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1. An object moves in a straight line, with velocity $v(t) = 98 - 2t^2$, where v is in m/s and t in s. At the time when it is momentarily at rest (when $v = 0$), its acceleration is:

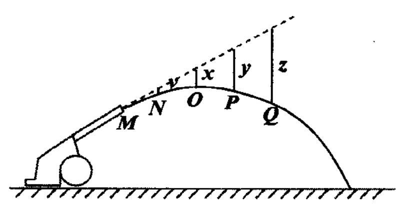
(1) -28 m/s^2 (2) 0 (3) -4.0 m/s^2 (4) -9.8 m/s^2 (5) 49 m/s^2

$$v = 98 - 2t^2$$
$$a = \frac{dv}{dt} = -4t$$

$v = 0$ when $98 = 2t^2$, $t = 7$ sec

$\Rightarrow a = -28 \text{ m/s}^2$

2. A cannon fires a projectile whose trajectory is the solid line shown. The dashed line shows the trajectory in the absence of gravity. Points $M\ N\ O\ P\ Q$ are separated by intervals of 1 s. Take $g = 10$ m/s^2. The lengths v, x, y (in m) are:



(1) 5,20,45 (2) 5,10,15 (3) 10,40,90 (4) 10,20,30 (5) 0.2,0.8,1.6

(from quickie)

Deviations from straight (dashed) lines are caused by gravitational term in equation of motion

$$y = \underbrace{y_0 + V_{oy}t}_{\text{straight line}} - \underbrace{\tfrac{1}{2}gt^2}_{\text{deviation}}$$

t	$\frac{1}{2}gt^2$
0	0
1	5
2	20
3	45

stationary

3. A large truck collides head on with a small compact car. During the collision:

(1) the car exerts the same force on the truck as the truck does on the car
(2) the truck exerts a greater amount of force on the car than the car exerts in the truck
(3) the car exerts a greater amount of force on the truck than the truck exerts in the car
(4) the center of mass does not move
(5) kinetic energy is conserved

Newton's 3rd law $\Rightarrow$

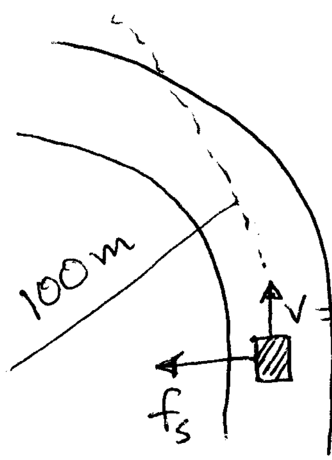
$$\vec{F}_{truck-car} = -\vec{F}_{car-truck}$$

same magnitude!

Small car accelerated more due to smaller mass, of course

4. The driver of a 1000 kg car driving at a constant speed of 10 m/s tries to turn through a circle of radius 100 m on an unbanked, level curve. The coefficient of static friction between the tires and the slippery road is $\mu_s = 0.092$. The car will:

(1) slide off to the outside of the curve
(2) slide into the inside of the curve
(3) stay on the road around the curve
(4) slow down due to the friction
(5) stay on the road around the curve only if it speeds up



Force of static friction (no sliding) can provide maximum force $f_{s,max} = \mu_s m g = 902\text{ N}$

Force required to keep car going in circle of given radius is $\dfrac{mv^2}{R} = \dfrac{(1000)(10)^2}{100} = 1000\text{ N}$

i.e., there's not enough friction to make car turn $\Rightarrow$ it spins out to outside of curve.

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5. At time $t = 0$, a 2 kg object has a velocity in m/s of $4\hat{\imath} - 3\hat{\jmath}$. At $t = 3$ s its velocity in m/s is $3\hat{\imath} + 3\hat{\jmath}$. During this time the work done on it was:

(1) -7 J (2) 4 J (3) -4 J (4) 40 J (5) $4\hat{\imath} + 36\hat{\jmath}$ J

$\Delta K = W$ (Work - KE theorem)

$$v_f^2 = \vec{v}_f \cdot \vec{v}_f = 9 + 9 = 18$$

$$v_i^2 = \vec{v}_i \cdot \vec{v}_i = 9 + 16 = 25$$

$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = 18 - 25 = -7\,\text{J}$$

6. The potential energy of a particle moving on the postive x axis is given by $U(x) = Cx^3$ where C is a constant. The force $F(x)$ exerted on the particle is given by:

(1) $F = -3Cx^2$ (2) $F = 9.8\text{N} - 3Cx^2$ (3) $F = (1/4)Cx^4$ (4) $F = Cx^2$ (5) $F = -Cx$

For conservative forces, where one can define a potential energy

$$F = -\frac{d}{dx} U(x) = -3Cx^2$$

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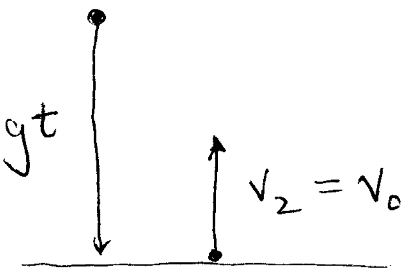
7. A 0.50 kg ball is dropped from a height of 25 m above the ground. At the same time, a second ball, mass equal to 0.25 kg, is thrown vertically from the ground surface with a speed of 15 m/s. The balls pass each other without colliding. After 2.0 s, the velocity of the center of mass is:

(1) 15 m/s, down (2) 15 m/s, up (3) 11 m/s, up (4) 9.8 m/s, down (5) 20 m/s, down

$$y_{cm} = \frac{1}{M}\, m_1 y_1 + M_2 y_2$$

total $\to M$

$$v_1 = -gt$$



$$v_2 = v_0 - gt$$

$$v_{y\,cm} = \frac{1}{M}\left(m_1 v_{y1} + m_2 v_{y2}\right)$$

$$= \frac{1}{M}\left(m_1(-gt) + m_2(v_0 - gt)\right)$$

$$= \frac{1}{0.75}\left(-0.5(9.8)(2) + 0.25(15 - (9.8)(2))\right) = -14.6\,m/s$$

8. A horizontal moving walkway is moving at 3 m/s. On the average, during each second, four stationary people step onto it and four other people step off it. Assuming that each person has a mass of 60 kg, what is the average force required to maintain the constant speed of the walkway?

(1) 720 N (2) 80 N (3) 1080 N (4) 0 N (5) 1200 N

Each person who steps on must be accelerated by the drive's motor up to speed. On the other hand, each person who steps off is decelerated by friction on ground, i.e. this has nothing to do with walkway.

Total change in momentum in 1 sec

$$\Delta P_{tot} = 4 \cdot m \cdot \Delta v = 4 \cdot 60 \cdot 3 = 720\,\frac{kg\,m}{s}$$

$$\text{Avg force} = \frac{\Delta P_{tot}}{\Delta t} = \frac{720\,\frac{kg\cdot m}{s}}{1\,s} = 720\,N$$

9. Two astronauts are floating together with zero relative speed in a gravity-free region of space. The mass of astronaut A is 120 kg and the mass of astronaut B is 90 kg. Astronaut A pushes astronaut B such that B's speed is 0.5 m/s. ~~The final~~ speed of A is:

(1) 0.38 m/s (2) 0 (3) 0.51 m/s (4) 0.67 m/s (5) 0.98 m/s

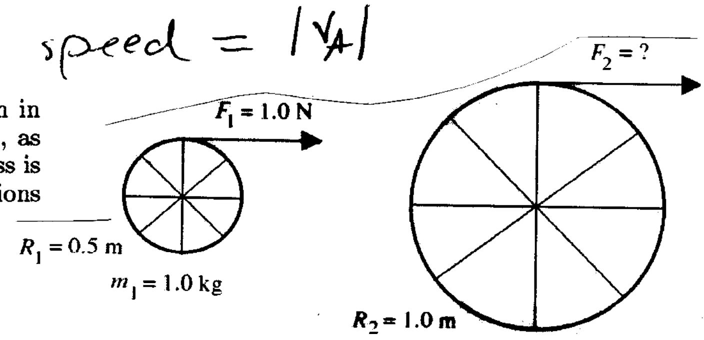
conserve momentum =>

$$P_{before} = 0 = M_A V_A + M_B V_B = P_{after}$$

$$V_A = -\frac{M_B V_B}{M_A} = -\frac{90 \cdot 0.5}{120} = -0.375 \text{ m/s}$$

Question asks for speed $= |V_A|$

10. Two bicycle wheels are mounted with their axles fixed, as shown in the diagram. Starting with the wheels at rest, forces are applied, as shown in the diagram. Assume the spokes are massless, i.e. all mass is concentrated on the rim. In order to impart equal angular accelerations to the wheels, F_2 must be:



(1) 2 N (2) 0.25 N (3) 0.5 N (4) 1 N (5) 4 N

$$\tau = I\alpha = RF$$

$$\alpha = \frac{RF}{I} = \frac{RF}{MR^2} = \frac{F}{MR}$$

for hoop

If radius doubles, Force must double to keep α const.

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11. An astronaut in an orbiting spacecraft feels "weightless" because she:

(1) has the same acceleration as the spacecraft
(2) is pulled outwards by centrifugal force
(3) has no acceleration
(4) is beyond the range of gravity
(5) is outside the Earth's atmosphere

Weightlessness is the same as "free fall."

12. A simple pendulum has a period of 8.0 s on earth. What is the period of this pendulum on a planet with 5.0 times the Earth's mass and 2.0 times its radius?

(1) 7.2 s (2) 5.1 s (3) 8.9 s (4) 12.6 s (5) 18.3 s

Pendulum $T = 2\pi\sqrt{L/g}$ $g = \frac{GM}{r^2}$

for both Earth and planet

$$\frac{T_P}{T_E} = \sqrt{\frac{g_E}{g_P}} = \sqrt{\frac{M_E}{r_E^2}\frac{r_P^2}{M_P}} = \frac{r_P}{r_E}\sqrt{\frac{M_E}{M_P}}$$

$= 7.16\,s$

13. A particle moves in simple harmonic motion with a frequency of 0.60 Hz and an amplitude of 10 cm. What is the speed of the particle when it is at a point 6.0 cm from the equilibrium position?

(1) 0.3 m/s (2) 0.008 m/s (3) 30 m/s (4) 0.8 m/s (5) 0.008 m/s

write

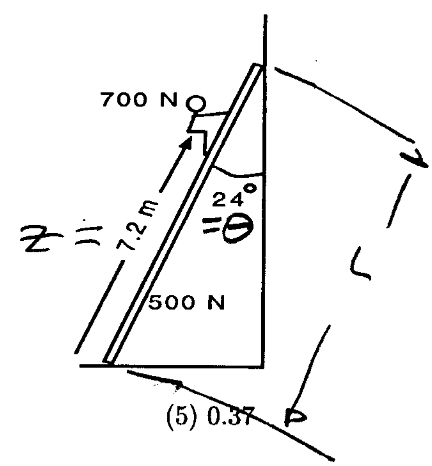
$x = A\sin(\omega t)$ $\omega = 2\pi f$

$\omega t_0 = \sin^{-1}\left(\frac{x_0}{A}\right)$ $x_0 = 6\text{cm}$ $A = 10\text{cm}$

$v = \omega A\cos(\omega t_0) = 2\pi f A\cos\left[\sin^{-1}\left(\frac{x_0}{A}\right)\right]$

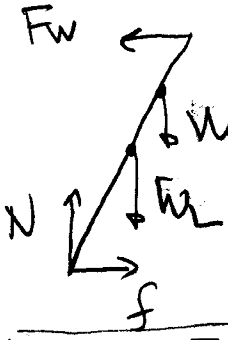
$= 0.30\ \text{m/s}$ $(30\ \text{cm/s})$

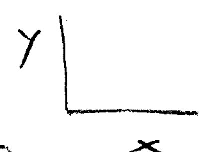
14. A uniform 10 m long ladder weighing 500 N rests against a smooth (assumed frictionless) vertical wall. The angle the ladder makes with the wall is 24°. When a 700 N person climbs the ladder, the ladder begins to slip when the person has climbed 7.2 m *along* the ladder. Find the coefficient of static friction, μ_s, between the ladder and the floor.



(1) 0.28 (2) 0.32 (3) 1.41 (4) 0.63 (5) 0.37

Free body diagram;





$\Sigma F_y = 0 = N - W_L - W_p$

$\Sigma F_x = 0 = f - F_W$

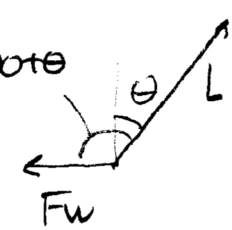
$f = \mu_s N = \boxed{\mu_s(W_L + W_p) = F_W} \Rightarrow \boxed{\mu_s = \frac{F_W}{W_L + W_p}}$

$\Sigma\tau = 0$ any point; use base

$\tau_p = -\ z W_p \sin(180 - \theta)$ ← $\sin\theta$

$\tau_L = -\ \frac{L}{2} W_L \sin(180 - \theta)$

$\tau_W = +\ L F_W \sin(90 + \theta)$ ← $\cos\theta$



$0 = LF_W\cos(\theta) - (zW_p + \frac{L}{2}W_L)\sin(\theta)$

$\mu_s = \frac{(zW_p + \frac{L}{2}W_L)\sin(\theta)}{LF_W\cos(\theta)(W_L + W_p)}$

$= 0.28$

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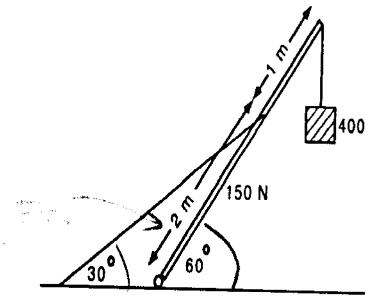
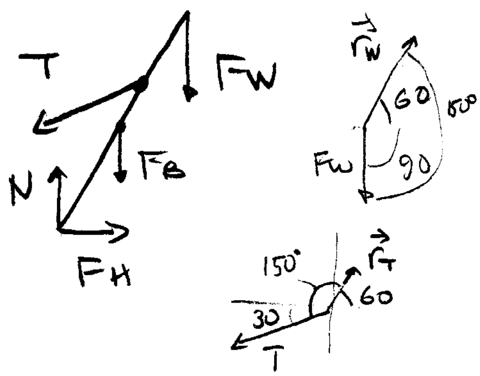
15. A 3 m long uniform beam weighing 150 N is maintained in static equilibrium by a rope attached 1 m from its upper end and by a hinge at its bottom end. A 400 N weight is attached to the upper end of the beam, as shown in the figure. Find the tension in the rope connecting the beam and the ground.



$= F_W$

$L = 3m$

(1) 712 N (2) 350 N (3) 750 N (4) 411 N (5) 866 N



$\Sigma \vec{\tau} = 0$ ($\Sigma \vec{F} = 0$ too but not needed)

$= \vec{\tau}_W + \vec{\tau}_B + \vec{\tau}_T$ (about hinge)

$\vec{\tau}_W = \vec{r}_W \times \vec{F}_W = -L F_W \sin(150°)$

$\vec{\tau}_B = \vec{r}_B \times \vec{F}_B = -\frac{L}{2} F_B \sin(150°)$

$\vec{\tau}_T = \vec{r}_T \times \vec{T} = +\frac{2}{3} L T \sin(150°)$

$0 = \frac{2}{3} L T \sin(150) - \frac{L}{2} F_B \sin(150) - L F_W \sin(150)$

$\frac{2}{3} T = \frac{1}{2} F_B + F_W$; $T = 712$ N

16. A 1.80 kg particle undergoing simple harmonic motion along the y (vertical) axis oscillates 250 times per minute. When a measurement of its motion is started (i.e., at $t = 0$) the particle is passing through its equilibrium position moving upwards. The particle is found to be located 31.0 cm above the equilibrium point 28.0 ms later. What kinetic energy did the particle have when it passed through its equilibrium position?

(1) 132 J (2) 1040 J (3) 970 J (4) 331 J (5) 114 J

Take $y = A \sin(\omega t)$ so at $t=0$ $y=0$

$v_y = A\omega \cos(\omega t)$ so at $t=0$ $v_y = A\omega > 0$

$K = \frac{1}{2} m v^2 = \frac{m}{2} A^2 \omega^2 \cos^2(\omega t)$

$K_{max} = \frac{m A^2 \omega^2}{2}$

What is A : $x_0 = A \sin(\omega t_0)$ $x_0 = .31$ m $t_0 = 0.028$ s

so $A = \frac{x_0}{\sin \omega t_0}$ with $\omega = 2\pi f = 2\pi\left(\frac{250}{60}\right)$

$K_{max} = \frac{m x_0^2 \omega^2}{2 \sin^2(\omega t_0)} = 132$ J

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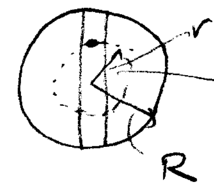
17. Mars has a mass of 6.42×10^{23} kg and a diameter of 6.79×10^{6} m. Engineers design a train connecting the north and south poles via a straight tunnel through the center of the planet. The train moves only because of the gravitational attraction of the planet, and friction is negligible. How long is the North Pole-South Pole transit time? (Assume the density of Mars is uniform.)

(1) 0.83 hours (2) 1.35 hours (3) 2.4 hours (4) 4.3 hours (5) 5.3 hours

Only mass "inside" counts

$$F = -\frac{GM'm}{r^2} = -\frac{GMm}{R^3} r = -kr$$



$$M' = \rho \frac{4\pi r^3}{3} \qquad \rho = \frac{3M}{4\pi R^3} \qquad M' = M\frac{r^3}{R^3}$$

SHO

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{R^3}{GM}}$$

Transit time is $T/2$

$$\text{transit} = \pi\sqrt{\frac{R^3}{GM}} = 0.83\,\text{hr} = \pi\sqrt{\frac{(3.4\times10^{6})^3}{(6.6\times10^{-11})(6.4\times10^{23})}}$$

18. You are waiting at the train station and a train is approaching at 88 km/hr. If it blows a whistle which has a frequency of 3300 Hz in the rest frame of the train, what frequency do you hear? (The speed of sound in air is 330 m/s.)

(1) 3564 Hz (2) 3052 Hz (3) 3105 Hz (4) 3157 Hz (5) 3262 Hz

Will be a higher frequency.

$$f' = f\frac{c}{c-v} = 3564\,\text{Hz}$$

$c = 330$ m/s

$v = 88$ km/hr $= 24.4$ m/s

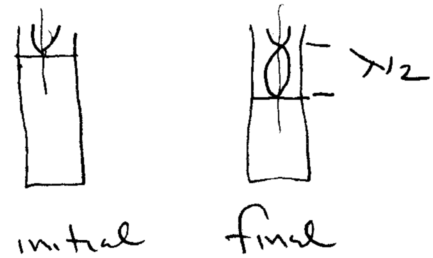
19. A tuning fork is held over the opening of a long vertical pipe partially filled with water. The fork's resonant frequency of 880 Hz matches the fundamental frequency of the pipe filled with water at this level. The water level is now lowered slowly until the pipe resonates again. Taking the speed of sound in air to be 330 m/s, how far was the water lowered?

(1) 0.19 m (2) 0.21 m (3) 0.23 m (4) 0.25 m (5) 0.27 m

$\Delta z = \frac{\lambda}{2} = \frac{c}{2f}$

$c = 330$ m/s

$\Delta z = 0.19$ m



20. A weight is hung over a pulley and attached to a string composed of two parts. Each part of the string has the same diameter, but string 1 is composed of a material having 1.5 times the mass density of string 2. What is the ratio v_1/v_2 of waves on the strings?

(1) 0.82 (2) 0.66 (3) 1.5 (4) 1.22 (5) 2.00

$V = \sqrt{\frac{T}{\mu}}$

T is the same

$\mu = \frac{M}{\ell} = \frac{\rho V}{\ell} = \rho A$

$V = \sqrt{\frac{T}{\rho A}}$

$\frac{V_1}{V_2} = \sqrt{\frac{\cancel{T_1}}{\rho_1 \cancel{A_1}} \frac{\rho_2 \cancel{A_2}}{\cancel{T_2}}} = \sqrt{\frac{\rho_2}{\rho_1}} = 0.82$