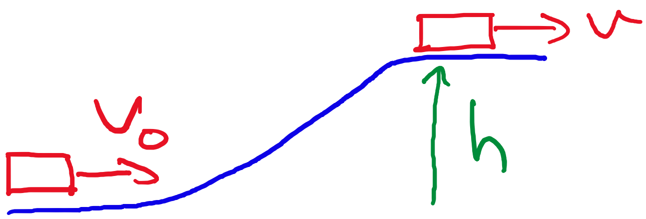
PHY2048 Spring 2019 Exam 2 Solutions

1. A sled and rider having a combined mass of 100 kg travels over a perfectly smooth icy surface. The sled has a speed of 10 m/s initially when traveling over a horizontal surface. It then encounters an upward sloping icy hill of vertical height 3 m. What is the speed of the sled in m/s when it reaches the top of the hill?

(1) 6.4 (2) 4.6 (3) 8.4 (4) 7.8 (5) 0 (or does not reach top)

2. A sled and rider having a combined mass of 100 kg travels over a perfectly smooth icy surface. The sled has a speed of 10 m/s initially when traveling over a horizontal surface. It then encounters an upward sloping icy hill of vertical height 4 m. What is the speed of the sled in m/s when it reaches the top of the hill?

(1) 4.6 (2) 6.4 (3) 8.4 (4) 7.8 (5) 0 (or does not reach top)



$v_0 = 10$ m/s

$h = 3$m (or 4m)

conservation of Mechanical Energy

$$E_{mec} = \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh$$

$$v_0^2 = v^2 + 2gh$$

$$\rightarrow v = \sqrt{v_0^2 - 2gh} = 6.4 \text{ m/s} \quad (\text{or } 4.6 \text{ m/s})$$

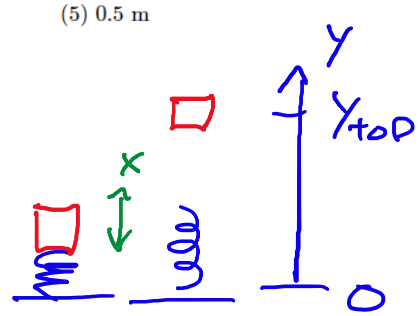
3. A marble with a mass of 0.05 kg is placed on a vertical spring of negligible mass and a spring constant equal to 400 N/m that is compressed by a distance of 7 cm. When the spring is released, how high does the marble rise from the release position? (The marble and the spring are not attached.)

(1) 2.0 m (2) 40 m (3) 0.07 m (4) 20 m (5) 0.5 m

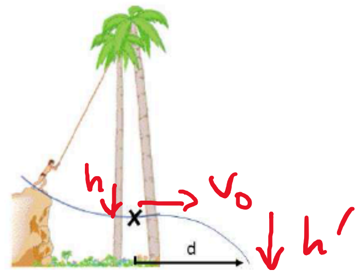
$$E_{mec,i} = \frac{1}{2}kx^2$$

$$= E_{mec,f} = mgh$$

$$\Rightarrow h = \frac{\frac{1}{2}kx^2}{mg} = 2m$$



4. Tarzan, who weighs 700 N, swings from a cliff at the end of a convenient vine that is 20 m long. From the top of the cliff to the bottom of the swing, he descends by 2.5 m. Unfortunately, at the bottom of his swing, the vine breaks and he falls another 5 m vertically. How far in the horizontal direction, d, does Tarzan travel from the point at which the vine breaks?



(1) 7.1 m
(2) ~~14 m~~
(3) 20 m
(4) 5 m
(5) 2.5 m

$h = 2.5\,m$

$h' = 5\,m$

velocity at bottom of swing:

$$\frac{1}{2} m v_0^2 = mgh \Rightarrow v_0 = \sqrt{2gh} = 7.0\ m/s$$

time to fall rest of distance:

$$h' = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h'}{g}} = 1.01\,s$$

horizontal distance

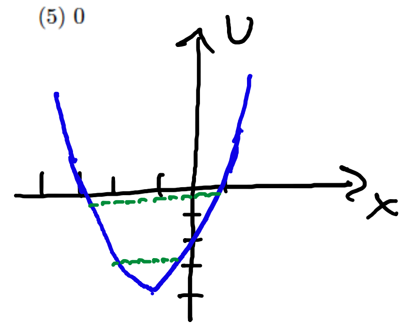
$$d = v_0 t = 7.1\,m$$

5. A potential energy function from a conservative force is given by $U(x) = x^2 + 2x - 3$, with the result measured in Joules. If a mass is released from rest at $x = 1$, what is the maximum kinetic energy (in Joules) in its subsequent motion?

(1) 4.0 (2) 1.0 (3) 2.0 (4) 3.0 (5) 0

6. A potential energy function from a conservative force is given by $U(x) = x^2 + 2x - 3$, with the result measured in Joules. If a mass is released from rest at $x = 0$, what is the maximum kinetic energy (in Joules) in its subsequent motion?

(1) 1.0 (2) 4.0 (3) 2.0 (4) 3.0 (5) 0



$$U(x) = x^2 + 2x - 3 = (x+3)(x-1)$$

crosses zero at $x = -3, 1$

minimum at $\frac{dU}{dx} = 2x + 2 = 0$

$\Rightarrow x = -1$

$\Delta U = -\Delta K \quad \Rightarrow \Delta K = -\Delta U$

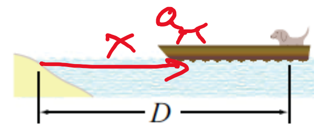
$U(1) = 0 \qquad = U(1) - U(-1) = 4$

$U(0) = -3 \qquad$ or $\quad = U(0) - U(-1) = 1$

$U(-1) = -4$

$U(-3) = 0$

7. A 10 kg dog stands at one end of a 3.0 m long boat of mass 20 kg, a total distance of $D = 8$ m from the shore. He then walks to the other end of the boat. Assuming no friction between the boat and the water, how far is the dog now from the shore?



(1) 6.0 m (2) 5.0 m (3) 2.0 m (4) 7.0 m (5) 1.0 m

initially: $$X_{com} = \frac{M_D \cdot D + M_B \cdot (D - \frac{L}{2})}{M_D + M_B}$$

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finally: $$X_{com} = \frac{M_D \cdot x + M_B (x + \frac{L}{2})}{M_D + M_B}$$

$$\Rightarrow M_D \cdot D + M_B \cdot D - M_B L/2 = M_D \cdot x + M_B \cdot x + M_B \frac{L}{2}$$

$$\frac{(\underset{=10}{M_D} + \underset{=20}{M_B}) \underset{=8}{D} - \underset{=20}{M_B} \underset{=3}{L}}{\underset{=10}{M_D} + \underset{=20}{M_B}} = x = 6.0\,\text{m}$$

8. A professor is standing at rest on a frictionless sheet of ice that covers a parking lot in Chicago, and there is negligible friction between his feet and the ice. He catches a ball of mass 0.4 kg that is tossed to him with a horizontal velocity of 10 m/s. What is the final kinetic energy in Joules of the professor+ball system? The professor's mass is 80 kg.

(1) 0.1 (2) 4000 (3) 0.4 (4) 20 (5) 0.0005

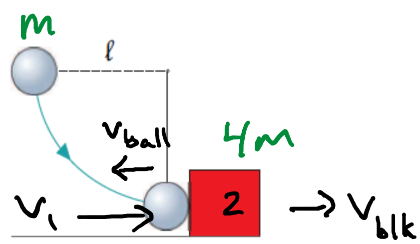
momentum conservation for inelastic collision:

$$m_b v_b = (m_b + m_p) v_f \Rightarrow v_f = \frac{m_b}{m_b + m_p} v_b = 0.050 \frac{m}{s}$$

$$K_f = \frac{1}{2}(m_p + m_b) v_f^2$$

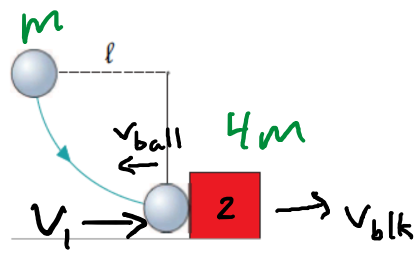
$$= \frac{1}{2} \frac{m_b^2}{m_p + m_b} v_b^2 = 0.1\,\text{J}$$

9. A steel ball of mass m is fastened to a cord of length ℓ and fixed at the far end. The ball is then released when the cord is horizontal, as shown in the figure. At the bottom of its path, the ball strikes a steel block of mass $4m$ that is initially at rest on a frictionless surface. The collision is elastic. What is the speed of the **ball** just after the collision?



(1) $3\sqrt{2g\ell}/5$ (2) $2\sqrt{2g\ell}/5$ (3) $\sqrt{2g\ell}/5$ (4) $\sqrt{2g\ell}$ (5) $8\sqrt{2g\ell}/5$

10. A steel ball of mass m is fastened to a cord of length ℓ and fixed at the far end. The ball is then released when the cord is horizontal, as shown in the figure. At the bottom of its path, the ball strikes a steel block of mass $4m$ that is initially at rest on a frictionless surface. The collision is elastic. What is the speed of the **block** just after the collision?



(1) $2\sqrt{2g\ell}/5$ (2) $3\sqrt{2g\ell}/5$ (3) $\sqrt{2g\ell}/5$ (4) $\sqrt{2g\ell}$ (5) $8\sqrt{2g\ell}/5$

First find speed of ball at bottom of swing:

$$mgh = \frac{1}{2} m v_1^2 \Rightarrow v_1 = \sqrt{2gh}$$

Now use formulas for elastic collisions:

$$V_{ball} = v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_1 = \frac{-3m}{5m}\sqrt{2gh} = -\frac{3}{5}\sqrt{2gh}$$

$$V_{blk} = v_{2f} = \frac{2m_1}{m_1 + m_2} v_1 = \frac{2m}{5m}\sqrt{2gh} = \frac{2}{5}\sqrt{2gh}$$

11. The velocity of a pitched baseball has a magnitude of 40 m/s before it is struck by a bat, and it is returned with a velocity of 60 m/s in the opposite direction. If the ball remains in contact with the bat for 1.5 milliseconds, find the magnitude of the average force applied by the bat. The baseball has a mass of 0.15 kg.

(1) 10,000 N (2) 15 N (3) 2000 N (4) 3 N (5) 100 N

$\leftarrow$o -40 m/s

o$\rightarrow$ $+60$ m/s

$$J = \Delta p = F_{av}\,\Delta t$$

$$\rightarrow F_{av} = \frac{\Delta p}{\Delta t} = \frac{(0.15\,\text{kg})(60-(-40)\,\frac{m}{s})}{(1.5\times 10^{-3}\,s)} = 10^4\,N$$

12. A high-speed flywheel in a motor is spinning at 50 rad/s when a power failure suddenly occurs. The power is off for $t = 40$ s and during this time the flywheel slows uniformly due to friction in its axle bearings. During the time the power is off, the flywheel makes 200 complete revolutions. At what rate is the flywheel spinning when the power comes back on?

(1) 13 rad/s (2) 27 rad/s (3) 0 rad/s (4) 63 rad/s (5) 113 rad/s

$$\Delta\theta = \omega_{av} \cdot t \qquad \omega_{av} = \frac{1}{2}(\omega + \omega_0)t$$

$$\omega = \frac{2\Delta\theta}{t} - \omega_0 = \frac{2}{(40\,s)}(200\,rev)\left(\frac{2\pi\,rad}{rev}\right) - 50\,\frac{rad}{s}$$

$$= 13\ rad/s$$

13. The angular position of one blade of a windmill after a wind begins to blow can be described by $\theta(t) = 10t + 4t^3$, with θ measured in radians and with t measured in seconds starting from 0 when the wind starts. What is the average angular velocity of the blade in the first two seconds after the wind starts?

(1) 26 rad/s (2) 52 rad/s (3) 58 rad/s (4) 48 rad/s (5) 10 rad/s

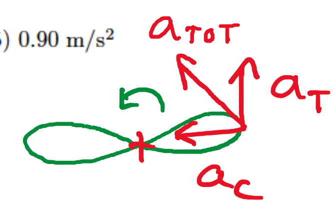
Note that this is asking for average velocity

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta(2) - \theta(0)}{2}$$

$$= \frac{10\cdot 2 + 4\cdot 2^3 - 0}{2} = 26\ \frac{rad}{s}$$

14. A large fan blade with a center-to-tip length of 1.0 m starts from rest and then speeds up with a uniform angular acceleration of 0.4 rad/s^2. Calculate the magnitude of the total linear acceleration of the tip of the blade at $t = 2$ s.

(1) 0.75 m/s^2 (2) 0.64 m/s^2 (3) 0.40 m/s^2 (4) 1.0 m/s^2 (5) 0.90 m/s^2



There is both tangential + centripetal acceleration.

$$a_T = r\alpha = 1\cdot(0.4)\ m/s^2$$

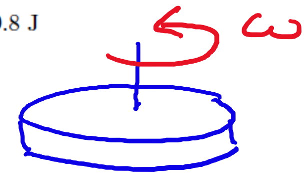
$$a_c = \frac{v^2}{r} = r\omega^2$$

$$\omega = \alpha t \Rightarrow a_c = r\alpha^2 t^2 = 0.64\ m/s^2$$

$$a_{TOT} = \sqrt{a_T^2 + a_c^2} = 0.75\ m/s$$

15. A solid disk of uniform thickness has a mass of 10 kg and a radius of 0.25 m. It is initially at rest and then rotates about a stationary axis through its center with a constant acceleration of 1 rad/s^2. What is the kinetic energy of the disk after it has turned through 10 revolutions?

(1) 20 J (2) 39 J (3) 3.1 J (4) 78 J (5) 9.8 J



$K_{rot} = \frac{1}{2} I \omega^2$

$\omega^2 = \underbrace{\omega_0^2}_{0} + 2\alpha \Delta\theta$

$\omega^2 = 2(1\ \text{rad}/s^2)(10\ \text{rev})\left(2\pi \frac{\text{rad}}{\text{rev}}\right) = 126\ \frac{\text{rad}^2}{s^2}$

For disk, $I = \frac{1}{2} M r^2 = \frac{1}{2}(10\ \text{kg})(0.25\ \text{m})^2 = 0.31\ \text{kg}\cdot\text{m}^2$

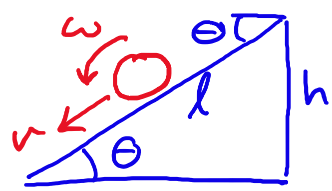
$\Rightarrow K_{rot} = 20\ \text{J}$

16. A solid sphere of mass 2 kg and radius 10 cm starts from rest and rolls without slipping down a surface that is sloped downward at an angle of 37° from horizontal. What is the translational speed of the sphere after it has rolled a distance of 5 m along the surface?

(1) 6.5 m/s (2) 7.7 m/s (3) 5.9 m/s (4) 5.4 m/s (5) 12 m/s

$h = \ell \sin\theta = 5 \sin 37° = 3.0\ \text{m}$

cons v. of mech. energy for rolling



$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

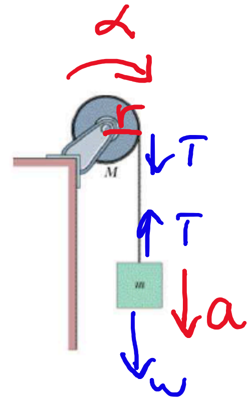
$\cancel{m} gh = \frac{1}{2} \cancel{m} v^2 \left(1 + \underbrace{\frac{I}{mr^2}}_{= 2/5}\right)$

$I = \frac{2}{5} Mr^2$ for a sphere

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}} = 6.5\ \text{m/s}$$

17. A block of mass $m = 0.5$ kg hangs vertically by a rope that is wrapped many times around the rim of a pulley that is in the shape of a solid disk of mass $M = 5$ kg and radius 0.1 m, as shown in the figure. When the block is released and the rope unspools without slipping, the angular acceleration of the block "pulley" (in rad/s^2) is:

(1) 16
~~(2) 93~~
(3) 8.9
(4) 20
(5) 28



Newton's 2nd law in rotational form for the pulley:

1) $I\alpha = rT$

Newton's 2nd law for falling mass:

2) $mg - T = ma \Rightarrow T = mg - ma$

plug into (1):

$I\alpha = mrg - mra$ but $a = r\alpha$

$= mrg - mr^2\alpha$

$$\Rightarrow \alpha = \frac{mrg}{I + mr^2} = \left(\frac{g}{r}\right)\frac{1}{1 + \frac{I}{mr^2}}$$

$I = \frac{1}{2}Mr^2$

$$\Rightarrow \alpha = \frac{g}{r}\frac{1}{1 + \frac{1}{2}M/m} = 16\ \frac{\text{rad}}{s^2}$$

18. Two identical masses are connected by a massless bar of length $2r$ and rotate about a perpendicular axis at the midpoint between the two masses at an angular velocity ω. If the radial distance r each mass lies from the axis is tripled, what would be the new value for the angular velocity?

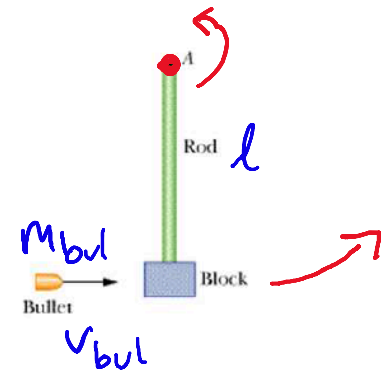
(1) $\omega/9$
~~(2) 9ω~~
(3) $\omega/3$
(4) 3ω
(5) ω

$I_1\omega_1 = I_2\omega_2$

$$\omega_2 = \frac{I_1}{I_2}\omega_1$$

$$= \frac{2mr^2}{2m(3r)^2} = \frac{\omega_1}{9}$$

19. In the figure a bullet of mass 0.01 kg is fired into a 1 kg block attached to the end of a massless rod of length 0.5 m, and the bullet becomes embedded into the block. The other end of the rod is attached at point A, and is free to rotate about an axis there. If the initial bullet speed is 100 m/s, what is the angular velocity of the bullet-block-rod system just after the impact, assuming that the block was initially at rest?



(1) 2.0 rad/s
(2) 200 rad/s
(3) 1.0 rad/s
(4) 4.0 rad/s
(5) 0.5 rad/s

Conservation of angular momentum:

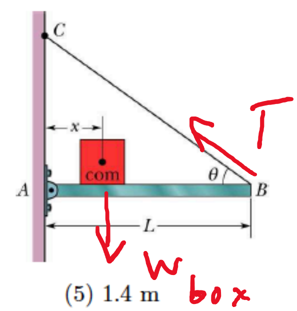
Before: $L_i = M_{bul} \cdot V_{bul} \cdot \ell$

After: $L_f = I\omega$

$I = (M_{bul} + M_{BLK})\,\ell^2$

$$\rightarrow \omega = \frac{\overset{=0.01}{M_{bul}} \cdot \overset{=100\,m/s}{V_{bul}}}{(\underset{0.01}{M_{bul}} + \underset{1}{M_{BLK}}) \cdot \underset{=0.5}{\ell}} = 2 \text{ rad/s}$$

20. In the figure, a massless bar of length $L = 2$ m is attached by a hinge to a wall at one end and is supported by a massless cable at the other end (which is also attached to the wall). The cable makes an angle of $\theta = 30°$ with respect to the bar and has a maximum tension of 350 N that it can withstand. The bar is intended to support a block with a weight of 500 N. What is the maximum possible distance x that the block can be placed along the bar from the wall without the cable breaking?



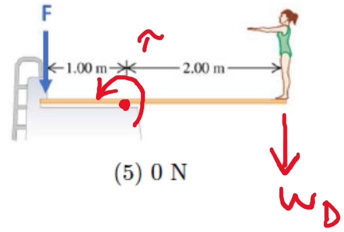
(1) 0.7 m (2) 2.0 m (3) 0 m (4) 1.2 m (5) 1.4 m

Take torque about hinge:

$$\tau_{net} = L T \sin\theta - x W_{Box} = 0$$

$$x = \frac{\overset{2m}{L}\,\overset{350N}{T}\,\sin\overset{=0.5}{\theta}}{W_{box} = 500N} = 0.7\,m$$

21. A diving board of length 3.0 m is supported at a point 1.0 m from the far end, and a diver weighing 450 N stands at the free end. The diving board is taken to be massless. Find the magnitude of the force F that must be made on the board at the far end.



(1) 900 N (2) 450 N (3) 225 N (4) 1350 N (5) 0 N

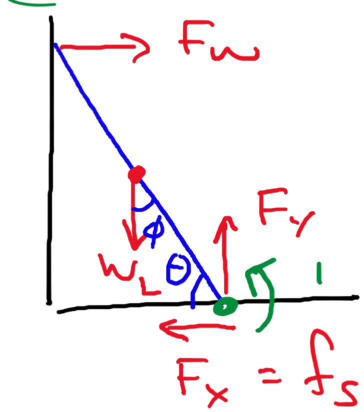
Balance torque about support point:

$$\tau_{net} = F \cdot l_1 - W_D \cdot l_2 = 0$$

$$\Rightarrow F = \frac{l_2}{l_1} W_D = \frac{2}{1}(450\,N) = 900\,N$$

22. A uniform ladder of length 5 m and weight 200 N leans against a frictionless vertical wall. The foot of the ladder is placed 3 m from the base of the wall. What must be the magnitude of the force of static friction supplied by the floor to keep the ladder from slipping?

(1) 75 N (2) 130 N (3) 150 N (4) 200 N (5) 100 N



$$\cos\theta = \frac{3\,m}{5\,m} = 0.6$$

$$\Rightarrow \theta = 53^\circ$$

$$\phi = 90^\circ - \theta$$

Force balance in x: $F_w - f_s = 0 \Rightarrow F_w = f_s$

in y: $F_y - W_L = 0 \Rightarrow F_y = W_L$

torque balance about foot of ladder:

$$\tau_{net} = \frac{l}{2} W_L \sin\phi - l\, F_w \sin\theta = 0$$

$$\frac{1}{2} W_L \cos\theta = F_w \sin\theta$$

$$\rightarrow f_s = F_w = \frac{W_L}{2\tan\theta} = \frac{200\,N}{2\cdot(1.33)} = 75\,N$$

23. A long cylindrical copper wire attached to a wall at one end has a radius of 0.5 mm and a length of 10 m. What force must be applied to the free end of the wire to stretch the wire by 2.5 mm? The Young's modulus of copper is 1.20×10^{11} N/m^2.

(1) 24 N (2) 3×10^7 N (3) 94 N (4) 240 N (5) 0.006 N

$$\frac{F}{A} = Y \frac{\Delta x}{L}$$

$$F = (\pi r^2) Y \frac{\Delta x}{L} = \pi (5 \times 10^{-4} m)^2 \left(1.2 \times 10^{11} \frac{N}{m^2}\right) \times \left(\frac{2.5 \times 10^{-3} m}{10 m}\right)$$

$$= 24 N$$