SPRING 2018 SOLUTIONS

1. 1 yd^2 = $(36\ in)^2$ = $(36 \times 2.54\ cm)^2$ = 8,361 $cm^2 \cong 8.4 \times 10^3\ cm^2$

2. Average velocity is displacement/time = $\Delta \mathbf{r}/\Delta t$
   The displacement can be seen as the hypotenuse of a right angle triangle of sides 500 and 1200, and is therefore 1300 m. The time taken for the first leg is 50s, and for the seconds is 60 seconds, so the magnitude of the average velocity is:
   1300/110 = 11.8 m/s

3. To be moving at right angles to both axes tells is that the velocity has equal x and y components.
   Differentiating (each component separately), gives:
   $\mathbf{v} = (20t)\mathbf{i} + 15t^2\mathbf{j}$
   Setting the two components to be the same, that gives
   $20t = 15t^2$ and therefore t = 1.33 s

4. If it is fired at an angle above the horizontal, it must have a positive component of velocity at t=0. That component will then decrease uniformly as a function of time (that is to say that it will be a straight line sloping down on a v_y versus t graph). Thus AE is the answer.

5. As explained in class (and one of the HW problems), the actual path followed is the vector sum of the boat with respect to the water, and the path of the water velocity with respect to the bank. Here we know the water velocity (1 m/s, right to left), the actual direction (straight up the page), and they make two sides of a right-angle triangle, with the velocity with respect to

the water being magnitude 2m/s which makes the hypotenuse. Thus, sin(theta) = 0.5, and theta = 30 degrees.

6. If there are two forces acting on it, but it goes at uniform velocity, the two forces must (by Newton's First Law) add to zero. The only way two vectors can add to zero is if they are equal in magnitude and opposite in direction. Note that mass and the value of the velocity do not enter the discussion.

7. Let's first find the acceleration of the blocks. It must by F/(m+M) by Newton's 2nd Law. Now find the force of the big block on the small block. As we know the acceleration of the small block (as above), and its mass, therefore we know that the horizontal force on it is mF/(m+M). Newton's third Law tells us that the force of the small block on the big block is that same magnitude.

8. We find the position (let's call the ground y=0, have up as the positive y direction in meters), and we see that for the first ball
   $y_1 = 15t + 0.5at^2$
   For the second ball
   $y_2 = 15(t-1) + 0.5a(t-1)^2$
   Equate the two and we get (noting that $a = -9.8\ m/s^2$)
   $15t - 4.9t^2 = 15t - 15 - 4.9t^2 + 9.8t - 4.9$
   $15 + 4.9 = 9.8t$
   $t = 19.9/9.8 = 2.031$ s
   Now we can plug into either of equations for y, and find that
   y = 10.3 meters.
   This is very similar to an example done in class.

9. The dot product is the product of the magnitudes multiplied by the cosine of the angle between them. We know the cosine of 30 degrees is sqrt(3)/2, so that gives sqrt(3)L^2/2

10. Once way, as we found in class, is to find how it takes to drop, and this depends only on the vertical motion. It is given by $\Delta y=0.5gt^2$ where Δy is 100 meters (measured down, so there are no minus signs). That means that t = sqrt(200/9.8) = 4.51 seconds. Its speed in the –y direction is 9.8*4.51 = 44.2 m/s. The horizontal component of the velocity is a fixed 50 m/s, and so the total velocity is the vector sum of these, and is given by $v^2 = 44.2^2 + 50^2$
v = 66.7 m/s

11. There is no information on the velocity. The laws of physics are equally valid in all inertial rest frames. It DOES matter if it is accelerating. If it is accelerating up then the scale reads a higher number (because the normal force does not balance is greater than the weight). So "a" is up, and we know nothing about v.

12. The question says that the block is stationary. Unfortunately, with the numbers as they are, the block would not be stationary. In other words, it was a MISTAKE. We therefore grade everyone to be correct.

Let us change the question so that it DOES make sense. Let's change the coefficient of static friction to be 0.9.

Now, we know that the component of the gravitational force on it along the direction of the slope must be mgsin(37^0) which is 2*9.8*0.6 = 11.8 N. Therefore the frictional force must be equal and opposite to this force and so is also 11.8 N.

Note that the coefficient of static friction does not enter this PROVIDED IT IS BIG ENOUGH TO NOT ALLOW THE BLOCK TO SLIP.

13. The key here is to remember that the force that makes it go into a circle (the centripetal force) is due to static friction. Therefore the static friction coefficient must be big enough. $mv^2/R \leq \mu_s mg$, and the mass cancels. This makes the minimum value of $\mu_s = v^2/Rg$ = 30*30/(150*9.8) = 0.61

14. There are many ways of doing this, but one of them is using work. At the top, it is stationary. At the bottom, it is stationary. So the net work done must be zero. Therefore, the work done by gravity is equal in magnitude to the work done by the spring – where the latter is negative. So, if the distance dropped is y, $Mgy = 0.5ky^2$ and this gives y = 0.098 meters = 9.8 cm.
This was the question that caused the most problems. Many people may have found the case when the forces (gravity and spring forces) cancel. That would give the final equilibrium point, but that is not what the question asks for, which is the maximum extension before "bouncing" back.

15. First, as so often, we find the time. The time taken to reach the ground is independent of anything happening in the horizontal plane, and so is given by:
$0.5gt^2 = 2$ meters
So therefore t = sqrt(4/g) = 0.638 seconds.

After release (at which point there is no acceleration in the horizontal plane) it travels 2 meters. Therefore its velocity in the horizontal plane is 2/.638 = 3.13 m/s
This velocity is the same magnitude as it had when it was going around in a circle. Therefore the centripetal acceleration was $v^2/R = 3.13^2/2 = 4.9$ m/s^2

16. This figure appears frequently in exams, but the question varies. This one is very similar to a homework question. Here we ask about what is the maximum speed that it can go without losing contact. First, let us assume that it is in contact. It is then going in a circle with speed v and so its centripetal acceleration is v^2/R . Therefore it must have an acceleration down of mv^2/R. The only force that can give this downward acceleration is gravity. Therefore the limiting case is when mg = mv^2/R and so v = sqrt(gR) = 24.2 m/s

17. We know the block's acceleration, Therefore we can work out what F_{EXT} is. We can do that by looking at the horizontal component of the force and finding that $F_{EXT}\cos(37^0)$ = Ma, and so F_{EXT} = 2*2/0.8 = 5 N. Therefore the vertical component of F_{EXT} is 5*sin(37^0) = 3 N.
The total vertical forces on the block are then given by
F_N + 3 –Mg = 0 (as there is no vertical motion).
So F_N = 2*9.8 – 3 = 16.6 N
This the upwards force of the ground on the block, and so is equal to the downwards force of the mass on the ground (Newton's Third Law).

18. As the object starts at rest, its velocity is given by v=at, and its acceleration is $a = 10/5 = 2\ m/s^2$

So, at t =2 s, the velocity is v = at = 4m/s.

The force is 10N and the velocity is 4m/s in the direction of the force, and so therefore the power is 10x4 = 40W

19. The force is not a constant, so there is no alternative to doing an integral. What we need is the integral of F with respect to distance from 0 to 3 meters.

$$Work = \int_0^3 (6x + 1)dx$$

(The force and the direction of motion are the same, so there is no need for a dot product to be shown)

So, Work = $(3.3^2 + 3) - (0) = 30$ J

20. Here we use the usual kinematic equations for distance, noting that the initial velocity is 3 m/s (up) but the acceleration before dropping is of no concern, accelerations only happen if there are forces and the only force after dropping is gravity. So, having y as positive up:

$\Delta y = v_0 t + 0.5at^2 = 3*3 - 0.5*9.8*3^2$

$= 9 - 44.1 \cong -35$ meters (i.e. 35 meters dropped).