Exam 2 – Fall 2019 – Solutions

1. Energy conservation. Similar to HW5 Q6 and Lecture 9-25
   Taking the equilibrium point of the spring to be the zero of PE. Then the PE before release
   equals the sum kinetic and potential energy afterward.
   $\frac{1}{2} kd^2 = \frac{1}{2} k(d/2)^2 + \frac{1}{2} mv^2$
   $\frac{3}{4} kd^2 = mv^2$
   $v^2 = 3kd^2/4m$ and so $v = d\sqrt{(\frac{3k}{4m})}$
   Unfortunately, and extra d was put in the equation at some point in the editing of the exam. Thus everyone was awarded a point!
2. Work (Chapter 7) An easier version of that done in class 9-23.
   Constant velocity, so net force is zero.
   Look at horizontal forces, we have $F_{netx} = F_{appx} - f_k = 0$
   Therefore $f_k = -F_{appx} = -20*\cos(37^0) = -16$ N
   Work done = $\mathbf{f_k.d}$ = -48 J

3. Easier even than the example done in class 9-25. Remember that KE is a scalar.
   Net work done = change in kinetic energy = $\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$
   = 0 J

4. Similar to the example done in class 10-2.

   $$0 = \Delta U + \Delta K + \Delta E_{TH}$$

   $0 = -mgh + \frac{1}{2} mv^2 + 100$
   $0.50 \times 9.8 \times 50 - 100 = \frac{1}{2} \times 0.5\ v^2$
   v = 24 m/s

5. Similar to HW6 Q5 and bowling ball example done in class 9-30 and the icecube example on 10-2. We use energy conservation.
   $mgh = \frac{1}{2} mv^2$ and so $v^2 = 2gh$ where h = length of rope = radius of circle it is moving in. Acceleration when moving in a circle = $v^2/r = 2g$

6. Similar to HW6 – Q7.
   $\Delta U + \Delta K = 0$
   We see the change in the potential energy is
   $32 - 48 + \Delta K = 0$
   $\frac{1}{2} mv^2 = 16J$
   $v = 8$ m/s

7. Impulse (see HW7, Q6). The change in momentum, (taking up to be positive), is:
   $(5 \times 2) - -(10 \times 2) = 30$ kg m/s = Impulse
   Impulse = (average force) x time, so average force = 30/0.4 = 75 N

8. See class of 10-9. Momentum Conservation.
   We have to add the momenta we are given, then that will tell us the momentum of the third piece. As the two pieces are at right angles to each other. Total Momentum (pieces 1 + 2) = $\sqrt{(10^2 + 10^2)} = 14.1$ kg m/s
   So, the third piece must have that same magnitude of momentum, and its velocity is 14.1/3 = 4.7 m/s

9. HW7 – Q5 and turtle example in class. The center-of-mass cannot move.
   Take starting point to be at position x = 0. Then the center-of-mass original situation taking care to put the com of barge at its center:
   $X_{com} = (0m + ML/2)/(m + M)$
   Now we are going to allow barge to move to distance *x* as the mass moves along the barge to its end.
   $X_{com} = (0m + ML/2)/(m + M) = (m(L-x) + M(L/2-x))/(m + M)$
   So, $ML/2 = m(L-x) + ML/2 - Mx$
   $x = mL/(m+M)$
   and compared with the bank, he moves $L(1 - m/(m+M))$

10. As promised in class 10-14

The total linear acceleration is the vector sum of the tangential and radial components. Differentiate to get ω=0.4t, and so α=0.4 rad/s^2

Tangential component: rα = 0.5 x 0.4 = 0.2 m/s^2
At t =2s, radial component: is rω^2 = 0.5 x 0.8^2 = 0.32 m/s^2
Add the two by the Pythagorean formula, and we get a = 0.38 m/s^2

11. HW8 – Q2 deals with this plugging into equations
$\Delta\theta = \omega t + ½ \alpha t^2 = 0.5 \times 2 \times t^2$

= 9 radians
That is 9/(2π) = 1.43 revolutions

12. A standard cross-product calculation as in HW9 Q9 (which uses a plum, but a change of fruit should not matter). Make sure you do it the correct way around (**r** x **F**). I notice immediately that as the position vector and Force vector are in the xy plane, that the torque will be along the (+ or -) z direction. So I can use the formula on the formula sheet, that it is
$(a_x b_y - a_y b_x)\hat{k}$ = (6 x 4 – 7 x 3) = 3$\hat{k}$

13. Very similar to example in class on 10-23
Angular momentum conservation. We need to lookup the rotational inertia of a solid disk
$I_1\omega_1 = I_2\omega_2$
$(0.5MR^2 + MR^2)\omega_1 = 0.5MR^2\omega_2$
$\omega_2 = 3\ \omega_1$

14. Discussed in class 10-21
KE = $0.5mv^2 + 0.5I\omega^2$

Remembering the v=Rω and looking at the formulae for the ball and cylinder, we get:
$(0.5mv^2 + 0.5*0.4mv^2)/(0.5mv^2 + 0.5*0.5mv^2) = 0.93$

15. HW 9- Q1 and a simpler version of that shown in class 10-18.

We need to look at the mass that is hanging and do a F=ma (positive down):

$mg - T = ma$

Then look at torque around the pulley. We can take positive as being clockwise to be consistent with the mass going down.

$TR = I\alpha = (1/2)MR^2\alpha$

Then we relate the angular acceleration of the pulley to the acceleration of the mass.

$a = R\alpha$

Now we have 3 equations and we can solve.

$mg - T = ma$

$T = 0.5Ma$

$mg = ma+0.5Ma$

$a = mg/(m + M/2)$

16. See HW 10 – Q2

We need to consider the torques around the point of contact of the wheel and the step. If the clockwise torque due to the applied force is larger than the counter-clockwise torque due to the wheel's mass, it will be able to raise the wheel. So, we solve for equal magnitude torques.
$F(0.759-0.288) = 0.471F = MgL$ where L is given by geometry to be
$L = \text{sqrt}(r^2-(r-h)^2) = 0.595$ m
$M = (0.471*18.5)/(0.595*9.8)$

17. See HW 10-Q9 which uses this same set-up.

There are many ways of doing this. One way is to consider how much a 2.5 meter long wire of these dimensions would stretch if it was, by itself, supporting a 119 kg mass. That length is given by: $(F/A) = Y\Delta L/L$ and so

$\Delta L = (FL/AY) = 3.83$ mm. (We are careful to get the units right, and of course $A=\pi r^2$)

This amount of wire stretching is the stretch of A added to that of B.

Now we note that A stretches by 2.05 mm more than B. So therefore, B stretched by (3.83-2.05)/2 = 0.89 mm

18. Homework 11, Question has this set-up.
    $F = GMm/r^2$. Here we have the sum of two forces, one from the mass M of the complete lead sphere and the other due to a “negative mass” hollow. The latter is centered at a distance of (17-4.7/2) away and is of mass M/8 (because mass is proportional to the cube of the radius, and the radius of the hollow is half that of the big sphere)

So, $F/Gm = (1/17^2 – (1/8)/(17-2.35)^2)M \quad = 0.00288M$

$M = (1.67 \times 10^{-9})/[(6.67 \times 10^{-11} \times 35 \times 0.00288) = 248$ kg

19. The force is given by GMm/r^2 where r is the distance to the center and M is the mass inside the point in question. Because the mass is proportional to volume, which is proportional to r^3, we find that inside a planet $F \alpha r$. Therefore, the answer is $F_R/4$

20. F=ma. The force is proportional to $1/r^2$ (Newton’s Universal Law of Gravity) and the acceleration is proportional to 1/r (centripetal acceleration = v^2/r). Therefore, $v^2 \alpha 1/r$

So, let’s look at the answers.

Speed is clearly proportional to $1/\sqrt{r}$

Kinetic energy is proportional to 1/r

Period is distance/speed, so is proportional to $r\sqrt{r}$

Angular Momentum is mvr, so is proportion to $r/\sqrt{r}$ which is the same as $\sqrt{r}$, so that’s the answer.