Exam 1 Solutions

Note that there are several variations of some problems, indicated by choices in parentheses.

Problem 1

Let vector $\vec{a} = 4\hat{i} + 3\hat{j}$ and vector $\vec{b} = -\hat{i} + 2\hat{j}$ (or $\vec{b} = -\hat{i} + 4\hat{j}$). What is the opening angle between vectors $\vec{a}$ and $\vec{b}$?

(1) 80° (2) 0.2° (3) 67° (4) 10° (5) 0°

The opening angle can be determined from:

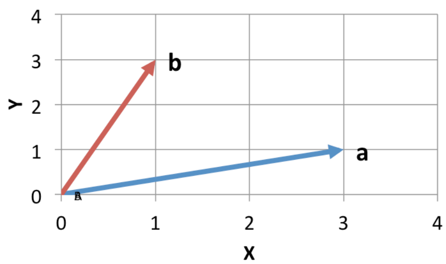
$$\cos\theta = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|a||b|}$$

$$= \frac{(-4+6)}{\sqrt{16+9}\sqrt{1+4}} \quad \text{or} \quad = \frac{(-4+12)}{\sqrt{16+9}\sqrt{1+16}}$$

Yielding 80° (or 67°)

Problem 2

Vectors $\vec{a}$ and $\vec{b}$ are shown in graphical form in the figure. What is the magnitude | $\vec{a}$ − $\vec{b}$ | in the same units as used in the graph?



(1) 2√2 (2) 2 (3) √2 (4) 4√2 (5) 2√10

$$\vec{\mathbf{a}} - \vec{\mathbf{b}} = \left(3\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) - \left(\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right) = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{a}} - \vec{\mathbf{b}}\right| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

Problem 3

Four vectors are expressed in terms of a magnitude and an angle measured counter-clockwise from the x-axis:

A: 6.0 m at 0°
B: 3.0 m at 60°
C: 9.0 m at 135°
D: 6.0 m at 240°

What is the x-component (or y-component) of the sum of these four vectors?

(1) 3.8m (2) -1.9m (3) 16.9m (4) 6.2m (5) -9.0m

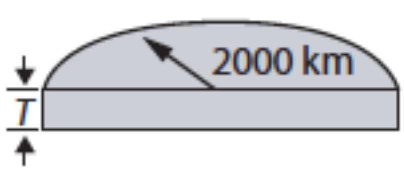
Separately compute the x and y components of each vector:

$$A_x = 6\cos 0 = 6$$
$$B_x = 3\cos 60^\circ = 1.5$$
$$C_x = 9\cos 135^\circ = -6.36$$
$$D_x = 6\cos 240^\circ = -3$$
$$Sum = -1.86$$

$$A_y = 6\sin 0 = 0$$
$$B_y = 3\sin 60^\circ = 2.6$$
$$C_y = 9\sin 135^\circ = 6.36$$
$$D_y = 6\sin 240^\circ = -5.2$$
$$Sum = 3.76$$

Problem 4

Antarctica is roughly semicircular, with a radius of 2000 km. In a particular year the new snow that falls on the continent has an average thickness of T = 5.0 cm (or 10cm, or 15cm) (see figure). How many cubic meters of snow fell in that year?



(1) 3.1×10^{11} (2) 6.3×10^{11} (3) 9.4×10^{11} (4) 3.1×10^{12} (5) 6.3×10^{12}

The volume is given by

$$V = \frac{1}{2}\pi R^2 T = \frac{\pi}{2}\left(2\times10^6 m\right)^2 (0.05m) = 3.1\times10^{11}\ \text{m}^3$$

Problem 5

A car, initially at rest, travels 40 m in time t = 4 s (or 5 s, or 6 s) along a straight line with constant acceleration. The acceleration of the car is:

(1) 5.0 m/s^2 (2) 3.2 m/s^2 (3) 2.2 m/s^2 (4) 1.1 m/s^2 (5) 6.1 m/s^2

$$d = \frac{1}{2}at^2$$
$$\Rightarrow a = \frac{2d}{t^2} = \frac{80}{\left(4^2, 5^2, 6^2\right)}$$

Problem 6

Two automobiles are a distance d = 200 km apart (or 250km, or 300 km) and traveling toward each other with one going at 60 km/h and the other at 40 km/h. In how much time (in h) do they meet?

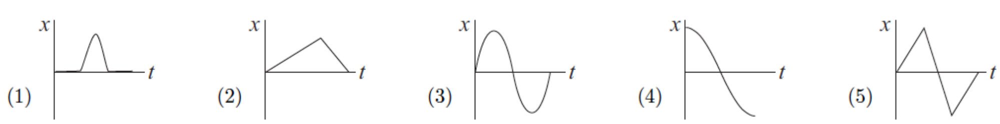
(1) 2.0 (2) 2.5 (3) 3.0 (4) 3.3 (5) 4.0

The closing speed between the vehicles is 60 + 40 = 100 km/h.
So the time to cover the distance is

$$t = \frac{d}{v_{\text{rel}}} = (2, 2.5, 3)\text{s}$$

Problem 7

A car accelerates from rest on a straight road. A short time later, the car decelerates to a stop and then returns to its original position in a similar manner, by speeding up and then slowing to a stop. Which of the following five coordinate versus time graphs best describes the motion?



Problem 8

One object is shot vertically upwards with an initial velocity of 100 m/s. Another object is shot vertically upwards with a velocity of 10 m/s. The first object climbs to a maximum height that is Q times the maximum height of the second object. Q is:

(1) 100 (2) 10 (3) 33.3 (4) 3.3 (5) 1000

Problem 9

A ball is thrown horizontally from the top of a cliff from a height h=23.0 m (or 32 m, 41 m) above the level plain below. It strikes the plain at an angle of 60° with respect to the horizontal. With what speed (in m/s) was the ball thrown?

(1) 12.3 (2) 14.5 (3) 16.4 (4) 10.1 (5) 7.9

Problem 10

A plane traveling north at 200 m/s turns and then travels south at 300 m/s (or 400 m/s, 500 m/s). The change in its velocity is:

(1) 500 m/s south (2) 600 m/s south (3) 700 m/s south (4) 100 m/s south
(5) 500 m/s north

$$\Delta v = v_f - v_i = 300 \text{ m/s} - (-200 \text{ m/s}) = 500 \text{ m/s south} \quad (\text{or } 600\text{m/s}, 700 \text{ m/s south})$$

Problem 11

Two objects are traveling around different circular orbits, each with a constant speed. They both have the same acceleration but object A is traveling twice as fast as object B. The radius for object A's orbit is how many times that of the radius for object B's?

(1) 4 (2) 2 (3) 1 (4) ½ (5) ¼

Centripetal acceleration is given by:

$$a_C = \frac{v^2}{r} = \frac{v_A^2}{r_A} = \frac{v_B^2}{r_B} = \frac{(2v_B)^2}{r_A}$$

$$\Rightarrow r_A = 4r_B$$

Problem 12

A ferry boat is sailing at 12 km/h 30° W of N with respect to a wide river that is flowing at 6.0 km/h E. As observed from the shore, the ferry boat is sailing:

(1) due N (2) 30° W of N (3) 45° E of N (4) 30° E of N (5) none of these

Problem 13

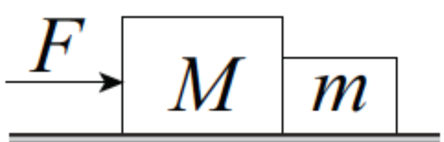
For a garden installation a series of flat stones are being stacked one atop the other. The first in contact with the ground weighs 50 N, the second 100 N, the third 150 N, and the fourth 200 N. The net force acting on the second stone (in N) is

(1) zero (2) 100 (3) 150 (4) 350 (5) 450

Since the stones are not moving, the net force must be zero!

Problem 14

Two blocks of different masses M and m, labeled as such in the figure, lie on a frictionless surface and are accelerated by the force labeled F which pushes on block M. The force acting on mass m is:



(1) mF/(m+M) (2) mF/M (3) mF/(m-M) (4) MF/(M+m) (5) MF/m

The acceleration of the combined system is:

$$a = \frac{F}{m+M}$$

Therefore the force on mass m is given by:

$$F_m = ma = \frac{mF}{m+M}$$

Problem 15

A 1200 kg elevator accelerates upwards at 3.00 m/s^2 (or 2.50, 2.00 m/s^2). The tension in the cable lifting the elevator (in N) has value closest to:

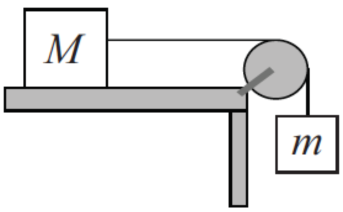
(1) 15,400 (2) 14,800 (3) 14,200 (4) 13,800 (5) 13,200

The net force acting on the elevator is:

$$F_{\text{net}} = T - Mg = Ma$$
$$\Rightarrow T = M(a+g)$$

Problem 16

A block of mass M=5.0 kg resting on a frictionless table is connected via a massless string, across a massless, frictionless pulley, to a hanging block of mass m=3.2 kg (or 4.2 kg, 5.2 kg). The system is let go to accelerate under Earth's gravity. The magnitude of that acceleration (in m/s²) is



(1) 3.8 (2) 4.5 (3) 5.0 (4) 3.2 (5) 6.1

Let's denote the acceleration of mass *M* to the right as *a*, and the acceleration of the mass m downward as the same *a*. We therefore have:

$T = Ma$ for the first mass, and
$mg - T = ma$ for the hanging mass.

Solving gives:

$$mg - Ma = ma$$
$$a = \frac{mg}{m+M}$$

Problem 17

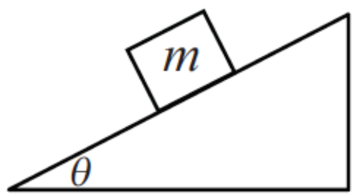
The speed of a 0.42-kg hockey puck, sliding across a level ice surface, decreases at the rate of 0.61 m/s² (or 0.72 m/s², 0.83 m/s²). The coefficient of kinetic friction between the puck and ice is:

(1) 0.062 (2) 0.074 (3) 0.085 (4) 0.091 (5) 0.051

$$f = \mu_k N = \mu_k mg = ma$$
$$\Rightarrow \mu_k = \frac{a}{g}$$

Problem 18

A block of unknown mass m is held on an incline for which θ =40°. The coefficient of kinetic friction μ_k =0.10 (or 0.20, 0.30) between the block and the incline. When the block is let go its acceleration down the incline (in m/s^2) is:



(1) 5.5 (2) 4.8 (3) 4.0 (4) 3.3 (5) mass of block needed to answer

Problem 19

While driving at constant speed around a traffic circle with a radius of 40 m you notice that the air freshener dangling from the rear view mirror is hanging at a steady angle of 20° (or 15°, 10°) with respect to the vertical. The speed of the car in m/s is:

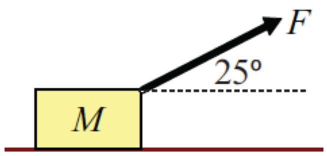
(1) 11.9 (2) 10.2 (3) 8.3 (4) 6.8 (5) 5.1

The tangent of the angle is given by the ratio of the horizontal centripetal acceleration to the vertical gravitational acceleration:

$$\tan\theta = \frac{v^2/r}{g}$$
$$\Rightarrow v = \sqrt{rg\tan\theta}$$

Problem 20

A crate of mass M = 40 kg is to be dragged across a floor by a rope pulled by a force F = 120 N (or 150 N, 180 N) acting at an angle of 25° from the horizontal. The coefficient of kinetic friction $\mu_k = 0.23$ between the crate and floor. The crate's acceleration (in m/s^2) is:



(1) 0.76 (2) 1.5 (3) 2.3 (4) 3.0 (5) 3.7

$$Ma = F\cos\theta - f = F\cos\theta - \mu_k\left(Mg - F\sin\theta\right)$$
$$a = \frac{F}{M}\cos\theta - \mu_k\left(g - \frac{F}{M}\sin\theta\right)$$