Physics 2048 - Spring 2019 - Exam 1 Solution

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Problem 1: Vectors $\vec{a}$ and $\vec{b}$ both lie in the xy-plane. Therefore, $\vec{a} \times \vec{b}$ must be perpendicular to the xy-plane (*i.e.*, parallel to the z-axis). We can tell that $\vec{a} \times \vec{b}$ must be in the positive z direction by using the right hand rule.

The angle between $\hat{k}$ and $\hat{\imath} + 2\hat{k}$ is $\arctan\frac{1}{2} = \boxed{26.6\,\text{deg}}$.

Problem 2: Vectors $\vec{a}$ and $\vec{b}$ both lie in the xy-plane. Therefore, $\vec{a} \times \vec{b}$ must be perpendicular to the xy-plane (*i.e.*, parallel to the z-axis). We can tell that $\vec{a} \times \vec{b}$ must be in the positive z direction by using the right hand rule.

The angle between $\hat{k}$ and $2\hat{\imath} + \hat{k}$ is $\arctan 2 = \boxed{63.4\,\text{deg}}$.

Problem 3: Write each step of travel as a vector:

$$\text{8.7 km, east:} \quad \Delta r_1 = (8.7\hat{\imath})\text{ km} \tag{1}$$

$$\text{4.1 km, 35° north of east:} \quad \Delta r_2 = (4.1\cos 35\hat{\imath} + 4.1\sin 35\hat{\jmath})\text{km} \tag{2}$$

$$\text{7.2 km, 25° south of west:} \quad \Delta r_3 = (7.2\cos 205\hat{\imath} + 7.2\sin 205\hat{\jmath})\text{ km} \tag{3}$$

The resultant displacement is just the sum of these displacements:

$$\Delta r = \Delta r_1 + \Delta r_2 + \Delta r_3 \tag{4}$$

$$= [(8.7 + 3.359 - 6.525)\hat{\imath} + (0 + 2.352 - 3.043)\hat{\jmath}]\text{ km} \tag{5}$$

$$= [(5.534)\hat{\imath} + (-0.691)\hat{\jmath}]\text{ km} \tag{6}$$

Now we calculate the angle from the components:

$$\theta = \arctan\frac{-0.691}{5.534} = \boxed{-7.1°} \tag{7}$$

Since both the x- component is positive and the y-component negative, we know that the angle is in the fourth quadrant, *i.e.*, "south of east."

Problem 4: Write each step of travel as a vector:

$$\text{8.7 km, east:} \quad \Delta r_1 = (8.7\hat{\imath})\text{ km} \tag{8}$$

$$\text{4.1 km, 35° north of west:} \quad \Delta r_2 = (4.1\cos 145\hat{\imath} + 4.1\sin 145\hat{\jmath})\text{km} \tag{9}$$

$$\text{7.2 km, 25° south of west:} \quad \Delta r_3 = (7.2\cos 205\hat{\imath} + 7.2\sin 205\hat{\jmath})\text{ km} \tag{10}$$

The resultant displacement is just the sum of these displacements:

$$\Delta r = \Delta r_1 + \Delta r_2 + \Delta r_3 \tag{11}$$
$$= [(8.7 - 3.359 - 6.525)\hat{\imath} + (0 + 2.352 - 3.043)\hat{\jmath}] \text{ km} \tag{12}$$
$$= [(-1.184)\hat{\imath} + (-0.691)\hat{\jmath}] \text{ km} \tag{13}$$

Now we calculate the angle from the components:

$$\theta = \arctan \frac{-0.691}{-1.184} = \boxed{30.3°} \tag{14}$$

Since both the x- and y-components are negative, we know that the angle is in the third quadrant, *i.e.*, "south of west."

Problem 5:

$$\vec{C} = p\vec{A} + q\vec{B} \tag{15}$$
$$\Longrightarrow -\hat{\imath} + 3\hat{\jmath} = p(3\hat{\imath} + \hat{\jmath}) + q(\hat{\imath} + 2\hat{\jmath}) \tag{16}$$
$$= (3p + q)\hat{\imath} + (p + 2q)\hat{\jmath} \tag{17}$$
$$\Longrightarrow 3p + q = -1 \quad \text{and} \quad p + 2q = 3 \tag{18}$$

We have two equations and two unknowns, so we can solve for p and q.

$$\Longrightarrow \boxed{p = -1 \quad \text{and} \quad q = 2} \tag{19}$$

Problem 6:

$$320 \cdot \frac{\text{furlongs}}{\text{fortnight}} \cdot \frac{\text{fortnight}}{2\,\text{weeks}} \cdot \frac{\text{week}}{7\,\text{days}} \cdot \frac{\text{day}}{24\,\text{hours}} \cdot \frac{0.125\,\text{miles}}{\text{furlong}} \tag{20}$$
$$= 320 \cdot \frac{0.125\,\text{miles}}{2 \cdot 7 \cdot 24\,\text{hour}} \tag{21}$$
$$= \boxed{0.12\,\text{mph}} \tag{22}$$

Problem 7: If the box is moving at constant velocity, then the acceleration must be zero. Newton's First Law tells us that if the acceleration is zero, the net force must be zero. For the two forces shown, the net force is $8\,\text{N} - 5\,\text{N} = 3\,\text{N}$ (to the right). So, the friction force must cancel out the $3\,\text{N}$ force to the right. Hence the friction force is:

$$\Longrightarrow \boxed{3\,\text{N}, \quad \text{to the left}} \tag{23}$$

Problem 8: First, we solve for the y-component of the initial velocity:

$$v_{y,0} = v \sin 63^\circ = 120\,\mathrm{m/s} \sin 63^\circ = 106.921\,\mathrm{m/s}. \tag{24}$$

Now, using the kinematic expression for constant acceleration:

$$y = y_0 + v_{y,0}t + \frac{1}{2}at^2. \tag{25}$$

Since y and y_0 are both zero and $a = -g$:

$$0 = 0 + v_{y,0}t - \frac{1}{2}gt^2 \tag{26}$$

$$\implies v_{y,0} = \frac{1}{2}gt \tag{27}$$

$$\implies t = \frac{2v_{y,0}}{g} = \frac{2 \cdot 106.921\,\mathrm{m/s}}{9.8\,\mathrm{m/s^2}} = \boxed{22\,\mathrm{s}} \tag{28}$$

Problem 9: We know that the horizontal component of the the velocity is constant. We need to find the vertical component of the velocity:

$$v_y^2 = v_{y,0}^2 + 2a\Delta y = -2g\Delta y = -2 \cdot 9.8\,\mathrm{m/s^2} \cdot 10\,\mathrm{m} \tag{29}$$

$$\implies v_y = 14\,\mathrm{m/s}. \tag{30}$$

Now we can find the angle:

$$\tan\theta = v_y/v_x \tag{31}$$

$$\implies v_x = v_y/\tan\theta = \frac{-14\,\mathrm{m/s}}{\tan(180^\circ - 39^\circ)} = \boxed{17\,\mathrm{m/s}} \tag{32}$$

Problem 10: For uniform circular motion, we must have a centripetal force:

$$F_c = mv^2/r. \tag{33}$$

We need to write the velocity in terms of the period of the motion (t) and the radius:

$$T = 2\pi r/v \implies v = 2\pi r/t \tag{34}$$

$$\implies F_c = m(2\pi r/t)^2/r = \frac{4\pi^2 mr}{t^2}. \tag{35}$$

The net force on the mass is the centripetal force, which is equal to the sum of the tension and the gravitational force. At the top of the circle, the gravitational force, the tension, and the centripetal force all point downwards:

$$F_c = F_g + T \implies \frac{4\pi^2 mr}{t^2} = mg + T \tag{36}$$

$$\implies T = \frac{4\pi^2 mr}{t^2} - mg = \frac{4\pi^2 \cdot 0.2\,\text{kg} \cdot 0.6\,\text{m}}{(0.8\,\text{s})^2} = \boxed{5.4\,\text{N}} \tag{37}$$

Problem 11: This is a relative motion problem. First, we need to define our notation:

$$\text{“T”: Tarzana} \tag{38}$$

$$\text{“W”: Water} \tag{39}$$

$$\text{“G”: Ground} \tag{40}$$

$$\tag{41}$$

The equation that describes the relative motion is:

$$\vec{v}_{TG} = \vec{v}_{TW} + \vec{v}_{WG}. \tag{42}$$

Next we write out each velocity vector in terms of the x- and y-components and then solve for $v_{TG,x}$ (which is unknown).

$$\vec{v}_{WG} = (0.3\,\text{m/s})\hat{j} \tag{43}$$

$$\vec{v}_{TG} = v_{TG,x}\hat{i} \tag{44}$$

$$\vec{v}_{TW} = (0.5\,\text{m/s} \cdot \cos\theta)\hat{i} + (0.5\,\text{m/s} \cdot \sin\theta)\hat{j}. \tag{45}$$

From Equation 42, we know that:

$$v_{TW,y} + v_{WG,y} = 0 \quad \text{and} \quad v_{TG,x} = v_{TW,x} \tag{46}$$

$$\implies 0.5\,\text{m/s} \cdot \sin\theta + 0.3\,\text{m/s} = 0 \quad \text{and} \quad v_{TG,x} = 0.5\,\text{m/s} \cdot \cos\theta \tag{47}$$

The line above has two equation and two unknowns, so we can solve for $v_{TG,x} = 0.4\,\text{m/s}$. Finally we know that the time is given by the distance divided by the speed:

$$t = \frac{120\,\text{m}}{0.4\,\text{m/s}} = \boxed{300\,\text{s}} \tag{48}$$

Problem 12: Work is the given by the integral:

$$W = \int F\,dx = 3\,\text{N/m}^3 \int_{1.8\,\text{m}}^{2.0\,\text{m}} x^3 dx \tag{49}$$

$$= 3\,\text{N/m}^3 x^4/4|_{1.8\,\text{m}}^{2.0\,\text{m}} = \boxed{4.1\,\text{J}}. \tag{50}$$

Problem 13: Newton's second law:

$$\vec{F}_{net} = ma = 0 \Longrightarrow kx - mg = 0 \Longrightarrow kx = mg \tag{51}$$

$$\Longrightarrow m = \frac{kx}{g} = \frac{100\,\mathrm{N/m} \cdot 0.12\,\mathrm{m}}{9.8\,\mathrm{m/s^2}} = \boxed{1.2\,\mathrm{kg}}. \tag{52}$$

Problem 14: First, solve for the acceleration of the entire assembly:

$$F = ma \Longrightarrow a = \frac{F}{3M}. \tag{53}$$

Now, use Newton's second law for mass C:

$$F_{net} = ma \Longrightarrow F - T = M \cdot \frac{F}{3M} = \frac{F}{3} \tag{54}$$

$$\Longrightarrow T = F - \frac{F}{3} = \boxed{\frac{2F}{3}}. \tag{55}$$

Problem 15: The mass of the car is not relevant here:

$$v^2 = v_0^2 + 2a\Delta x \Longrightarrow 0 = (34\,\mathrm{m/s})^2 + 2 \cdot a \cdot 90\,\mathrm{m} \tag{56}$$

$$\Longrightarrow a = \frac{-(34\,\mathrm{m/s})^2}{2 \cdot 90\,\mathrm{m}} = \boxed{6.4\,\mathrm{m/s^2}}. \tag{57}$$

Problem 16: The acceleration of mass A is zero, so the net force must be zero. We will write Newton's second law for the components of force that are parallel to the incline:

$$F_{net} = 0 = T - mg\sin\theta - f_s. \tag{58}$$

Since the mass B is hanging by the string and we know the tension in the string has the same magnitude everywhere, we know that:

$$T = 11\,\mathrm{N} \tag{59}$$

$$\Longrightarrow 11\,\mathrm{N} - 22\,\mathrm{N}\sin\theta - f_s = 0 \tag{60}$$

$$\Longrightarrow f_s = 11\,\mathrm{N} - 22\,\mathrm{N}\sin 33^\circ = -0.98\,\mathrm{N}. \tag{61}$$

$$\Longrightarrow \boxed{|f_s| = 0.98\,\mathrm{N}} \tag{62}$$

Problem 17: The maximum static friction force is:

$$f_{s,max} = \mu_s F_N = 0.6 \cdot 31\,\mathrm{N} = 18.6\,\mathrm{N}. \tag{63}$$

The gravitational force has a magnitude of:

$$1.5\,\text{kg} \cdot 9.8\,\text{m/s}^2 = 14.7\,\text{N}. \tag{64}$$

Since the gravitational force is smaller in magnitude than the maximum static frictional force, the object will not move. Hence:

$$a = 0 \Longrightarrow ma = \boxed{F_{net} = 0} \tag{65}$$

Problem 18: The centripetal acceleration is always directed towards the center of rotation. From $x = 1$, $y = 0$ to the origin is in the:

$$\boxed{-x \text{ direction}} \tag{66}$$

Problem 19: Velocity is the time derivative of position:

$$v = \frac{dx}{dt}, \tag{67}$$

which is the slope on an x vs t plot. There are only two positions where the slope is zero:

$$\boxed{\text{B and D.}} \tag{68}$$

Problem 20: From the definition of acceleration:

$$a = \frac{dv}{dt} \Longrightarrow adt = dv \tag{69}$$

$$\Longrightarrow \int adt = \Delta v. \tag{70}$$

Setting $\Delta v = 0$ and making the substitution $a = F/m$:

$$\int \frac{F}{m} dt = 0 \Longrightarrow \int F dt = 0. \tag{71}$$

The integral represents the area under the curve on an F vs t plot. So the velocity will return to zero only when the area under the curve is zero, which happens at only one time:

$$\boxed{\text{4 minutes}} \tag{72}$$

Problem 21: From the definition of acceleration:

$$\vec{a} = d\vec{v}/dt = (2 \cdot 10 \cdot t)\hat{\imath} - (3 \cdot 4 \cdot t^2)\hat{\jmath} \quad (73)$$

$$= (40\hat{\imath} - 48\hat{\jmath}) \text{ m/s} \quad (74)$$

$$|\vec{a}| = \sqrt{40^2 + 48^2} \text{ m/s} = \boxed{62\,\text{m/s}} \quad (75)$$

Problem 22: From the definition of acceleration:

$$\vec{a} = d\vec{v}/dt = (2 \cdot 10 \cdot t)\hat{\imath} - (3 \cdot 3 \cdot t^2)\hat{\jmath} \quad (76)$$

$$= (40\hat{\imath} - 36\hat{\jmath}) \text{ m/s} \quad (77)$$

$$|\vec{a}| = \sqrt{40^2 + 36^2} \text{ m/s} = \boxed{54\,\text{m/s}} \quad (78)$$

Problem 23:

$$K = \frac{1}{2}mv^2 \quad (79)$$

$$\frac{K_f}{K_i} = \frac{5\,\text{joules}}{4\,\text{joules}} = 1.25 = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}mv_i^2} = \frac{v_f^2}{v_i^2} \quad (80)$$

$$\Longrightarrow \frac{v_f}{v_i} = \sqrt{1.25} = 1.12 \quad (81)$$

$$\Longrightarrow \boxed{12\%\text{ increase}} \quad (82)$$

Problem 24:

$$K = \frac{1}{2}mv^2 \quad (83)$$

$$\frac{K_f}{K_i} = \frac{6\,\text{joules}}{5\,\text{joules}} = 1.2 = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}mv_i^2} = \frac{v_f^2}{v_i^2} \quad (84)$$

$$\Longrightarrow \frac{v_f}{v_i} = \sqrt{1.2} = 1.095 \quad (85)$$

$$\Longrightarrow \boxed{9.5\%\text{ increase}} \quad (86)$$