Physics 2048 - Spring 2018 - Final Exam Solution

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Problem 1: First set up a vector equation representing the relative motion. B: Boat, G: Ground, W: Water.

$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$.

For example, $\vec{v}_{BG}$, stands for "the velocity of the boat with respect to the ground."

Since the boat make headway directly north (up):
$v_{BG,x} = 0 = v_{BW,x} + v_{WG,x} \Longrightarrow v_{BW,x} = -v_{WG,x} = -(-1\,\mathrm{m/s}) = 1\,\mathrm{m/s}$.

Since the boat's heading is at 45°, we know that $v_{BW,x} = v_{BW,y}$.

$v_{WG,y} = 0 \Longrightarrow v_{BG,y} = v_{BW,y} = 1\,\mathrm{m/s}$.

$t = 5\,\mathrm{m}/v_{BG,y} = 1\,\mathrm{s}$.

Problem 2: These two forces form an action-reaction pair. So the force of the earth on the moon is equal in magnitude and opposite in direction to the force of the moon on the earth. Therefore $F_1/F_2 = 1$, where F_1 and F_2 are the magnitude of the forces.

Problem 3: First, solve for the acceleration of the entire assembly:
$F = ma \Longrightarrow a = F/m = 20\,\mathrm{N}/(8\,\mathrm{kg} = 2\,\mathrm{kg}) = 2\,\mathrm{m/s^2}$.

The only force acting on the 2 kg mass is the force applied by the other mass. Therefore, we can just solve for the net force on the 2 kg mass: $F_{net} = ma = 2\,\mathrm{kg} \cdot 2\,\mathrm{m/s^2} = 4\,\mathrm{N}$.

Problem 4: Since the block is stationary, we know that the net force on the block is zero. Therefore, the component of the gravitational force parallel to the plane is equal to the frictional force.

$f_s = F_{G//} = mg\sin(20°) = 2\,\mathrm{kg} \cdot 9.8\,\mathrm{m/s^2} \cdot \sin(20°) = 6.7\,\mathrm{N}$.

Problem 5: Since the block is stationary, we know that the net force on the block is zero. Therefore, the component of the gravitational force parallel to the plane is equal to the frictional force.

$f_s = F_{G//} = mg\sin(20°) = 3\,\mathrm{kg} \cdot 9.8\,\mathrm{m/s^2} \cdot \sin(20°) = 10.1\,\mathrm{N}$.

Problem 6: Conservation of mechanical energy: $\Delta K + \Delta U = 0$.

Since the mass is stationary at the beginning and also stationary at the point where it turns from going down to going up: $\Delta K = 0$.

$\Longrightarrow \Delta U = 0 \Longrightarrow U_f = U_i$.

In the initial state, all of the energy is gravitational potential energy ($U_i = mgh$). In the final state (at the turning point) all of the energy is spring potential energy ($U_f = kh^2/2$).

$$\Longrightarrow mgh = \frac{1}{2}kh^2 \Longrightarrow h = \frac{2mg}{k} = \frac{25\,\mathrm{kg} \cdot 9.8\,\mathrm{m/s^2}}{500\,\mathrm{N/m}} = 0.196\,\mathrm{m} = 19.6\,\mathrm{cm}$$

Problem 7: We want to find a minimum in the potential energy.

$\Delta U + \Delta K = 0$, so where the potential passes through a minimum the kinetic energy (and hence velocity) will be maximum.

$dU/dx = 2x - 4 \Longrightarrow dU/dx = 0 = 2x - 4 \Longrightarrow 2x = 4 \Longrightarrow x = 2$.

We might want to double check that this corresponds to a minimum in the potential energy by taking a second derivative: $d^2U/dx^2 = 2$. Positive curvature means that $x = 2$ corresponds to a minimum, not a maximum.

Problem 8: The acceleration is proportional to the sum of the forces, which consist of the force due to gravity and the buoyant force:
$\sum F = ma = -mg + F_b$.

The buoyant force $F_b = M_f g$, where $M_f = \rho V$ is the mass of the displaced fluid: $\Longrightarrow ma = -mg + \rho V g$

$$\Longrightarrow V = \frac{m(a+g)}{\rho g} = \frac{5\,\mathrm{kg} \cdot (-4\,\mathrm{m/s^2} + 9.8\,\mathrm{m/s^2})}{1000\,\mathrm{kg/m^3} \cdot 9.8\,\mathrm{m/s^2}} = 0.003\,\mathrm{m^3}$$

Problem 9: Pascal's law tells us that a pressure change in one part of the container is transmitted without loss to every portion of the fluid and to the walls of the container.

$P = F/A \Longrightarrow F = PA$

$$\frac{F_1}{F_2} = \frac{PA_1}{PA_2} = \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{2}{1}\right)^2 = 4$$

$$\Longrightarrow \frac{F_1}{F_2} = 4 \Longrightarrow F_1 = 4 \cdot F_2 = 4 \cdot 16\,\mathrm{N} = 64\,\mathrm{N}$$

Problem 10: Pascal's law tells us that a pressure change in one part of the container is transmitted without loss to every portion of the fluid and to the walls of the container.

$P = F/A \Longrightarrow F = PA$

$$\frac{F_1}{F_2} = \frac{PA_1}{PA_2} = \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{2}{1}\right)^2 = 4$$

$$\Longrightarrow \frac{F_1}{F_2} = 4 \Longrightarrow F_1 = 4 \cdot F_2 = 4 \cdot 8\,\text{N} = 32\,\text{N}$$

Problem 11: Use conservation of momentum: $P_i = P_f$.

$P_i = mv_i = 0.02\,\text{kg} \cdot 100\,\text{m/s} = 2\,\text{kg}\,\text{m}\,\text{s}^{-1}$

$P_f = Mv_f = (m_{nail} + m_{block})v_f = (0.02\,\text{kg} + 1\,\text{kg})v_f = 1.02\,\text{kg} \cdot v_f$

$$P_i = P_f \Longrightarrow 2\,\text{kg}\,\text{m}\,\text{s}^{-1} = 1.02\,\text{kg} \cdot v_f \Longrightarrow v_f = \frac{2\,\text{kg}\,\text{m}\,\text{s}^{-1}}{1.02\,\text{kg}} = 2.0\,\text{m/s}$$

Problem 12: Use conservation of angular momentum: $L_i = L_f$

$L = I\omega$

For a solid disk: $I_{disk} = M_{disk}R^2/2 = 200\,\text{kg} \cdot (2\,\text{m})^2/2 = 400\,\text{kgm}^2$.

The child is modeled as a point: $I_{child} = M_{child}r^2$

Initially the child is at the center so $r = 0$ and hence $L_i = I_{disk}\omega_i$

After the child has crawled to the outside: $r = R$
so $I_{child} = 25\,\text{kg}(2\,\text{m})^2 = 100\,\text{kgm}^2$ and $L_f = (I_{disk} + I_{child})\omega_f$

$L_i = L_f \Longrightarrow I_{disk}\omega_i = (I_{disk} + I_{child})\omega_f$

$$\Longrightarrow \omega_f = \frac{I_{disk}}{I_{disk} + I_{child}} \cdot \omega_i = \frac{400\,\text{kgm}^2}{400\,\text{kgm}^2 + 100\,\text{kgm}^2} \cdot 1\,\text{rev/s} = 0.8\,\text{rev/s}$$

Problem 13: We need to apply Newton's Second Law ($\sum F = ma$) to each mass and the circular motion equivalent ($\sum \tau = I\alpha$) to the disk. We'll have to be careful to use consistent signs in order to arrive at the correct answer.

Masses m_1 and m_2 have the same magnitude of acceleration, a, but are accelerating in opposite directions. We will define up as positive and down as negative. Hence, $a_1 = a = -a_2$

Each mass is subject to a gravitational force and tension, T. Since the pulley is not massless, the tensions in the two sides of the machine could be different.

(1) For mass 1: $\sum F = m_1a_1 = m_1a = T_1 - m_1g$

(2) For mass 2: $\sum F = m_2a_2 = -m_2a = T_2 - m_2g$

(3) For the pulley: $\sum \tau = T_2r - T_1r = (T_2 - T_1)r = I\alpha$ and $\alpha = a/r$
$\Longrightarrow (T_2 - T_1)r = Ia/r$

Note, it looks like we have defined the signs correctly, since if T_2 is finite and $T_1 = 0$ we get a positive acceleration, consistent with the way that we have defined the sign of acceleration. From here, we just need to solve this system of equations for a.

We could start by finding the quantity $T_2 - T_1$ which we can obtain by subtracting (1) from (2): $T_2 - m_2 g - T_1 + m_1 g = -m_2 a - m_1 a$

$\Longrightarrow T_2 - T_1 = -m_2 a - m_1 a + m_2 g - m_1 g = g(m_2 - m_1) - a(m_2 + m_1)$

$\Longrightarrow g(m_2 - m_1)r - a(m_2 + m_1)r = Ia/r$

$\Longrightarrow a[I/r + r(m_1 + rm_2)] = gr(m_2 - m_1)$

$$\Longrightarrow a = \frac{gr(m_2 - m_1)}{I/r + r(m_1 + m_2)} = \frac{9.8 \cdot 0.25 \cdot 1}{0.5/0.25 + 0.25 \cdot 3}\,\mathrm{m/s^2} = 0.9\,\mathrm{m/s^2}$$

Problem 14: Since the system is in equilibrium, we know that $\sum \tau = 0$. We will consider the pivot point as the point where the strut is fixed to the wall. There are two different torques, the one generated by the hanging mass τ_{mass} and the one generated by the tension in the cable τ_{cable}.

Torque: $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$

$\tau_{mass} = -mgL/2$, where, L is the length of the strut. This torque alone would cause the system to rotate clockwise, hence the negative sign.

$\tau_{cable} = TL \sin\theta$

$\sum \tau = \tau_{mass} + \tau_{cable} = -mgL/2 + TL \sin\theta$

$$\Longrightarrow mgL/2 = TL\sin\theta \Longrightarrow T = \frac{mg}{2\sin\theta} = \frac{500\,\mathrm{kg} \cdot 9.8\,\mathrm{m/s^2}}{2\sin 45^\circ} = 3465\,\mathrm{N}$$

Problem 15: Use phasors to solve this problem. The phasor is a vector, where the length of the vector is the amplitude of the wave and the angle of the vector is the phase of the wave. To find the resulting wave, we simply add together to two vectors (phasors).

So, we need to add together two vectors that are perpendicular to each other, one with a length of 3.5 mm and one with a length of 5 mm.

$\Longrightarrow s_{m'} = \sqrt{(3.5\,\mathrm{mm})^2 + (5\,\mathrm{mm})^2} = 6.1\,\mathrm{mm}$

Problem 16: This is a physical pendulum, so we can use the following expression for the period:

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

Here I is the moment of inertia about the pivot, so we need to use the parallel axis theorem with the distance to the center of mass (h) being the radius of the sphere:

$$I = I_{cm} + Mh^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$$\implies I = 2\pi\sqrt{\frac{\frac{7}{5}MR^2}{MgR}} = 2\pi\sqrt{\frac{7R}{5g}} = 2\pi\sqrt{\frac{7 \cdot 0.1\,\mathrm{m}}{5 \cdot 9.8\,\mathrm{m/s^2}}} = 0.7501\,\mathrm{s}$$

$$f = 1/T = 1/0.7501\,\mathrm{s} = 1.3\,\mathrm{Hz}$$

Problem 17: The velocity of A is twice that of B: $v_A = 2v_B$.

The gravitational force is given by: $F_G = \dfrac{GmM}{r^2}$.

The centripetal force is given by: $F_c = \dfrac{mv^2}{r}$.

Following the hint, for a circular orbit, the gravitational force is equal to the centripetal force:

$$F_G = F_c \implies \frac{GmM}{r^2} = \frac{mv^2}{r} \implies \frac{GM}{r^2} = v^2 \implies r = \frac{GM}{v^2}$$

$$\implies \frac{r_A}{r_B} = \frac{v_B^2}{v_A^2} = \frac{1}{4}$$

Problem 18: The frequency of the third harmonic on string A is the same as the frequency of the first harmonic on string B:

$$f_{A,3} = f_{B,1}$$

Allowed resonant frequencies on a string: $f = \dfrac{nv}{2L}; n = 1, 2, 3...$

The velocity v is given by $v = \sqrt{\tau/\mu}$.

$$\implies f = \frac{n\sqrt{\tau/\mu}}{2L}$$

$$f_{A,3} = f_{B,1} \implies \frac{3\sqrt{\tau/\mu_A}}{2L_A} = \frac{1\sqrt{\tau/\mu_B}}{2L_B} \implies \sqrt{\frac{\mu_A}{\mu_B}} = \frac{3L_B}{L_A} = \frac{3 \cdot 40\,\mathrm{cm}}{60\,\mathrm{cm}}$$

$$\implies \frac{\mu_A}{\mu_B} = 4$$

Problem 19: The amplitude, A, is given by: $A(t) = x_m e^{\frac{-bt}{2m}}$

The problem states that $A(0) = 3\,\mathrm{mm}$ so $x_m = 3\,\mathrm{mm}$.

The problem also states that $A(10\,\mathrm{s}) = 1.5\,\mathrm{mm}$.

$A(10\,\mathrm{s}) = (3\,\mathrm{mm})e^{\frac{-b10\,\mathrm{s}}{2m}} = 1.5\,\mathrm{mm} \implies e^{\frac{-b\cdot 10\,\mathrm{s}}{2m}} = 0.5 \implies \dfrac{-b\cdot 10\,\mathrm{s}}{2m} = \ln 0.5$

$\implies \dfrac{b}{2m} = 0.069\,315\,\mathrm{s}^{-1}$

Now that we've solved for $b/(2m)$, we can simply substitute that value into our expression for the amplitude to find the amplitude at any time:

$A(25\,\mathrm{s}) = (3\,\mathrm{mm})e^{-25\,\mathrm{s}\cdot 0.069\,315\,\mathrm{s}^{-1}} = 0.53\,\mathrm{mm}$

Problem 20: The velocity of the source (the police car) $v_s = 0$. The detector (the motorcycle driver) is moving in the same direction as the sound waves so, according the equation sheet, we use a minus sign:

$$f_{obs} = f_s \frac{v_{sound} - v_D}{v_{sound}} = 800\,\mathrm{Hz} \cdot \frac{760\,\mathrm{mph} - 120\,\mathrm{mph}}{760\,\mathrm{mph}} = 674\,\mathrm{Hz}$$

Problem 21: The velocity of the source (the police car) $v_s = 0$. The detector (the motorcycle driver) is moving in the same direction as the sound waves so, according the equation sheet, we use a minus sign:

$$f_{obs} = f_s \frac{v_{sound} - v_D}{v_{sound}} = 900\,\mathrm{Hz} \cdot \frac{760\,\mathrm{mph} - 120\,\mathrm{mph}}{760\,\mathrm{mph}} = 758\,\mathrm{Hz}$$

Problem 22: The allowed resonant modes for a tube that is open at both ends is given by:

$f = \dfrac{nv}{2L}, n = 1, 2, 3...$, where v is the speed of sound and L is the length of the tube. The $n = 1$ value corresponds to the fundamental frequency.

The beat frequency occurs at the difference of the two original frequencies: $f_{beat} = f_1 - f_2$.

$$\implies f_{beat} = \frac{nv}{2L_1} - \frac{nv}{2L_2} = \frac{nv}{2}\left(\frac{1}{L_1} - \frac{1}{L_2}\right)$$

$$\implies f_{beat} = \frac{340\,\mathrm{m/s}}{2}\left(\frac{1}{0.199\,\mathrm{m}} - \frac{1}{0.200\,\mathrm{m}}\right) = 4.3\,\mathrm{Hz}$$

Problem 23: The allowed resonant modes for a tube that is open at both ends is given by:

$f = \dfrac{nv}{2L}, n = 1, 2, 3...$, where v is the speed of sound and L is the length of the tube. The $n = 1$ value corresponds to the fundamental frequency.

The beat frequency occurs at the difference of the two original frequencies: $f_{beat} = f_1 - f_2$.

$$\implies f_{beat} = \frac{nv}{2L_1} - \frac{nv}{2L_2} = \frac{nv}{2}\left(\frac{1}{L_1} - \frac{1}{L_2}\right)$$

$$\implies f_{beat} = \frac{340\,\text{m/s}}{2}\left(\frac{1}{0.299\,\text{m}} - \frac{1}{0.300\,\text{m}}\right) = 1.9\,\text{Hz}$$

Problem 24: The shell theorem tells us that the force due to a solid sphere or spherical shell on a mass located outside of the sphere is the same as if all of the mass were concentrated at the center point. Thus, we can solve the problem as if $4M$ were all concentrated at $x = 0$ and $2M$ were all concentrated at $x = 2L$. The radii of the spheres (or the cavity in the left sphere) does not effect the solution.

The gravitational force is given by: $F_G = \dfrac{GmM}{r^2}$

For the mass on the left, the relevant mass is $4M$ at a distance L from mass m. For the mass on the right, the relevant mass is $2M$ at a distance L from mass m.

$$F_{net} = \sum F = -\frac{Gm \cdot 4M}{L^2} + \frac{Gm \cdot 2M}{L^2} = -\frac{2GmM}{L^2}$$