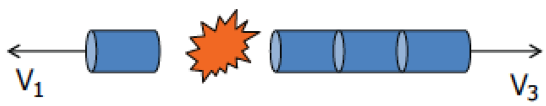
Exam 2 Solutions

Note that there are several variations of some problems, indicated by choices in parentheses.

Problem 1

Four identical soda cans initially at rest have a firecracker explode between them such that one can recoils against the other three which are still stuck together. With V_1 and V_3 the velocities of the single and 3 cans, respectively, what is the ratio of the magnitude of the speed of the single can to the magnitude of the speed of the other three (V_1/V_3)?



(1) 3 (2) 2 (3) 4 (4) 1/3 (5) 1/2

Use conservation of momentum, initial (where the cans are at rest) equals final:

$$0 = 3mV_3 - mV_1$$
$$\Rightarrow \frac{V_1}{V_3} = 3$$

Problem 2

A cylindrical bar aligned along the x-axis has a mass density that increases linearly along its length: $\lambda(x) = (4\ \text{kg/m}^2)\ x$. If the bar extends from x=0 to x=1.5 m (or 0.5m or 2 m), at what position along the x-axis (in m) is the center-of-mass of the bar?

(1) 1 (2) ½ (3) 1/3 (4) 4/3 (5) 3/2

To calculate the center of mass in x for a continuous mass distribution in 1D of length L use:

$$x_{\text{com}} = \frac{1}{M}\int_0^L x\lambda(x)\,dx$$

The total mass M is given by:

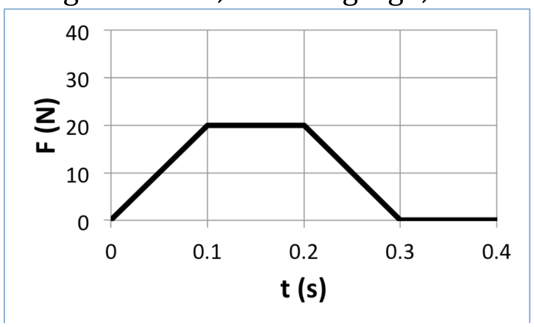
$$M = \int_0^L \lambda(x)\,dx = \int_0^L 4x\,dx = \left(2\ \text{kg/m}^2\right)L^2$$

$$x_{\text{com}} = \frac{1}{M}\int_0^L 4x^2\,dx = \frac{1}{2L^2}\frac{4}{3}L^3 = \frac{2L}{3}$$

So, 2/3 of the total length

Problem 3

Consider a ball of mass 0.5 kg that travels only in one dimension along the x axis. Its initial velocity at time t=0 is v=−5.0 m/s. A force is then applied to the ball in the x direction as a function of time as shown by the graph. What is the velocity of the ball along the x-axis, including sign, at a time of 0.4 s?



(1) 3 m/s (2) 8 m/s (3) -13 m/s (4) -1 m/s (5) 0 m/s

The impulse gives the change in momentum:

$$J = \Delta p = \int F(t)\,dt$$

The area under the curve therefore gives the momentum change:

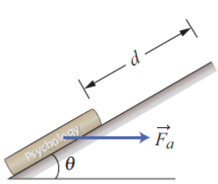
$$\int F(t)\,dt = \frac{1}{2}(20)(0.1) + (20)(0.2-0.1) + \frac{1}{2}(20)(0.3-0.2) = 4 \text{ kg m/s}$$

So

$$\Delta p = m\left(v_f - v_i\right) \Rightarrow v_f = \frac{J}{m} + v_i = \frac{4}{0.5} - 5 = 3 \text{ m/s}$$

Problem 4

In the figure, a horizontal force $\mathbf{F}_a$ of magnitude 20 N is applied to a 2.5 kg book as the book slides a distance d=1 m (or 4 m) up a frictionless ramp at angle θ=30°. What is the speed of the book at the end of the displacement if it starts initially at rest?



(1) 2.0 m/s (2) 4.0 m/s (3) 7.5 m/s (4) 16.0 m/s (5) 1.0 m/s

Find the net force along the incline acting on the book:

$$F_{net} = F_a \cos\theta - mg\cos\left(90^\circ - \theta\right) = 17.32 - 12.25 = 5.07\text{N}$$

The acceleration is therefore:

$$a = \frac{F_{net}}{m} = 2.0 \text{ m/s}$$

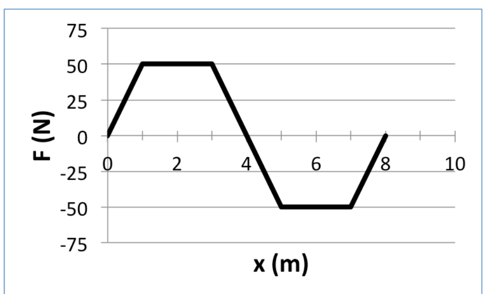
And the final velocity is given by:

$$v_f^2 = 2ad$$

$$\Rightarrow v_f = 2.0 \text{ m/s} \quad (\text{or } 4 \text{ m/s})$$

Problem 5

The figure gives the force acting on a 5.0 kg particle as it moves from rest along an x axis from x = 0 to x = 8.0 m. What is the particle's speed and direction (give a positive answer if the particle moves along the x axis in the positive direction and negative otherwise) of travel when it reaches x = 5 m (or 6 m)?



(1) +7.1 m/s (2) +5.5 m/s (3) -5.5 m/s (4) -4.5 m/s (5) 0 m/s

The work done by the force changes the kinetic energy of the particle:

$$W = \int F(x)dx = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

So the area under the graph up to the limit of the travel should be calculated.

$$W = \int_0^5 F(x)dx = \frac{1}{2}(50)(1) + (50)(3-1) + \frac{1}{2}(50)(4-3) - \frac{1}{2}(50)(5-4) = 125 \text{ J}$$

So the velocity is given by

$$v_f = \sqrt{\frac{2W}{m}} = 7.1 \text{ m/s}$$

Problem 6

A high jumper of mass 70 kg runs toward a high jump bar with a speed of 6.7 m/s. What would be the maximum bar height the center-of-mass of the jumper could clear if he/she were able to convert all of his/her initial kinetic energy into a vertical jump?

(1) 2.3 m (2) 4.6 m (3) 22.4 m (4) 0.4 m (5) 1.5 m

Use conservation of mechanical energy:

$$E_{mec} = K_i + U_i = K_f + U_f$$
$$\Rightarrow K_i = U_f$$
$$\Rightarrow \frac{1}{2}mv_i^2 = mgh$$
$$h = \frac{v_i^2}{2g} = 2.3 \text{ m}$$

Problem 7

A 0.25 kg mass is attached to the end of a relaxed, vertical hanging spring and the mass let go from rest. What is the maximum distance the spring stretches downward after the mass is released if the spring constant is k=50 N/m?

(1) 10 cm (2) 30 cm (3) 5 cm (4) 0 cm (5) 40 cm

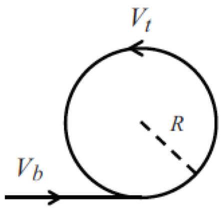
Apply conservation of mechanical energy, but there are two sources of potential energy: gravitational and elastic:

$$E_{mec} = K_i + U_i = K_f + U_f$$
$$E_{mec} = K_i + U_{i,grav} + U_{i,spring} = K_f + U_{f,grav} + U_{f,spring}$$
$$U_{i,grav} + U_{i,spring} = U_{f,grav} + U_{f,spring} \quad \text{(since the mass is at rest initially and finally)}$$
$$\Rightarrow 0 = -mgy + \frac{1}{2}ky^2$$
$$\Rightarrow y = \frac{2mg}{k} = \frac{2(0.25)(9.8)}{50} = 0.1\text{m}$$

Note that unless you do work on the system and take away some of the initial energy, the distance is NOT given by Hooke's law: $y = \frac{mg}{k} = 5 \text{ cm}$. Rather the spring stretches farther to 10cm before stopping, and in fact will oscillate.

Problem 8

A roller coaster is to go through a vertical loop as shown. What is the minimum speed at the bottom of the loop (V_b) necessary so that the cars do not fall off of the track at the top of the loop if the radius of the loop is R=10 m? Treat a roller coaster car as a point mass going through the loop.



(1) 22 m/s (2) 20 m/s (3) 10 m/s (4) 17 m/s (5) 500 m/s

Not only do we need enough initial kinetic energy to reach the top of the loop, we need enough centripetal acceleration for the car to stay on the track! Namely it should be at least as large as g:

$$a_{c,t} = \frac{V_t^2}{R} \geq g$$

Now use conservation of mechanical energy:

$$E_{mec} = K_i + U_i = K_f + U_f$$

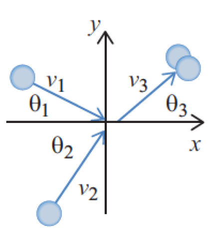
$$\Rightarrow \frac{1}{2}mV_b^2 = \frac{1}{2}mV_t^2 + mgy$$

$$\frac{1}{2}mV_b^2 = \frac{1}{2}m(gR) + mg(2R)$$

$$\Rightarrow V_b = \sqrt{5gR} = 22 \text{ m/s}$$

Problem 9

Two sticky equal mass air hockey pucks collide and stick together at the origin of the frictionless x-y plane shown in the figure. The shown angles are θ_1=30° and θ_2=60°. The initial speed of both pucks are equal: $v_1 = v_2 = 2$ m/s. Determine the outgoing angle θ_3 for the combined system of two pucks, measured counter-clockwise with respect to the x-axis.



(1) 15° (2) 60° (3) 0° (4) 75° (5) 30°

Since this is an inelastic collision, we can only use conservation of momentum, which we apply separately in x and y:

$$\hat{\mathbf{i}}: \quad mv_1\cos\theta_1 + mv_2\cos\theta_2 = 2mv_3\cos\theta_3$$
$$\hat{\mathbf{j}}: -mv_1\sin\theta_1 + mv_2\sin\theta_2 = 2mv_3\sin\theta_3$$

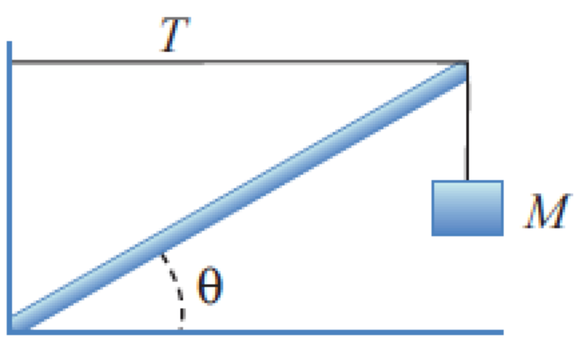
Take the ratio (and note that $v_1=v_2$)

$$\tan\theta_3 = \frac{-v_1\sin\theta_1 + v_2\sin\theta_2}{v_1\cos\theta_1 + v_2\cos\theta_2} = \frac{-\sin\theta_1 + \sin\theta_2}{\cos\theta_1 + \cos\theta_2} = \frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}+\frac{1}{2}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\Rightarrow \theta_3 = 15^\circ$$

Problem 10

The system depicted in the figure is in equilibrium. A block of mass M=300 kg hangs from the end of a massless strut which is fixed to the ground at the other end and makes an angle θ=30° (or 60°) with respect to the horizontal. Find the tension T in the massless, horizontal cable attaching the strut to the wall.



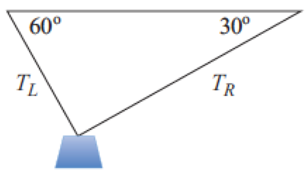
(1) 5100 N (2) 2900 N (3) 1700 N (4) 5900 N (5) 3400 N

Balance the torques from the mass and the tension *T* on the strut about an axis at the end fixed to the floor (L is the length of the strut, but it cancels):

$$LT\sin\theta - LMg\sin\left(90^\circ - \theta\right) = 0$$
$$T\sin\theta = Mg\cos\theta$$
$$\Rightarrow T = \frac{Mg}{\tan\theta} = 5100 \text{ N}$$

Problem 11

A mass is attached to, and hangs from, two ropes which are fixed at their other ends in the ceiling, as shown in the figure. The left rope makes an angle of 60° with respect to the horizontal, and the right rope 30°. What is the ratio of the tension in the left rope to that of the right (T_L / T_R)?



(1) √3 (2) 1/√3 (3) 1 (4) √3/2 (5) 2

Let's use force balance in the x and y directions:

$$\hat{\mathbf{i}}: \quad -T_L \cos 60^\circ + T_R \cos 30^\circ = 0$$
$$\hat{\mathbf{j}}: \quad T_L \sin 60^\circ + T_R \sin 30^\circ - Mg = 0$$

The first equation is enough for the ratio:

$$\frac{T_L}{T_R} = \frac{\cos 30^\circ}{\cos 60^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

Problem 12

A 5 kg object is suspended from the ceiling by a cylindrical copper wire with a diameter of 1 mm. If the wire initially has a length of 2 m, what is the increase in its length in millimeters (mm) after the object is attached if the Young's modulus of copper is 120×10^9 N/m^2?

(1) 1.0 (2) 5.2×10^{-4} (3) 0.25 (4) 8.2×10^{-7} (5) 1.2×10^{11}

Use the stress-strain relation for tensile stress:

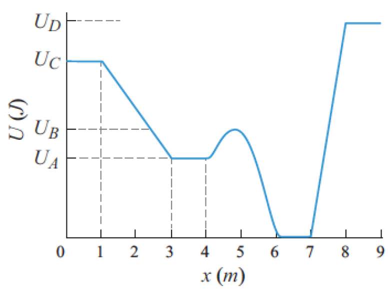
$$\frac{F}{A} = E\frac{\Delta L}{L}$$

So solving for the change in length:

$$\Delta L = \frac{FL}{EA} = \frac{MgL}{EA} = 1.0\text{mm}$$

Problem 13

The graph shows the potential energy of an object acted upon by a conservative force as a function of its position x. The potential values indicated are: U_A = 20 J, U_B=25 J, U_C=40 J, and U_D=50 J. At which of the listed positions in x is the force largest in the +x direction?



(1) 2 m (2) 7.5 m (3) 0.5 m (4) 5 m (5) 6.5 m

The relation between force and potential energy is:

$$F_x = -\frac{dU}{dx}$$

So we need the largest <u>negative</u> slope of the potential energy curve to have the largest force pointing in the +x direction. That occurs for x=2m.

Problem 14

The following objects (i) a solid sphere, (ii) a spherical shell and (iii) a cube all have the same mass. The objects all start to move, from rest, from the same elevation on an inclined surface (same incline), at the same time. The spheres each roll without slipping while the cube slides without friction. The order in which they get to the bottom of the incline, fastest first, is:

(1) (iii), (i), (ii) (2) (iii), (ii), (i) (3) (i), (ii), (iii) (4) (ii), (i), (iii) (5) (ii), (iii), (i)

Solution:
Since the objects fall from the same elevation they convert the same amount of potential energy to kinetic energy. For the cube which slides freely, without friction, all the potential energy becomes translational kinetic energy so it will be fastest arriving, at the bottom first. The rolling spheres will convert some of their initial potential energy into rotational kinetic energy, leaving less translational kinetic energy for each of them. That rotational kinetic energy is $K = \frac{1}{2} I\omega^2$. Since this depends linearly on the rotational inertial I, the one that has the greater rotational inertia will have converted more of its initial potential energy to rotational energy leaving it the less translational kinetic energy so it will be slowest and arrive at the bottom last. For the same mass the rotational inertia of a

spherical shell is greater than that of a solid sphere so it comes in last. The order, first to last, down the incline is thus cube, solid sphere, spherical shell or (iii), (i), (ii).

Problem 15

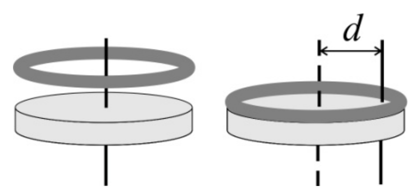
A wheel initially has an angular velocity of (30, 35, 40) rad/s but is slowing at a rate of 2 rad/s^2. By the time it stops, the number of revolutions it will have turned through is:
(1) 36 (2) 49 (3) 64 (4) 81 (5) 25

Solution:
Use the kinematic expression: $\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$ solved for $\theta - \theta_o$:

$$(\theta - \theta_o) = \frac{\omega^2 - \omega_o^2}{2\alpha}$$

where, $\omega = 0$, $\omega_o = (30, 35, 40)$, $\alpha = -2\,\frac{\text{rad}}{\text{s}^2}$. The results will be in radians and must be converted to revolutions by multiplying with the conversion factor 1 Rev/2π rad. Giving solutions (36, 49, 64) revolutions, respectively.

Problem 16



A thin hoop of radius R and mass M is welded to a thin uniform disk of the same mass and the same outer radius such that the hoop and disk are coaxial (dotted line). The rotational inertia for this object about an axis that is $d = (\frac{3}{4}, \frac{7}{8})R$ from the dotted axis, as shown in the sketch is:
(1) 2.63MR^2 (2) 3.03MR^2 (3) 2.06MR^2 (1) 2.27MR^2
(1) 1.63MR^2

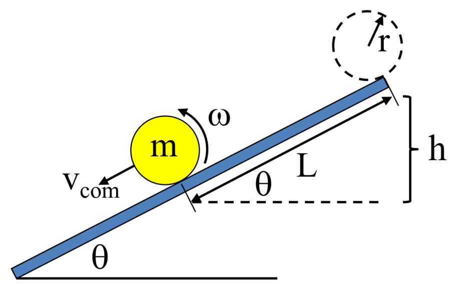
Solution:
From the Table the rotational inertia of a disk and a hoop about their relevant center of mass (com) axes are $I = \frac{1}{2}MR^2$, and $I = MR^2$, respectively. Since rotational inertias about the same axes are simply additive, the rotational inertia of the hoop and disk welded together is $I_{com} = \frac{3}{2}MR^2$. To find the rotational inertia about the new, parallel, axis a distance $h = (\frac{3}{4}, \frac{7}{8})R$ away from the first, we use the parallel theorem:
$I = I_{com} + M'h^2$ (where we must use $M' = 2M$ as the total mass in the 2nd term since the hoop and disk each contribute a mass M) so that:

$$I = \tfrac{3}{2}MR^2 + (2M)(\tfrac{3}{4}R, \tfrac{7}{8}R)^2 = [\tfrac{3}{2} + 2(\tfrac{3}{4}, \tfrac{7}{8})^2]MR^2 = [\tfrac{3}{2} + 2(\tfrac{9}{16}, \tfrac{49}{64})]MR^2 = (2.63, 3.03)MR^2$$

Problem 17

A solid uniform sphere of mass 0.4 kg and diameter 10 cm rolls from rest without slipping down a 30◦ incline. After rolling a distance (8.0, 10, 12) m measured along the incline, its rotational kinetic energy is:
(1) 4.48 J (2) 5.60 J (3) 6.72 J (4) 7.84 J (5) 8.96 J

Solution:
Mechanical energy is conserved such that the spheres change in kinetic energy after rolling a distance L along the incline will equal the change in its potential energy of having fallen through height $h = L\sin\theta$. Since the sphere starts from rest its initial kinetic energy is zero so we have that,



$K_{trans,f} + K_{rot,f} = \Delta U = mgL\sin\theta$, or

$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 = mgL\sin\theta$. Since the sphere rolls without slipping we can write that $v = r\omega$, also using the rotational inertia for a solid sphere $I = \frac{2}{5}mr^2$. Then

$$\tfrac{1}{2}m(r\omega_f)^2 + \tfrac{1}{2}(\tfrac{2}{5}mr^2)\omega_f^2 = mgL\sin\theta$$

$$r^2\omega_f^2 + \tfrac{2}{5}r^2\omega_f^2 = 2gL\sin\theta$$

$$(1+\tfrac{2}{5})r^2\omega_f^2 = 2gL\sin\theta$$

$$\tfrac{7}{5}r^2\omega_f^2 = 2gL\sin\theta$$

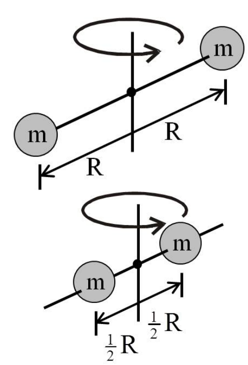
$\omega_f^2 = \dfrac{5}{7}\dfrac{2gL\sin\theta}{r^2}$. The question asks for the rotational kinetic energy which is

$K_{rot,f} = \frac{1}{2}I\omega_f^2 = \frac{1}{2}\frac{2}{5}mr^2\omega_f^2 = \dfrac{1}{2}\dfrac{2}{5}mr^2\dfrac{5}{7}\dfrac{2gL\sin\theta}{r^2} = \dfrac{2}{7}mgL\sin\theta$, or since the angle is 30° and sin(30°) = 0.5 we have,

$$K_{rot,f} = \frac{1}{7}mgL = \frac{1}{7}(0.4\text{kg})(9.8\text{m}\tfrac{\text{m}}{\text{s}^2})(8\text{ m}, 10\text{ m}, 12\text{ m}) = 4.48\text{J},\ 5.60\text{J},\ 6.72\text{J}$$

Problem 18

A dumbbell rotor consists of a massless rigid rod free to pivot around a vertical axis through the middle of the rod. Two small, equal masses, are attached to the rod at a distance R from the rotation axis. The rotor rotates with an angular velocity of (1.0, 2.0, 3.0) rad/s. An internal mechanism pulls the masses in symmetrically until they are each at (½)R from the axis. The angular velocity of the rotor after this move (in rad/s) is:
(1) 4 (2) 8 (3) 12 (4) 1/4 (5) 1/8



Solution:
Since no external torques act on the system angular momentum is conserved and requires that $L_f = L_i$ or $I_f\omega_f = I_i\omega_i$ so that, $\omega_f = \frac{I_i}{I_f}\omega_i$. Since the masses are small we treat them as point particles for which $I = \sum_i m_i r_i^2$ here we have initially $I_i = 2mR^2$ and finally $I_f = 2m\left(\frac{1}{2}R\right)^2 = \frac{1}{2}mR^2$ so that, $\omega_f = \frac{2mR^2}{\frac{1}{2}mR^2}\omega_i$, or $\omega_f = 4\omega_i$ then

$\omega_f = 4(1\frac{\text{rad}}{\text{s}}, 2\frac{\text{rad}}{\text{s}}, 3\frac{\text{rad}}{\text{s}}) = 4\frac{\text{rad}}{\text{s}}, 8\frac{\text{rad}}{\text{s}}, 12\frac{\text{rad}}{\text{s}}$

Problem 19

A spherical shell is free to rotate about its vertical axis having rotational inertia I = 40 N·m^2 about that axis. A motor causes a torque that gives the shell a constant angular acceleration of (1.2, 1.4, 1.6) rad/s^2. In rotating the shell through 15 revolutions the motor does an amount of work (in J) equal to,

(1) 4.5×10^3 (2) 5.3×10^3 (1) 6.0×10^3 (1) 0.84×10^3 (1) 2.1×10^3

Solution:
The work done by the motor will be, $W = \int_{\theta_1}^{\theta_2} \tau d\theta$, Since torque $\tau = I\alpha$, both of which are here constant, we can write, $W = \int_{\theta_1}^{\theta_2} I\alpha d\theta = I\alpha\int_{\theta_1}^{\theta_2} d\theta = I\alpha(\theta_2 - \theta_1) = I\alpha\Delta\theta$. The 15 revolutions must be converted to radians so that $\Delta\theta = (15\text{ rev})(2\pi\frac{\text{rad}}{\text{rev}}) = 94.25\text{ rad}$ then,

$W = I\alpha\Delta\theta = (40\text{ N}\cdot\text{m}^2)(1.2, 1.4, 1.6)\frac{\text{rad}}{\text{s}^2}(94.25) = (4.5, 5.3, 6.0)\times 10^3\text{ J}$

Problem 20

Tarzan, having a mass of 88.0 kg, swings on a 10.0 m long vine such that just before getting to the bottom of the swing his angular velocity is 1.40 rad/s. At the bottom of the swing he picks up Cheetah (mass of 20.0 kg) who was waiting there at rest. Just after the pick-up their angular velocity in (rad/s) was (hint: treat both Tarzan & Cheetah as point masses):
(1) 1.14 (2) 1.30 (3) 1.47 (4) 0.98 (5) 0.81

Solution: At the bottom of the swing the force of gravity is perpendicular to the direction of the motion (constrained to an arc by the vine) so it generates no torque there. With no external torques the angular momentum is conserved and requires that: $L_f = L_i$ or $I_f\omega_f = I_i\omega_i$ so that, $\omega_f = \frac{I_i}{I_f}\omega_i$. With Tarzan and Cheetah point particles $I_i = m_{Tarzan} r_{vine}^2$ and $I_f = m_{Tarzan} r_{vine}^2 + m_{Cheetah} r_{vine}^2 = (m_{Tarzan} + m_{Cheetah}) r_{vine}^2$. Then

$$\omega_f = \frac{m_{Tarzan} r_{vine}^2}{(m_{Tarzan} + m_{Cheetah}) r_{vine}^2}\omega_i = \frac{m_{Tarzan}}{m_{Tarzan} + m_{Cheetah}}\omega_i$$

$$\omega_f = \frac{88\text{ kg}}{88\text{ kg} + 20\text{ kg}}(1.2, 1.4, 1.6)\,\tfrac{\text{rad}}{\text{s}} = (1.14, 1.30, 1.47)\,\tfrac{\text{rad}}{\text{s}}$$