$ meters and mass $M$. He walks to the other end of the barge and stops. How far did he walk with respect to the land beside the canal? Ignore any water resistance.
$\boldsymbol{\&}(1) L-L m /(m+M)$
(2) $L-L M /(m+M)$
(3) $L(M-m) / M$
(4) L
(5) $L-L m / M$
10. A propeller blade has a radius (i.e. distance from the central shaft of the propeller to the end) of 0.5 meters. It starts from rest with an angle with respect to the horizontal given by $\theta=0.2 t^{2}$ where $t$ is the time in seconds. At a time of $t$ $=2 \mathrm{~s}$, what is the magnitude of the total linear acceleration (in $\mathrm{m} / \mathrm{s}^{2}$ ) of a point at the end of a blade?
\&(1) 0.38
(2) 0.32
(3) 0.20
(4) 0.64
(5) 0.42
11. A merry-go-round rotates from rest with an angular acceleration of $2.0 \mathrm{rad} / \mathrm{s}^{2}$. How many revolutions does it make in the first three seconds?
\&(1) 1.43
(2) 9.0
(3) 2.86
(4) 0.95
(5) 4.24
12. A force (in Newtons) $\vec{F}=3 \hat{i}+4 \hat{j}$ acts on an apple with position vector (in meters) $\vec{r}=6 \hat{i}+7 \hat{j}$ relative to the origin. What is the torque on the particle (in N.m)?
$\boldsymbol{\&}(1) 3 \hat{k}$
(2) $-3 \hat{k}$
(3) $18 \hat{i}+28 \hat{j}$
(4) $9 \hat{i}+11 \hat{j}$
(5) $52 \hat{k}$
13. A merry-go-round is a uniform disk of has a mass $M$ and radius $R$. A child (who can be considered to be point-like) also has (by coincidence) the same mass, M . This child is on the merry-go-round near its outside edge and the merry-go-round is turning with an angular velocity of $\omega_{1}$ The child then moves to a position at the axis of rotation. What is the merry-go-round's final angular velocity? Assume that there is no friction or other external torques.
$\boldsymbol{q}(1) 3 \omega_{1}$
(2) $2 \omega_{1}$
(3) $\omega_{1}$
(4) $1.5 \omega_{1}$
(5) $0.75 \omega_{1}$
14. A uniform solid sphere and a uniform solid cylinder of equal mass both roll without slipping on a horizontal surface. They have the same center-of-mass velocity. What is the ratio of their kinetic energies, $K E_{b a l l} / K E_{\text {cyl }}$ ?
\&(1) 0.93
(2) 1.00
(3) 1.07
(4) 0.70
(5) There is insufficient information
15. A mass $m$ hangs from a (mass-less) rope which is wound around a frictionless pulley which is a uniform disk of mass $M$ and radius $R$, and released. What is the magnitude of the acceleration of $m$ ?
(1) $\mathrm{mg} /(\mathrm{m}+\mathrm{M} / 2)$
(2) g
(3) $\mathrm{mg} /(\mathrm{m}+\mathrm{M})$
(4) $\mathrm{mg} /(\mathrm{m}+\mathrm{M} / 4)$
(5) $\mathrm{g}(\mathrm{m}-\mathrm{M} / 2) / \mathrm{m}$

## $m$

16. In the figure, a force of magnitude 18.5 N applied horizontally at the axle is just barely able to raise the wheel of radius $r=0.759 \mathrm{~m}$ over an obstacle of height $h=0.288 \mathrm{~m}$. What is the mass of the wheel?
\&(1) 1.49 kg
(2) 1.17 kg
(3) 1.89 kg
(4) 3.09 kg
(5) 2.43 kg

17. In the figure, a 119 kg uniform log hangs by two steel wires, $A$ and $B$, both initially of radius 1.10 mm . Initially, wire $A$ was 2.50 m long and 2.05 mm shorter than wire $B$. The $\log$ is now horizontal. Young's modulus, $Y$, for steel is $2.0 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$. By how much is wire $B$ stretched?
$\boldsymbol{\&}(1) 0.89 \mathrm{~mm}$
(2) 2.9 mm
(3) 3.8 mm

(4) 2.1 mm
(5) 1.6 mm
18. The figure shows a spherical hollow inside a lead sphere of radius $R=4.7 \mathrm{~m}$; the surface of the hollow passes through the center of the sphere and "touches" the right side of the sphere. The gravitational force on a small sphere of mass $m=35 \mathrm{~kg}$ that lies a distance $d=17 \mathrm{~m}$ from the center of the lead sphere is $F=1.67 \times 10^{-9} \mathrm{~N}$. What was the mass of the lead sphere before the spherical hollow was removed?
\&(1) 248 kg

(3) 236 kg
(4) 224 kg
(5) 215 kg
19. Assume a planet is a uniform sphere of radius $R$ that (somehow) has a narrow radial tunnel through its center. Let the gravitational force on the apple at the planet's surface be $F_{R}$. If we move the apple a distance $\frac{3}{4} R$ below the surface (i.e. so that it is $\frac{1}{4} R$ from the center), what is the gravitational force on it?
\&(1) $\frac{1}{4} F_{R}$
(2) $\frac{1}{2} F_{R}$
(3) $\frac{3}{4} F_{R}$
(4) $\frac{1}{16} F_{R}$
(5) $\frac{9}{16} F_{R}$
20. A satellite is in a circular Earth orbit of radius $r$. The area $A$ enclosed by the orbit depends on $r^{2}$ because $A=\pi r^{2}$. Which of the following properties of the satellite depends on $\sqrt{r}$ ?
\&(1) Angular momentum
(2) Period
(3) Kinetic energy
(4) Speed
(5) Insufficient information
