Q1. This is basically a repeat of Exam 2, number 10, and also a question in class, but the numbers are change. We remember that total linear acceleration has two components, one tangential and one radial. To find the tangential one we differentiate twice to find that α = 0.2 rad/s^2 Therefore the tangential component of the acceleration is rα = **0.1 m/s^2.** The angular velocity is ω = 0.2t = 0.6 rad/s at t = 3 s, so the tangential velocity is rω = 0.3 m/s. The radial acceleration is thus v^2/r = **0.18 m/s^2**

Adding the two in quadrature gives 0.21 m/s^2

Q2. Momentum conservation, like in Exam 2 number 8. If we consider the x-axis to bisect the two 1 kg pieces, we see that the y-coordinates of the momenta of the two pieces to be equal and opposite. The x-coordinate of momentum is (2 pieces) *(1 kg) *(10 m/s) *cos(37^0) and we know that cos(37^0) = 0.8

That gives the two pieces to have a total momentum of 16 kg.m/s in the positive x-direction. The other piece has therefore got to have a speed of (16 kg.m/s) /(2 kg) = 8 m/s.

Q3. Angular momentum conservation (Exam 2, number 13). We need to remember that the rotational inertia of a uniform disk is (1/2)MR^2 where R is the radius of the disk, whereas for a point-like man it is mr^2, where r is the distance from the axis of rotation.

$I_1\omega_1 = I_2\omega_2$

$(0.5MR^2 + 2MR^2)\omega_1 = (0.5MR^2+2M(R/2)^2)\omega_2$

$2.5\omega_1 = 1.0\omega_2$

Q4. We can look at the mass, m and call down is positive as it is going down.

F = mg – T = ma

Now look at the pulley wheel, which has a rotational inertia of (1/2)MR^2

Torque = Iα = (1/2)MR$^2\alpha$ = TR

Lastly, we remember that a = Rα, so the torque equation becomes (1/2)Ma = T

Combining equations, we have mg = ma + (1/2)Ma and a = mg/(m+M/2) which was the answer to Exam 2, Question 15. Plugging back into mg – T = ma, gets us T = mg[1-(m/(m + M/2))]

Q5. It is going up, but its acceleration is down. If you are on a scale that is accelerating downwards, it reads low. You have all felt this as you reach the top of a large building and the elevator slows down, you feel a little “lighter”. This is from Exam 1 number 14.

Formally, if up is positive, F_N-mg = ma (and a is negative). Therefore F_N < mg and it reads low.

Q6. The question on Newton’s Third Law changed from Exam 2. In fact, the solution for the question of Exam 2 gives the solution to this one. Remember Newton’s third law “action-reaction pairs” act on different bodies. The force of A on B is equal and opposite to the force of B on A. Therefore, if the action is the rope pulling the ball up, the reaction to this is the ball pulling the rope down.

Q7. A repeat of Exam 2, number 17 with the numbers changed. The acceleration is given by a = F/m = 25/25 = 1 m/s^2

Now just use F = ma to find that the horizontal force on the upper block is 5 Newtons.

Note that the coefficient of friction is not important provided that there is enough friction for this to happen. The maximum static friction force is 0.51*5*g which is more than enough to supply 5 Newtons.

Q8. (See HW12, Q5) The key here is Pascal's Principle. The added pressure on the left piston is transmitted undiminished to the right piston. $Mg/A_1 = F/(13*A_1) = kx/(13*A_1)$

So k = Mg*13/x = 10.9*9.8*13/0.063 = 22042 N/m

Q9. (See HW12, Q7) There are various ways of doing this. One way is energy conservation. It starts at rest and is at rest at the top of the trajectory, so the work done by forces must add to zero. Here the first force is buoyant force, and the second is the gravitational force.

$\rho_w Vg(0.43) - \rho_b Vg(0.43+1.72) = 0$

$\rho_b = \rho_w(0.43)/(0.43+1.72) = (1/5)\ \rho_w$

(You can also do it by F=ma, find the speed with which it exits the water, etc.)

Q10. (See HW12, Q9) Bernoulli's Equation tells us that when the y positions are the same,

and $\Delta p = P_2 - P_1$

$0.5\rho V^2 = \Delta p + 0.5\rho v^2$

Where the continuity equation tells us that v=(A/a)V, and rate of fluid flow = AV = av

So $0.5\rho V^2 = \Delta p + 0.5\rho v^2 = \Delta p + 0.5\rho(A/a)^2V^2$

$0.5 \times 1000 \times (0.014/0.0078)^2 = -12000 + 0.5 \times 1000 \times (0.014)^2/a^2$

$1607 + 12000 = 500*(0.014/a)^2$

$a^2 = 500*0.014^2/13607$

$a = 0.0027\ m^2 = 27\ cm^2$

Q11. (See HW15 Q2) $y = y_{max}(\omega t+\phi)$

$v = \omega y_{max}(\omega t+\phi)$

$\omega = 3.4/0.0012$ rad/s = 2833 rad/s

T = 2*3.14159/2833 = 0.0022 s = 2.2 ms

Q12. (See HW13, Q3, but this is just putting the right numbers into the formula and using the parallel axis theorem).

$T = 2*3.14159*\sqrt{((I_{com} + Mh^2)/Mgh)}$, where $I_{com} = (M/12)(a^2+b^2)$ from the formula sheet, and h = r = 0.2 m

= 1.1 s

Q13. (See HW13, Q4) This is a tough question. First find the spring constant, k. We find that k = 425*9.8/0.1 = 41650 N/m

Then find $\omega = \sqrt{(k/m)}$ = 9.89 rad/s

Period = T = 2*3.14159/9.89 = 0.635 s

Now we are ready to plug into the formula.

$e^{-bT/2m} = 0.452$

So it loses 55% per cycle

Q14. (See HW13, Q6). We can see that the wavelength (the distance in x over which the graph repeats itself) is 0.4 meters. The wave number is then k = 2*π/0.4 = 5π rad/m

Q15. (See HW13, Q7). Here we use the equation on the formula sheet. At the maximum value of the graphs, we know dK/dt, and that is when the cos term is set to 1. We also are given in this case y_{max}. To find v_{wave} we can look at the distance and time for one cycle on the graphs, and we find that v_{wave} = 100 m/s. Furthermore, we can see that the $\omega=2\pi/T$ and we have T = 2 ms. Now all we have to do is to put the numbers into the formula and solve. Note that some people might read the plot and think that T= 1ms, but that is because it is a plot of the $\cos^2$ not cos (this was in the homework).

$11 = (1/2).\mu.100.(2\pi/0.002)^2.(0.003)^2$ and solve to get μ = 0.0025 kg/m = 2.5 g/m

Q16. (See HW16, Q10)

f = nv/2L (n = 1,2,3 etc), and so $f^2 = n^2v^2/4L^2$

$v = \sqrt{(\tau/\mu)}$ where τ=mg

$f^2 = mgn^2/(4L^2\mu) = 0.58 \times 16 \times 9.8/(4*0.9^2 \times 1.1)$

f = 160 Hz

Q17. See the example in class Nov. 25. This was the easiest question on the exam.

Speed = wavelength x frequency

1300 = λ x 500,000

λ = 1300/500,000 = 0.0026 m = 2.6 mm

Q18. This follows the problem in class (Problem 17.13)

$S_A = S_m$ and so $(kx - \omega t + \phi) = 0$ (Where x = 2 meters)

$S_B = 0.25S_m$ and so cos(0.05 k) = 0.25 and so cos(2*3.14159*0.05/λ) = 0.25

That gives λ = 0.238 meters

f = v/0.238 = 1400 Hz

Q19. One end of the “tube” is closed and one end is open, so that means that ¼ of a wavelength fits into the tube and resonates. The wavelength is 343/8 = 42.8 meters. So the answer is 42.8/4.=10.7 meters

Q20. This is almost identical to the problem in class 12-4.

f_{BD} = 40(1+0.04)/(1-0.04) = 43 kHz