PHY 2048

Instructor: Profs. Acosta, Hagen, Hamlin

PHYSICS DEPARTMENT

Final Examination

April 29, 2019

Name (print, last first):

Signature:

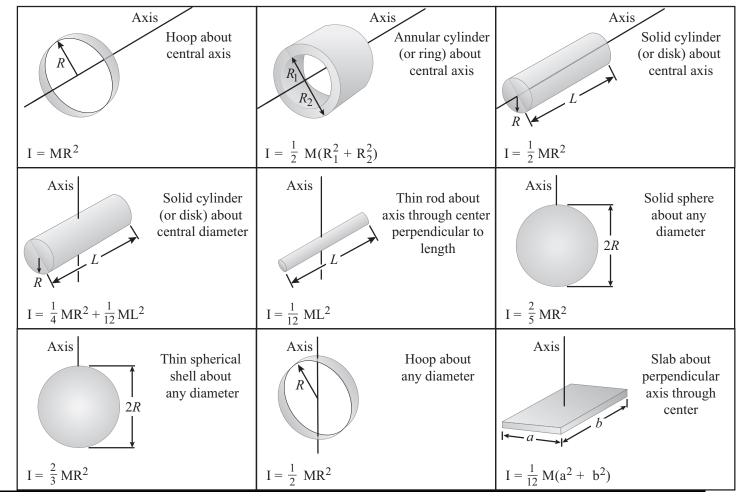
On my honor, I have neither given nor received unauthorized aid on this examination.

BEFORE YOU START

- (1) Print and sign your name on this QUESTION sheet (in the space provided above).
- Your TEST NUMBER is the 5-digit number at the top of this QUESTION SHEET.
- (3) Code your 5-digit test number onto lines 76 80 of your ANSWER SHEET (scan sheet).
 (4) Code your name and UFID on your ANSWER SHEET.
- (5) For each question, blacken the circle of your intended answer completely on the answer sheet. DARKEN CIRCLES COMPLETELY, using a No. 2 pencil or blue or black ink. Do not make any stray marks on the answer sheet.
- (6) You can do scratch work on this question sheet. Additional scratch paper is available if you want it.
- (7) For each question, choose the best answer. Many answers are rounded. Choose the most correct answer. Use g = 9.80 m/s^2 . All questions carry equal credit. There is no penalty for guessing.
- You may use a simple electronic calculator not a cell phone. No books, notes, or other resources are permitted.
- (9) You may separate the included formula sheets and question pages if you wish. There is no need to reattach or staple them later.

WHEN YOU FINISH

- (1) Check that you have coded your name, UFID and test number on your answer sheet (scan sheet). Check that your name and signature are on this question sheet.
- Turn in your answer sheet (scan sheet) AND question sheet. Please keep your scratch paper.
- (3) No credit will be given unless you turn in both the answer sheet and the question sheet with your name on each.



PHY2048 Formulas

Vectors

$$\begin{split} \vec{a} &= a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \quad \vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k} \qquad \text{Magnitudes:} \quad |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad |\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2} \\ \text{Scalar Product:} \quad \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z \qquad \text{Magnitude:} \quad \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \ (\theta = \text{angle between } \vec{a} \text{ and } \vec{b}) \\ \text{Vector Product:} \quad \vec{a} \times \vec{b} = (a_y b_z - a_z b_y) \hat{i} + (a_z b_x - a_x b_z) \hat{j} + (a_x b_y - a_y b_x) \hat{k} \\ \text{Magnitude:} \quad |\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta \ (\theta = \text{angle between } \vec{a} \text{ and } \vec{b}) \end{split}$$

Motion

Displacement: $\Delta \vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$ Average Velocity: $\vec{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$ Instantaneous Velocity: $\vec{v} = \frac{d\vec{r}(t)}{dt}$ Average Acceleration: $\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}$ Instantaneous Acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

Equations of Motion for Constant Acceleration

$$\begin{split} \vec{v} &= \vec{v}_0 + \vec{a}t \\ \vec{r} - \vec{r}_0 &= \vec{v}_0 t + \frac{1}{2} \vec{a}t^2 \\ v_x^2 &= v_{x0}^2 + 2a_x(x-x_0) \text{ (in each of 3 dim)} \end{split}$$

Newton's Laws

 $\vec{F}_{net} = 0 \Leftrightarrow \vec{v}$ is a constant (Newton's First Law) $\vec{F}_{net} = m\vec{a}$ (Newton's Second Law) "Action = Reaction" (Newton's Third Law)

Magnitude of the Frictional Force

Static: $f_s \leq \mu_s F_N$ Kinetic: $f_k = \mu_k F_N$

<u>Uniform Circular Motion (Radius R, Tangential Speed $v = R\omega$, Angular Velocity ω)</u>

Centripetal Acceleration: $a = \frac{v^2}{R} = R\omega^2$ Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$

Quadratic Formula

If:
$$ax^2 + bx + c = 0$$
 Then: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Work (W), Mechanical Energy (E, Kinetic Energy (K)), Potential Energy (U)

Kinetic Energy: $K = \frac{1}{2}mv^2$ Work: $W = \int_{\vec{r}_1}^{r_2} \vec{F} \cdot d\vec{r}$ When force is constant $W = \vec{F} \cdot \vec{d}$ Power: $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$ Work-Energy Theorem: $K_f = K_i + W$ Work done by gravity: $W_g = -mg\Delta y$ Potential Energy: $\Delta U = -W = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$ $F_x = -\frac{dU}{dx}$ Gravitational (y=up): $F_y = -mg$ U(y) = mgyMechanical Energy: $E_{mec} = K + U$ $E_{mec} = \text{constant for an isolated system with conservative forces}$ Work-Energy: $W(\text{external}) = \Delta K + \Delta U + \Delta E(\text{thermal})$

<u>Springs</u>

Spring force (Hooke's law): F = -kx Work done by spring: $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Momentum

Center of Mass: $\vec{r}_{com} = \frac{1}{M_{tot}} \sum_{i=1}^{N} m_i \vec{r}_i$ $M_{tot} = \sum_{i=1}^{N} m_i$ Linear Momentum: $\vec{p} = m\vec{v}$ Impulse: $\vec{J} = \Delta \vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt = \vec{F}_{av} \Delta t$ $\vec{F} = \frac{d\vec{p}}{dt}$ If $\vec{F} = \frac{d\vec{p}}{dt} = 0$ then $\vec{p} = \text{constant}$ and $\vec{p}_f = \vec{p}_i$ $\vec{P}_{tot} = M_{tot} \vec{v}_{com} = \sum_{i=1}^{N} \vec{p}_i$ $\vec{F}_{net} = \frac{d\vec{P}_{tot}}{dt} = M_{tot} \vec{a}_{com}$ Elastic Collisions of Two Bodies, 1D $v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$ $v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$

Rockets: Thrust = $Ma = v_{\rm rel} \frac{dM}{dt}$ $\Delta v = v_{\rm rel} \ln(\frac{M_i}{M_f})$

Rotational Variables

angular position: $\theta(t)$ angular velocity: $\omega(t) = \frac{d\theta(t)}{dt}$ angular acceleration: $\alpha(t) = \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2}$

Motion with constant angular acceleration α :

$$\omega = \omega_0 + \alpha t \qquad \qquad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \qquad \theta = \theta_0 + \frac{1}{2}(\omega + \omega_0)t$$

Angular to linear relationships for circular motion

arc length: $s = r\theta$ velocity: $v = r\omega$ tangential acceleration: $a_{\rm T} = r\alpha$ centripetal acceleration: $a_{\rm c} = r\omega^2$ Rotational Inertia: $I = \sum_{i=1}^{N} m_i r_i^2$ (discrete) $I = \int r^2 dm$ (continuous) Parallel Axis: $I = I_{\rm com} + M_{\rm tot} d^2$ (d is displacement from c.o.m.)

Rotational, Rolling Kinetic Energy: $K_{\rm rot} = \frac{1}{2}I\omega^2$ $K_{\rm roll} = \frac{1}{2}Mv_{\rm com}^2 + \frac{1}{2}I_{\rm com}\omega^2$ Rolling without slipping: $x_{\rm com} = R\theta$ $v_{\rm com} = R\omega$ $a_{\rm com} = R\alpha$

Rolling down a ramp: $a_{\text{com}} = \frac{gsin\theta}{\left(1 + \frac{I}{mR^2}\right)}$

Torque and Angular Momentum

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$ (where $\hat{k} = \hat{i} \times \hat{j}$ gives directions for cross product) $\tau = rF$ sin (angle between \vec{r} and \vec{F}) = rF_{\perp} Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$ $\vec{\tau} = \frac{d\vec{L}}{dt}$ L = rp sin (angle between \vec{r} and \vec{p}) = rp_{\perp} $L = I\omega$ If $\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$ then $\vec{L} = \text{constant}$ and $\vec{L}_f = \vec{L}_i$

Work done by a constant torque: $W = \tau \Delta \theta = \Delta K_{\rm rot} = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$

Power done by a constant torque: $P = \tau \omega$

For torque acting on a body with rotational inertia $I: \vec{\tau} = I\vec{\alpha}$

Gravitation

Weight (near the surface of the Earth) = mg (**use g=9.8** m/s^2) Newton's law of gravitation $\vec{F} = \frac{Gm_1m_2}{r^2}\hat{r}$ where $G = 6.67408 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ $U = -\frac{GMm}{r}$ Circular orbit $E = U + K = -\frac{GMm}{2r}$ Kepler's Area Law: $\frac{dA}{dt} = constant$ Kepler's Law of periods: $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ Elasticity

Stress and Strain (Y = Young's modulus, B = bulk modulus)

Linear: $\frac{F}{A} = Y \frac{\Delta L}{L}$ Volume: $P = \frac{F}{A} = -B \frac{\Delta V}{V}$

<u>Fluids</u>

Fluid density: $\rho = \frac{m}{V}$ Pressure: $P = \frac{F}{A}$ $P_2 = P_1 + \rho g(y_1 - y_2)$ Archimedes: $F_b = m_f g$ Flow of fluid: Av = constant

Bernoulli's Eqn: $P + \frac{1}{2}\rho v^2 + \rho gy = constant$

Waves and Oscillators

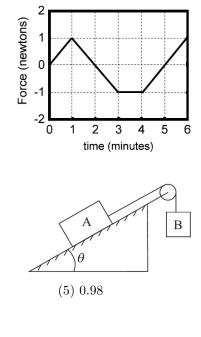
Simple harmonic motion $\frac{d^2x}{dt^2} = -\omega^2 x$ $x = x_m \cos(\omega t + \phi)$ Energy (undamped) $E = \frac{1}{2}kx_m^2$ Frequency (Hz) and period (s): $f = \frac{1}{T}$ Angular freq $\omega = 2\pi f$ Simple pendulum: $\omega = \sqrt{\frac{g}{L}}$ Torsional pendulum: $\omega = \sqrt{\frac{\kappa}{T}}$ Linear oscillator: $\omega = \sqrt{\frac{k}{m}}$ Physical pendulum: $\omega = \sqrt{\frac{mgh}{I}}$ Linear oscillator with damping force -bv: $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ $x(t) = x_m \exp(-bt/2m)\cos(\omega' t + \phi)$ Sinusoidal waves: $y(x, t) = y_m \sin(kx - \omega t)$ $k = \frac{2\pi}{\lambda}$ $\omega = \frac{2\pi}{T}$ $v = \frac{\omega}{k} = f\lambda$ Wave equation: $\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$ Wave speed on string (linear density μ) $v = \sqrt{\tau/\mu}$ and in fluid (bulk modulus B) $v = \sqrt{B/\rho}$ n^{th} harmonic resonance on string of length L: $f = \frac{v}{\lambda} = n\frac{v}{2L}$ Path difference and phase $\phi = \frac{2\pi\Delta L}{\lambda}$ Constructive: $\phi = 2m\pi$, Destructive: $\phi = (2m + 1)\pi$ Sound intensity $I = \frac{P}{A} = \frac{1}{2}\rho v\omega^2 s_m^2$ and sound level (decibels): $10 \log_10 \frac{I}{I_0}$ (where $I_0 = 10^{-12} \text{ W/m}^2$) Doppler effect for source moving at v_s and detector moving at v_D : $f' = f \frac{v \pm v_D}{v \pm v_s}$ 1. A transverse wave has the displacement $y(x,t) = (2.00 \text{ mm}) \sin[(2.0 \text{ m}^{-1})x - (40 \text{ s}^{-1})t]$. Its speed is

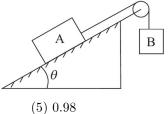
(1)
$$20 \text{ m/s}$$
 (2) 0.05 m/s (3) 80 m/s (4) 2.0 m/s (5) 0.2 m/s

- 2. A 6.0-kg cart is moving horizontally at 5.0 m/s. If 70 J of work is done to speed it up, what is its final speed in m/s?
 - (1) 7.0 (2) 4.8 (3) 48 (4) 9.6 (5) 8.0
- 3. The two arms of a U-shaped tube are not identical, one having four times the diameter of the other. The tube is filled with water and the wide arm is fitted with a piston. A force of 22 newtons is required to remove the cork from the narrow arm. The minimum force (in N) that must be applied to the piston to push out the cork is
 - (1) 352 N
 - (2) 5.5 N
 - (3) 1.4 N
 - (4) 88 N
 - (5) Cannot be calculated without knowing the volume of the fluid.
- 4. A solid sphere of mass m = 1.5 kg and diameter 4.2 cm floats in an unknown fluid. The fluid fills a vertical cylindrical container of diameter 6.1 cm. The sphere is removed and the fluid level drops by 9.3 mm. The density of the unknown fluid(g/cm³) is
 - (1) 55 (2) 55,000 (3) 14 (4) 14,000 (5) The fluid level should rise!
- 5. A nail gun shoots a 10 g nail horizontally with a speed of 50 m/s into a 300 g block of wood resting on a frictionless horizontal surface. The nail is embedded into the block after the collision and does not pass through it. The final speed of the nail+block system is most nearly
 - (1) 1.6 m/s (2) 1.7 m/s (3) 0.90 m/s (4) 1.5 m/s (5) 2.0 m/s
- 6. The mass of the Earth is M_E and the mass of the Moon is M_m . If F_1 is the magnitude of the gravitational force exerted on the Moon by Earth and F_2 is the magnitude of the force exerted on Earth by the Moon, then the ratio F_1/F_2 is
 - (1) 1 (2) M_E/M_m (3) $\sqrt{M_E/M_m}$ (4) M_m/M_E (5) $\sqrt{M_m/M_E}$
- 7. Three vectors are $\vec{a} = 2\hat{i} \hat{j}$, $\vec{b} = \hat{i} + 2\hat{j}$, and $\vec{c} = \hat{j} + 2\hat{k}$. The angle between the directions of \vec{c} and $\vec{a} \times \vec{b}$ is (in degrees)
 - (1) 26.6 (2) 107 (3) 66.4 (4) 60.0 (5) 63.4
- 8. A uniform solid sphere of mass 2 kg and radius 10 cm starts from rest and rolls without slipping down a surface that is sloped downward at an angle θ relative to the horizontal. After the sphere has rolled a distance of 7.0 m down the surface, its translational speed is 5.5 m/s. The angle θ is (in degrees)
 - (1) 18 (2) 12.7 (3) 5.1 (4) 25 (5) 37
- 9. A large fan blade with a center-to-tip length of 50 cm starts from rest at t = 0. It then speeds up with a uniform angular acceleration of 2.5 rad/s². At t = 1.0 s, the magnitude of the total linear acceleration (m/s²) of the tip of the blade is
 - (1) 3.4 (2) 3.1 (3) 1.3 (4) 4.4 (5) 6.3



- (1) At t = 3.5 minutes only
 (2) At t = 2 minutes only
 (3) At t = 2 minutes and 5 minutes
 (4) The velocity is never zero after t = 0
 (5) At t = 5 minutes only
- 11. Block A, with a weight of 13 N, rests on an incline of $\theta = 40$ degrees above the horizontal. A massless string attached to block A is parallel to the incline and passes over a massless, frictionless pulley at the top. Block B has a weight of 14 N. Both blocks are stationary (i.e., block A does not slide). The magnitude of the friction force (in newtons) is
 - (1) 5.6 (2) 8.5 (3) 4.0 (4) 2.3
- 12. Block A, with a weight of 13 N, rests on an incline of $\theta = 35$ degrees above the horizontal. A massless string attached to block A is parallel to the incline and passes over a massless, frictionless pulley at the top. Block B has a weight of 16 N. Both blocks are stationary (i.e., block A does not slide). The magnitude of the friction force (in newtons) is
 - (1) 8.5 (2) 5.6 (3) 4.0 (4) 2.3
- 13. A student holds a very thin, uniform rod of length 1.0 meter vertically with one end just touching the floor. The stick is then released (from rest) and begins to fall. If the end on the floor does not slip, what is the speed (m/s) of the other end when it strikes the floor? (Consider rotational energy.)
 - (1) 5.4 (2) 3.1 (3) 4.4 (4) 2.6 (5) 7.7
- 14. A student holds a very thin, uniform rod of length 2.0 meter vertically with one end just touching the floor. The stick is then released (from rest) and begins to fall. If the end on the floor does not slip, what is the speed (m/s) of the other end when it strikes the floor? (Consider rotational energy.)
 - (1) 7.7 (2) 3.1 (3) 4.4 (4) 2.6 (5) 5.4
- 15. A vertical spring supports a platform with mass $m_A = 0.50$ kg at rest. A mass $m_B = 0.50$ kg is then dropped from a height of 1.0 meter and sticks to the platform. This causes a brief compression of the spring, bringing the platform 5.5 mm below its rest position before it rises again. (Note that m_B has an inelastic collision with m_A .) The Hooke's law constant (N/m) of the spring is approximately
 - (1) 1.6×10^5 (2) 4.0×10^5 (3) 1.8×10^3 (4) 2800 (5) 3.2×10^5
- 16. A vertical spring supports a platform with mass $m_A = 0.50$ kg at rest. A mass $m_B = 0.50$ kg is then dropped from a height of 1.0 meter and sticks to the platform. This causes a brief compression of the spring, bringing the platform 3.5 mm below its rest position before it rises again. (Note that m_B has an inelastic collision with m_A .) The Hooke's law constant (N/m) of the spring is approximately
 - (1) 4.0×10^5 (2) 1.6×10^5 (3) 1.8×10^3 (4) 2800 (5) 8.0×10^5

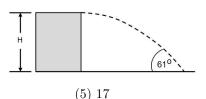


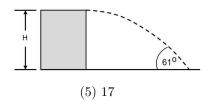


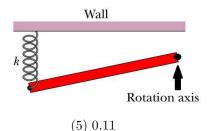
- 17. Planet R47A is a spherical planet where the gravitational acceleration on the surface is 3.45 m/s². A satellite orbits Planet R47A in a circular orbit of radius 5000 km and period 4.0 hours. What is the radius of Planet R47A?
 - (1) 2600 km (2) 3100 km (3) 1200 km (4) 4300 km (5) Cannot be determined
- 18. Planet R47A is a spherical planet where the gravitational acceleration on the surface is 4.35 m/s^2 . A satellite orbits Planet R47A in a circular orbit of radius 6000 km and period 4.0 hours. What is the radius of Planet R47A?

(1) 3100 km (2) 2600 km (3) 1200 km (4) 4300 km (5) Cannot be d	e determined
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- 19. A uniform ladder of length 6.0 m and weight 300 N leans against a frictionless vertical wall. The foot of the ladder is placed 3.0 m from the base of the wall. What must be the magnitude of the force of static friction supplied by the floor to keep the ladder from slipping?
 - (1) 87 N (2) 260 N (3) 130 N (4) 53 N (5) 75 N
- 20. A uniform ladder of length 6.0 m and weight 300 N leans against a frictionless vertical wall. The foot of the ladder is placed 2.0 m from the base of the wall. What must be the magnitude of the force of static friction supplied by the floor to keep the ladder from slipping?
 - (1) 53 N (2) 260 N (3) 130 N (4) 87 N (5) 75 N
- 21. A rock is thrown horizontally off a cliff that is H = 15 m high. The rock strikes the ground at an angle of 61 degrees. With what speed (in m/s) was the rock thrown? You may neglect air resistance.
 - (1) 9.5 (2) 12 (3) 5.5 (4) 25
- 22. A rock is thrown horizontally off a cliff that is H = 24 m high. The rock strikes the ground at an angle of 61 degrees. With what speed (in m/s) was the rock thrown? You may neglect air resistance.
 - (1) 12 (2) 9.5 (3) 5.5 (4) 25
- 23. A long uniform rod of mass 2.0 kg and length 1.5 m is free to rotate in a horizontal plane about a vertical axis at one end. At the other end, a spring with Hooke's law constant k = 1500 N/m is connected horizontally between the end of the rod and a fixed wall. The equilibrium position of the rod is parallel to the wall. If the rod is rotated slightly and released (to act like a torsion oscillator), the period (in seconds) of small oscillations about the rotation axis is:
 - (1) 0.13 (2) 0.23 (3) 0.087 (4) 0.15

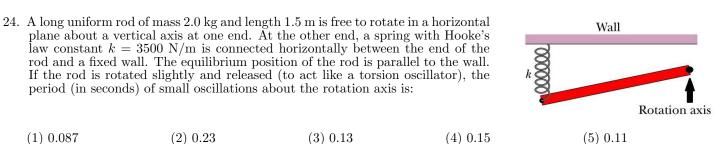






(1) 0.087

five identical lawn mowers would produce a sound level



- 25. From a distance of 40 m, one noisy lawn mower produces a sound level of 75 dB (decibels). From the same distance,
 - (1) 82 dB (2) 375 dB (3) 80 dB(4) 167 dB (5) 78 dB
- 26. Asteroid Zelda circles the Sun in a circular orbit with a radius that is three times the radius of the Earth's orbit. The period of Asteroid Zelda's orbit is most nearly (in Earth years)
 - (3) 9.0(4) 2.1(1) 5.2(2) 3.0 (5) 27
- 27. A car and its driver together have mass 1100 kg. The car is driven over a "washboard" road with regularly spaced bumps, and it undergoes maximum amplitude motion when driven at speed v = 27.0 m/s. The driver then picks up a hitchhiker. Continuing along the same road with the hitchiker, the car now undergoes its maximum amplitude motion when its speed is v = 26.0 m/s. The mass (kg) of the hitchhiker is
 - (1) 86(2) 42 (3) 74(4) 98 (5) 68
- 28. A car and its driver together have mass 1200 kg. The car is driven over a "washboard" road with regularly spaced bumps, and it undergoes maximum amplitude motion when driven at speed v = 26.0 m/s. The driver then picks up a hitchhiker. Continuing along the same road with the hitchiker, the car now undergoes its maximum amplitude motion when its speed is v = 25.0 m/s. The mass (kg) of the hitchhiker is
 - (1) 98(2) 42 (3) 74(4) 86(5) 68
- 29. A cello string has length 60.4 cm and oscillates in its second harmonic (n = 2). The wave speed on the string is 250 m/s and the speed of sound in air is 343 m/s. The wavelength of the resulting sound wave in the air is
 - (1) 0.83 m(2) 0.41 m (3) 1.7 m (4) 1.2 m (5) 0.60 m

Scratch paper

Scratch paper

 $\begin{array}{c} {\rm TYPE \ 1} \\ {\rm Q}\#\ {\rm S}\ 11 \\ {\rm Q}\#\ {\rm S}\ 12 \\ {\rm TYPE \ 2} \\ {\rm Q}\#\ {\rm S}\ 13 \\ {\rm Q}\#\ {\rm S}\ 14 \\ {\rm TYPE \ 3} \\ {\rm Q}\#\ {\rm S}\ 14 \\ {\rm TYPE \ 3} \\ {\rm Q}\#\ {\rm S}\ 16 \\ {\rm TYPE \ 4} \\ {\rm Q}\#\ {\rm S}\ 16 \\ {\rm TYPE \ 4} \\ {\rm Q}\#\ {\rm S}\ 17 \\ {\rm Q}\#\ {\rm S}\ 18 \\ {\rm TYPE \ 5} \\ {\rm Q}\#\ {\rm S}\ 19 \\ {\rm Q}\#\ {\rm S}\ 20 \\ {\rm TYPE\ 6} \end{array}$

 $\begin{array}{c} TYPE \ 6\\ Q\# \ S \ 21\\ Q\# \ S \ 22\\ TYPE \ 7\\ Q\# \ S \ 23\\ Q\# \ S \ 24\\ TYPE \ 8\\ O\# \ S \ 27\end{array}$

 $Q \# S 27 \\ Q \# S 28$