Chapter 9 - 12 review

Center of Mass $m_{com}=\ldots$, $ma=F$

Momentum: $p=mv$, $dp=Fdt$, *conservation*

Collisions: using momentum and kinetic energy conservation

Rotational kinematics

Moment of inertia $I=\ldots$, **parallel axis theorem** $I=I_{com}+mh^2$

Torque $\tau=r\times F$, $\tau=I\alpha$

Rolling: ω **wrt center =** ω **wrt touching point**

Angular momentum: $L=r\times p$, $dL=\tau dt$, *conservation*

Using angular momentum conservation

Gyroscope precession

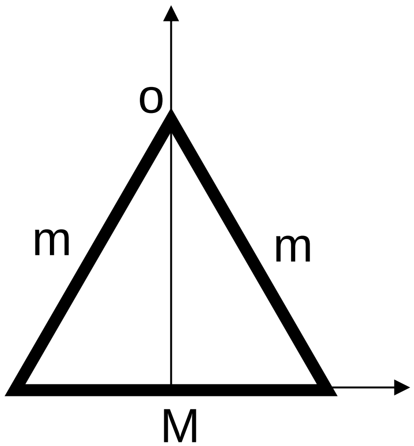
Equilibrium

Finding COM, Moment of Inertia

-**COM:** $\vec{r}_{com} = \frac{1}{m_{TOT}} \sum m_i \vec{r}_i$

-**Use symmetry**

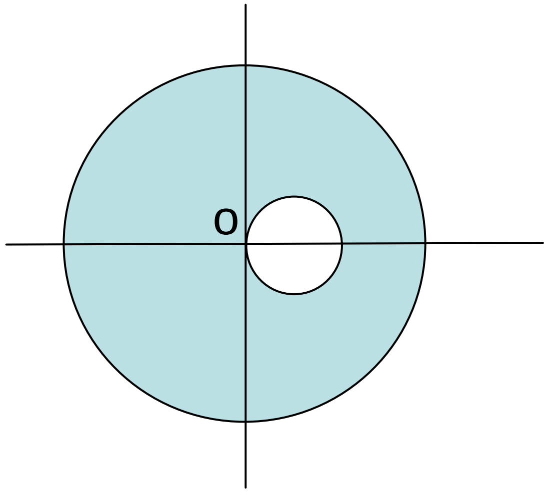
-**Break up object in simpler parts**



$$y_{com} = \frac{2m}{M+2m}\frac{\sqrt{3}L}{4}$$

$$I_0 = 2\left(\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2\right) + \left(\frac{1}{12}ML^2 + M\left(\frac{\sqrt{3}L}{2}\right)^2\right)$$

- **Moment of Inertia:** $I = \sum m_i r_i^2$
- **Use parallel axis theorem**
- **Break up object in simpler parts**



$$x_{com} = -\frac{r^2}{R^2 - r^2} r$$

$$I_0 = \frac{1}{2}MR^2 - \left(\frac{1}{2}mr^2 + mr^2\right) = \frac{1}{2}\left(\pi R^2 \rho - \pi r^2 \rho\right)R^2 - \frac{3}{2}\left(\pi r^2 \rho\right)r^2$$

Momentum

Gun of mass M fires a bullet of mass m with velocity v.

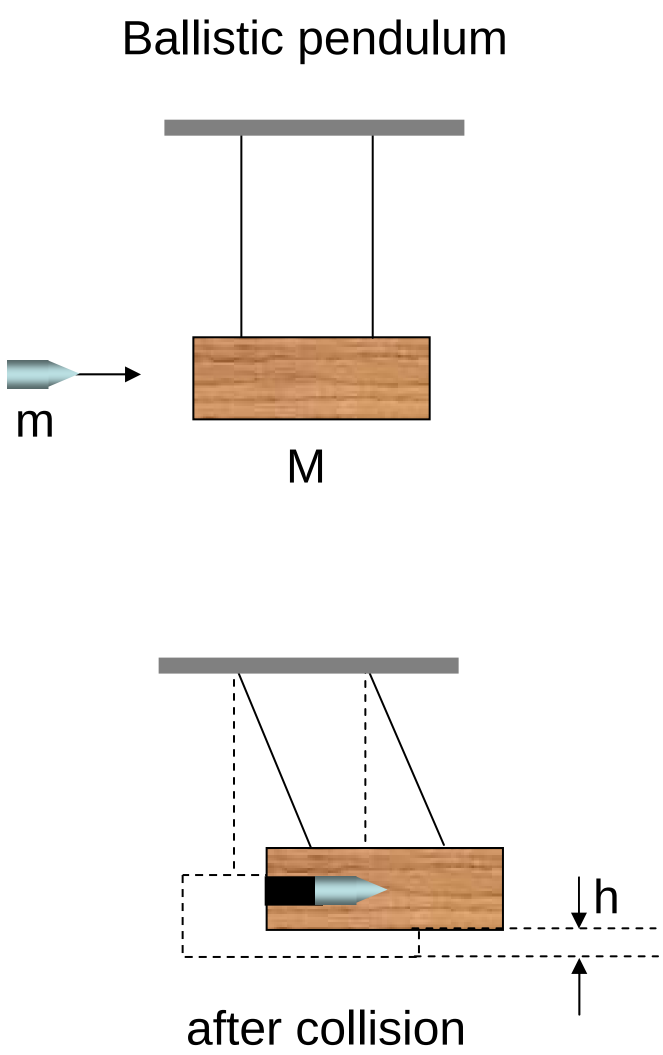
Find recoiling velocity of gun u.

Find the force F on your shoulder, if your shoulder deforms by distance d as it stops the recoil. Assume that the force, while it acts, is constant.

$$u = \frac{m}{M} v$$

$$F = \left(\frac{mv^2}{2} \right) \frac{1}{d} \frac{m}{M}$$

Inelastic collision



Given you know the maximum height of the block as it swings to the right, find initial velocity of a bullet.

Elastic collisions

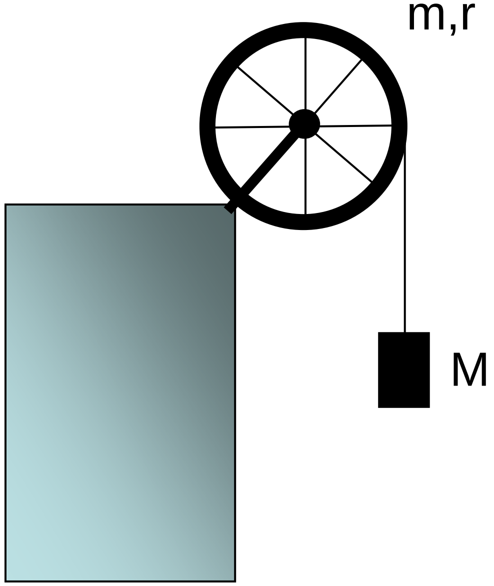
A large ball (#1) of mass M and moving with velocity v along an x-axis collides elastically with a small ball (#2) of mass m at rest.

Find velocities of the two balls after the collision.

$$u_1 = \frac{M - m}{M + m} v$$

$$u_2 = \frac{2M}{M + m} v$$

Rotation

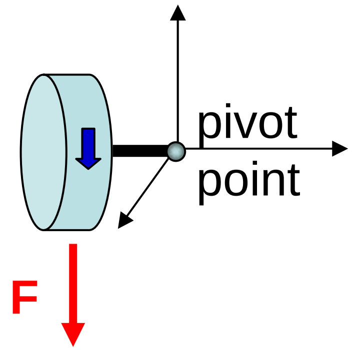


Find tension in the rope T, as the rope wound on the wheel of mass m and radius r unwinds under the weight M

$$T = \frac{Mg}{1 + \frac{Mr^2}{I}} = \frac{Mm}{M + m} g$$

Gyroscope

$$\Omega = \frac{\tau}{L} = \frac{rF_{\perp}}{I\omega}$$



The gyroscope (disk) spins around its axis and is free to rotate around the pivot point.

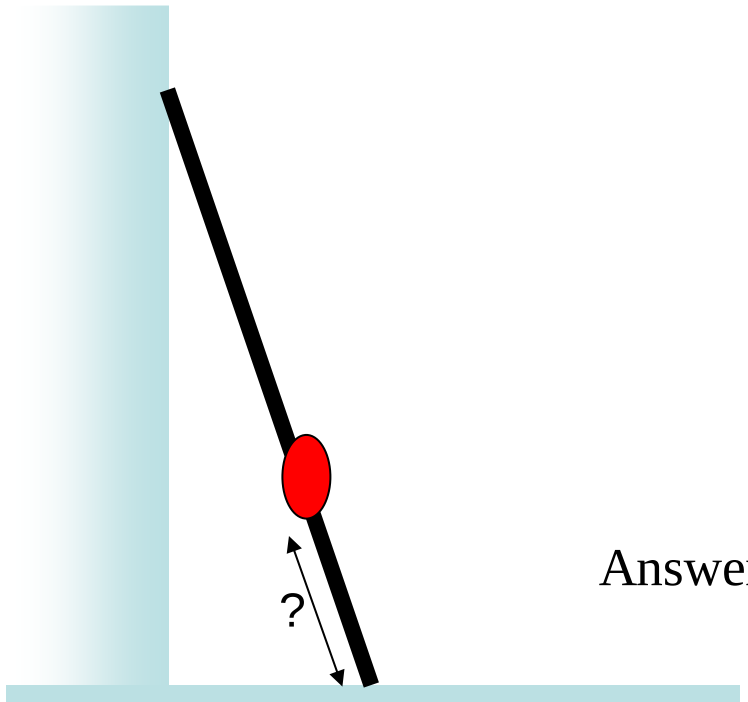
Direction of spinning is shown with ➡
Force is applied downward in plane of the screen

Which way will the gyroscope start precessing?

Away from you into the screen (around z-axis)

Equilibrium

A ladder of length L makes an angle 30° with a wall.
Max static friction coefficient between the ladder and the floor μ_{max}=0.25 and there is no friction between the ladder and the wall.
How far can you climb the ladder before it starts sliding along the floor?



Answer: $$l_{\max} = \mu_{\max}\frac{\cos\alpha}{\sin\alpha}L = 0.25\sqrt{3}L \approx 0.4L$$