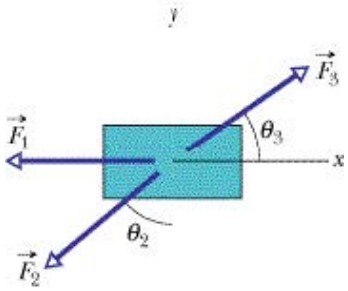
Exam 2 Solutions

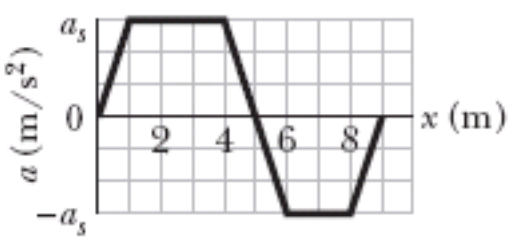
1. The figure shows an overhead view of three horizontal forces acting on a cargo canister that was initially stationary but that now moves across a frictionless floor. The force magnitudes are F_1 = 3.0 N, F_2 = 10.0 N, and F_3 = 10.0 N, and the indicated angles are θ_2 = 30 degrees and θ_3 = 30 degrees What is the net work done on the canister by the three forces during the first 4.0 m of displacement?



(1) 12 J (2) 92 J (3) 57 J (4) 28 J (5) 0 J

The x and y components of F_2 and F_3 cancel out, leaving only F_1 as the net force. So the work done is $F_1 d$ = 12 J

2. The figure gives the acceleration of a 4 kg object as an applied force moves it from rest along an x-axis from x=0 to x=9 m. The scale of the figure's vertical axis is set by as=10.0 m/s^2. What is the object's speed when it reaches x=5 m?



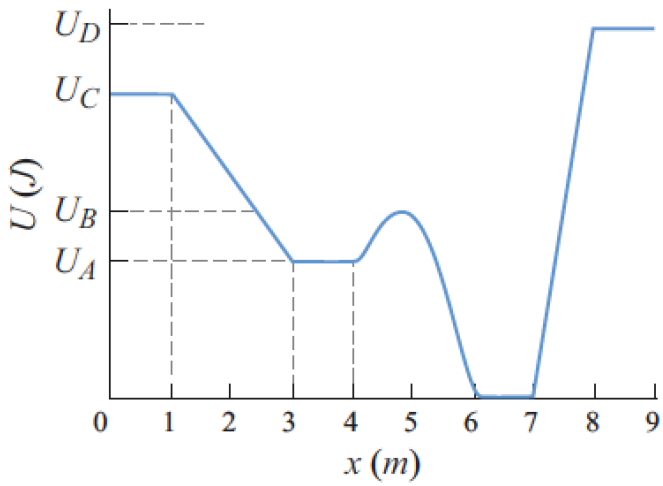
(1) 8.9 m/s (2) 0 m/s (3) 4.5 m/s (4) 10 m/s (5) 18 m/s

Multiplying the acceleration a by the mass is equal to the force applied. So the integral (area) of the force with distance is the work done, which changes the kinetic energy:

$$W = \Delta K = m\int a\,dx = 4ma_s = 160 \text{ J}$$

$$v = \sqrt{2K/m} = \sqrt{8a_s} = 8.9 \text{ J}$$

3. The graph shows the potential energy of an object acted upon by a conservative force as a function of its position x. The potential values indicated are: U_A = 20 J, U_B=25 J, U_C=40 J, and U_D=50 J. At which of the listed positions in x is the force pointing in the +x (or –x) direction with the largest magnitude?



(1) 7.5 m (2) 2 m (3) 0.5 m (4) 5 m (5) 6.5 m

$$F_x = -\frac{dU}{dx}$$

The steepest negative slope gives the largest force in +x (at x=2m), and the steepest positive slope the largest force in the –x direction (at x=7.5 m)

4. A piece of cheese with a mass of 0.5 kg is placed on a vertical spring of negligible mass and a spring constant = 1600 N/m that is compressed by a distance of 10 cm. When the spring is released, how high does the cheese rise from the release position? (The cheese and the spring are not attached.)

(1) 1.6 m (2) 33 m (3) 0.1 m (4) 16 m (5) 0.4 m

Use conservation of mechanical energy before the spring launch and at the maximum height:

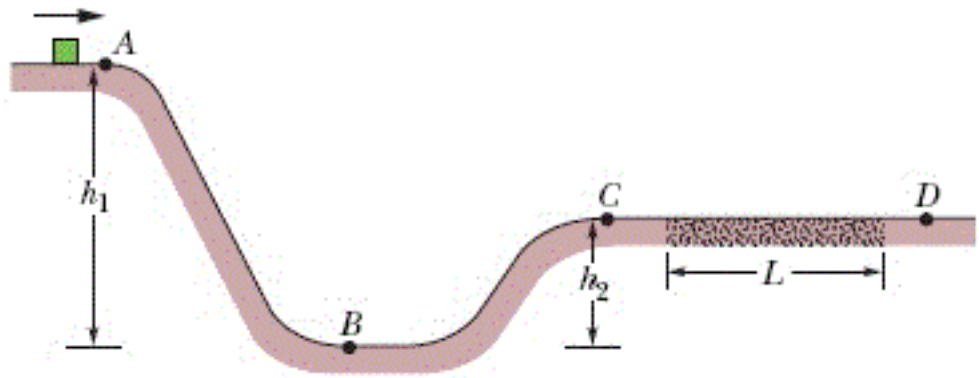
$$E_{mec} = K + U_s + U_g$$

$$\text{initially}: E_{mec} = \frac{1}{2}kx^2$$

$$\text{finally}: E_{mec} = mgh$$

$$h = \frac{kx^2}{2mg} = 1.6 \text{ m}$$

5. In the figure here, a small block is sent through point A with a speed of 4.4 m/s. Its path is without friction until it reaches the section of length L, where the coefficient of kinetic friction is 0.5. The indicated heights are h_1 = 5.0 m and h_2 = 3.0 m. What is the minimum length of L such that the block comes to rest?



(1) 6.0 m (2) 4.0 m (3) 2.0 m (4) 3.0 m (5) 12.0 m

First find the speed at point C using conservation of mechanical energy

$$K_A + U_A = K_C + U_C$$
$$\frac{1}{2}mv_A^2 + mgh_1 = mgh_2 + \frac{1}{2}mv_C^2$$
$$v = \sqrt{v_A^2 + 2g(h_1 - h_2)}$$

The deceleration is caused by friction. We can solve for the length:

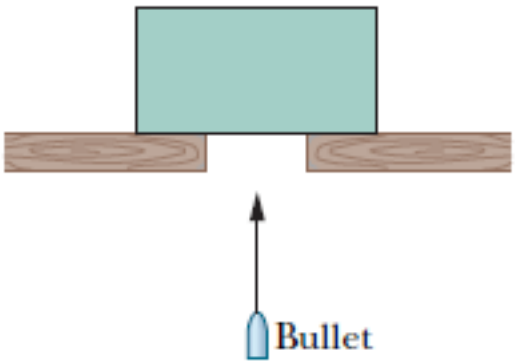
$$0 = v^2 + 2aL \Rightarrow v^2 = -2(-\mu_K g)L$$
$$L = \frac{v_A^2 + 2g(h_1 - h_2)}{2\mu_K g} = \frac{v_A^2}{2\mu_K g} + \frac{(h_1 - h_2)}{\mu_K} = 6.0 \text{ m}$$

6. A 0.5 kg ball moving horizontally at 10.0 m/s strikes a vertical wall and rebounds in the opposite direction at 10.0 m/s. If the collision took place in 0.1 s, what was the magnitude of the average force on the wall?

(1) 100 N (2) 200 N (3) 0 N (4) 50 N (5) 10 N

$$J = \Delta p = F_{av}\Delta t$$
$$\frac{m(v_2 - v_1)}{\Delta t} = \frac{(0.5kg)(10 - (-10)m/s)}{0.1 \text{ s}} = 100 \text{ N}$$

7. In the figure shown, a 10 g bullet moving directly upward at 1000 m/s strikes and becomes lodged within the center of a 10 kg block initially at rest. To what maximum height does the block and embedded bullet then rise above its initial position?



(1) 0.05 m (2) 0.1 m (3) 0.5 m (4) 1 m (5) 50 m

Use momentum conservation to solve for the initial vertical velocity of the block and bullet, which is an inelastic collision:

$$m_b v_b = \left(M_{blk} + m_b\right)V$$

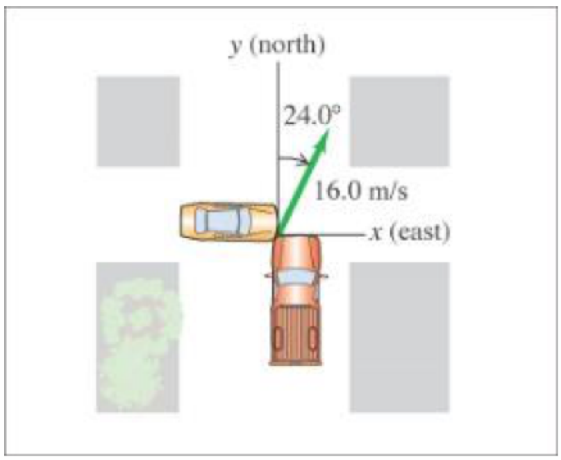
$$V = \frac{m_b v_b}{\left(M_{blk} + m_b\right)} = 1 \text{ m/s}$$

Then use conservation of mechanical energy to find the maximum height

$$\frac{1}{2} M_{blk} V^2 = M_{blk} g h$$

$$h = \frac{V^2}{2g} = 0.05 \text{ m}$$

8. At the intersection of 13th Street and University Avenue, a subcompact car with mass 900 kg traveling east on University collides with a pickup truck with mass 2700 kg that is traveling north on 13th St. and ran a red light . The two vehicles stick together as a result of the collision and, after the collision, the wreckage is sliding at 16.0 m/s in the direction 24° east of north as shown in the figure. The collision occurs during a heavy rainstorm; you can ignore friction forces between the vehicles and the wet road. Calculate the speed of the car (or truck) before the collision.



(1) 20 m/s (2) 26 m/s (3) 58 m/s (4) 9 m/s (5) 16 m/s

$$m_{\text{car}} v_{\text{car}} = \left(m_{\text{car}} + m_{\text{truck}}\right) v_f \sin\theta$$

$$v_{\text{car}} = \frac{\left(m_{\text{car}} + m_{\text{truck}}\right)}{m_{\text{car}}} v_f \sin 24^\circ = 26 \text{ m/s}$$

$$m_{\text{truck}} v_{\text{truck}} = \left(m_{\text{car}} + m_{\text{truck}}\right) v_f \cos\theta$$

$$v_{\text{truck}} = \frac{\left(m_{\text{car}} + m_{\text{truck}}\right)}{m_{\text{truck}}} v_f \cos\theta = 20 \text{ m/s}$$

9. A bomb at rest explodes into two fragments, one of mass m_1 and one of mass m_2, that travel in opposite directions. What is the ratio of the kinetic energy of the fragment of mass m_1 to the kinetic energy of the fragment of mass m_2 (or the inverse for some exams)?

(1) m_2/m_1 (2) m_1/m_2 (3) m_2^2/m_1^2 (4) m_1^2/m_2^2 (5) 1

Use conservation of momentum, then calculate the ratio of kinetic energies.

$$m_1 v_1 + m_2 v_2 = 0$$

$$v_2 = -\frac{m_1}{m_2} v_1$$

$$\frac{K_1}{K_2} = \frac{\frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_2 v_2^2} = \frac{m_1}{m_2} \frac{v_1^2}{\frac{m_1^2}{m_2^2} v_1^2} = \frac{m_2}{m_1}$$

10. An electric fan is turned off, and its angular velocity decreases uniformly from 500 revolutions per minute to 200 revolutions per minute in a time interval of length 3 s. How many more seconds are required for the fan to come to rest?

(1) 2 (2) 3 (3) 5 (4) 1 (5) 4

Find the angular acceleration:

$$\omega_2 = \omega_1 + \alpha t \Rightarrow \alpha = \frac{(\omega_2 - \omega_1)}{t} = \frac{(200 - 500 \text{ rev/min})(1 \text{ min/ } 60 \text{ s})}{3s} = 1.67 \text{ rev/s}^2$$

Then the total time to rest:

$$t_3 = \frac{(\omega_3 - \omega_1)}{\alpha} = \frac{(0 - 500 \text{ rev/min})(1 \text{ min/ } 60 \text{ s})}{1.67 \text{ rev/s}^2} = 5 \text{ s}$$

so the additional time was 2 s

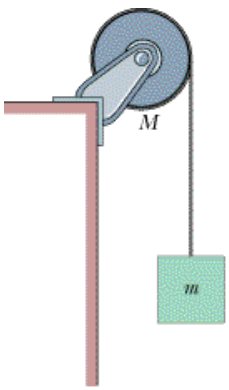
11. A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one time it is rotating at 10 rev/s; 50 revolutions later, its angular speed is 25 rev/s. Calculate the angular acceleration in rev/s^2.

(1) 5.25 (2) 1.0 (3) 6.25 (4) 15.0 (5) 0.15

$$\omega_2^2 - \omega_1^2 = 2\alpha\Delta\theta$$

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\Delta\theta} = \frac{(25 \text{ rev/s})^2 - (10 \text{ rev/s})^2}{2(50 \text{ rev})} = 5.25 \text{ rev/s}^2$$

12. A light string is wrapped around the edge of a metal disk and a 0.5 kg block is suspended from the free end. The radius of the disk is 0.20 m and its rotational inertia is 0.016 kg m^2. If the block is released from rest 2.0 m above the floor, what is the speed just before it strikes the floor?



(1) 4.7 m/s (2) 50 m/s (3) 6.3 m/s (4) 22 m/s (5) 7.0 m/s

Energy arguments can be used:

$$K_{disks} + K_B + U_B = \text{const.}$$

$$m_B gh = \frac{1}{2} I_{disks}\omega^2 + \frac{1}{2} m_B v_B^2 \quad \text{where } v_B = R_1\omega$$

$$m_B gh = \frac{1}{2} m_B v_B^2 \left(1 + \frac{I_{disks}}{m_B R_1^2}\right)$$

$$v_B = \sqrt{\frac{2gh}{\left(1 + \frac{I_{disks}}{m_B R_1^2}\right)}} = 4.7 \text{ m/s}$$

13. A solid cylinder of mass 10 kg rolls up an incline at an angle of 30°. At the bottom of the incline the center of mass of the cylinder has a translational speed of 5.0 m/s. How far does the cylinder travel up along the path of the incline?

(1) 3.8 m (2) 2.6 m (3) 1.3 m (4) 1.9 m (5) 5.1 m

Energy arguments can be used:

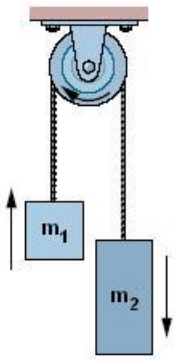
$$K + U = \text{const.}$$

$$\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = mgh$$

$$\frac{1}{2} mv^2 \left(1 + \frac{I}{mr^2}\right) = mg\ell \sin\theta$$

$$\ell = \frac{1}{2g\sin\theta} v^2 \left(1 + \frac{\frac{1}{2}mr^2}{mr^2}\right) = \frac{3v^2}{4g(0.5)} = 3.8 \text{ m}$$

14. The figure illustrates an Atwood's machine, where two masses m_1 and m_2 are suspended by a cord over a pulley. The larger mass is m_2. In terms of the acceleration a of the masses, which is in the direction of the arrows shown, what is the tension of the cord connected to mass m_2 (or m_1)?



(1) $m_2(g-a)$ (2) $m_1(g+a)$ (3) $m_2(g+a)$ (4) $m_1(g-a)$ (5) m_2 a

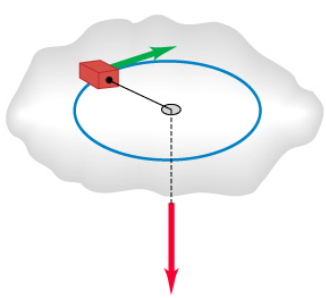
Forces acting on m_2:

$$m_2 g - T_2 = m_2 a$$
$$T_2 = m_2 (g - a)$$

Forces acting on m_1:

$$T_1 - m_1 g = m_1 a$$
$$T_1 = m_1 (g + a)$$

15. A small block on a frictionless, horizontal surface has a mass of 0.1 kg. It is attached to a massless cord passing through a hole in the surface. The block is originally revolving at a distance of 0.3 m from the hole with an angular speed of ω_1. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.15 m. Model the block as a particle. What is the new angular speed of the block?

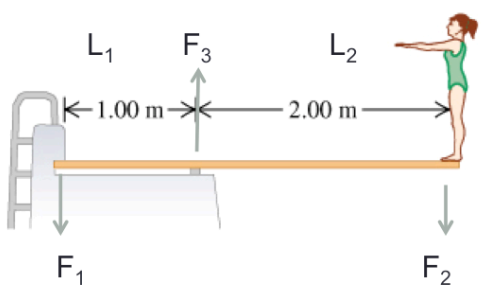


(1) $4\omega_1$ (2) $2\omega_1$ (3) ω_1 (4) $\omega_1/2$ (5) $\omega_1 / 4$

Conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$
$$mr_1^2\omega_1 = m\left(r_1^2/4\right)\omega_2$$
$$\Rightarrow \omega_2 = 4\omega_1$$

16. A diving board of length 3.0 m is supported at a point 1.0 m from the end, and a diver weighing 600 N stands at the free end. The diving board is taken to be massless. Find the magnitude and direction of the force that the support point makes on the board.



(1) 1800 N, up
(2) 1200 N, up
(3) 1200 N, down
(4) 1800 N, down
(5) 600 N, down

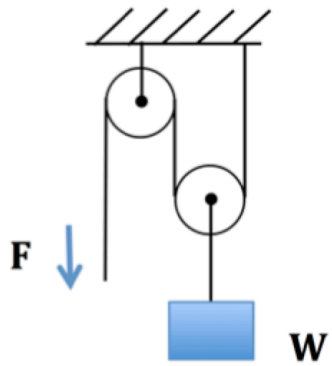
Torque balance across support point:

$$F_1L_1 - F_2L_2 = 0$$
$$F_1 = \frac{L_2}{L_1}F_2$$

Force balance in vertical direction:

$$F_3 - F_1 - F_2 = 0$$
$$F_3 = F_1 + F_2 = F_2\left(1 + \frac{L_2}{L_1}\right) = 3F_1 = 1800\text{ N, up}$$

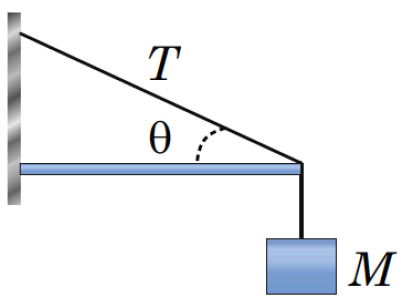
17. For the pulley system shown, determine the force F required to balance a weight W=500 N.



(1) 250 N (2) 500 N (3) 1000 N (4) 167 N (5) 125 N

The tension on both sides of the cable routed through the pulley lifting W is W/2 to balance forces. The left pulley routes the tension of one side of that cable to F, so F=W/2 = 250 N

18. The system depicted in the figure is in equilibrium. A block of mass M=500 kg hangs from the end of a massless, horizontal strut which is fixed to the wall at the other end. A massless cable is attached to the wall and to the end of the strut, and makes an angle of \theta=30^\circ with respect to the horizontal as shown. What is the horizontal component of the force that the wall makes on the horizontal strut?



(1) 8490 N (2) 9800 N (3) 5660 N (4) 4900 N (5) 2450 N

Torque about the strut's attachment point at the wall:

$$TL\sin\theta - MgL = 0$$

$$T = \frac{Mg}{\sin\theta}$$

Force in x for the strut:

$$F_x - T\cos\theta = 0$$

$$F_x = T\cos\theta = \frac{Mg}{\tan\theta} = 8490 \text{ N}$$

19. After a fall, a 50 kg rock climber finds herself dangling from the end of a rope that had been 20 m long and 5 mm in radius but has stretched by 2 cm. What is the Young's modulus for the rope?

(1) 6.2 x 10^9 N/m^2
(2) 0.001 N/m^2
(3) 6.2 x 10^6 N/m^2
(4) 1.6 x 10^{-10} N/m^2
(5) 1.6 x 10^7 N/m^2

Tensile stress and strain relationship:

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$Y = \frac{F/A}{\Delta L/L} = \frac{\dfrac{(50\text{ kg})g}{\pi(0.005\text{ m})^2}}{\dfrac{(0.02\text{ m})}{(20\text{ m})}} = 6.2\times10^9\ \text{N/m}^2$$

20. A solid cylinder, a solid sphere, and a hoop all have the same mass and radius. They all roll without slipping the same distance down an inclined plane starting from rest. In what order, from fastest to slowest, do they reach the bottom of the incline?

(1) sphere, cylinder, hoop
(2) hoop, cylinder, sphere
(3) sphere, hoop and cylinder tie
(4) cylinder, sphere, hoop
(5) sphere, hoop, cylinder

The acceleration is fastest for the object with the smallest rotational inertia, and slowest for the object with greatest rotational inertia. So the order is given by

Sphere: I=2/5 Mr^2
Cylinder: I=1/2 Mr^2
Hoop: I= Mr^2

If this was not known, one can solve for the acceleration down the inclined plane as done in lecture and get

$$a = \frac{g\sin\theta}{1+\dfrac{I}{Mr^2}}$$