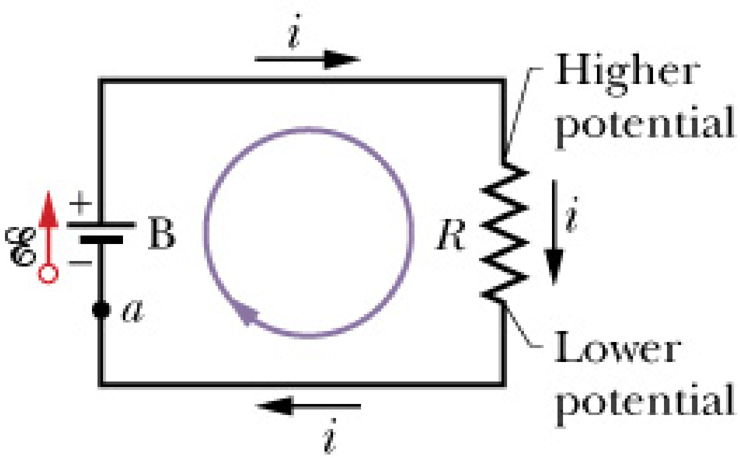
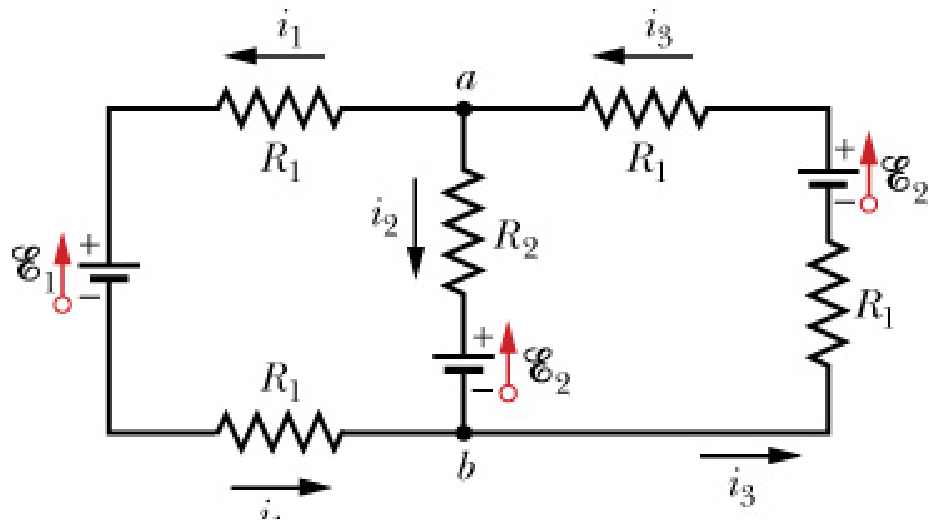
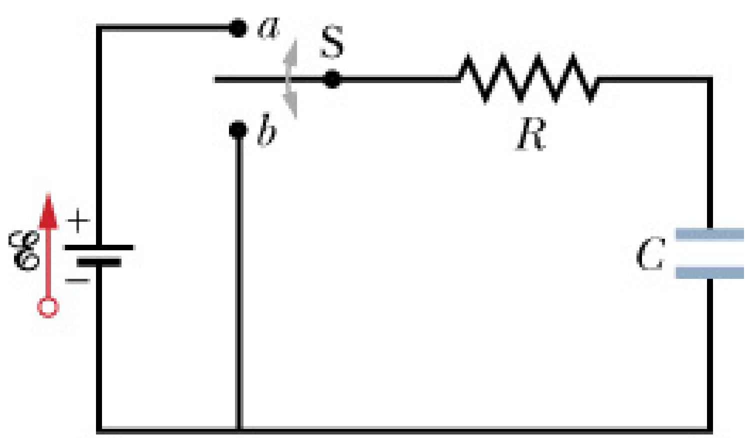
Circuits



i
Higher potential
+
−
ℰ
B
a
R
i
Lower potential
i



i_1
i_3
a
R_1
R_1
+
−
$ℰ_2$
$ℰ_1$
i_2
R_2
R_1
R_1
$ℰ_2$
b
i_3



a
S
b
R
ℰ
+
−
C

What You Already Know

- ➔Nature of current
- ➔Current density
- ➔Drift speed and current
- ➔Ohm's law
- ➔Conductivity and resistivity
- ➔Calculating resistance from resistivity
- ➔Power in electric circuits

Chapter 27: Electric Circuits

- ➔Work, energy and EMF
- ➔Single loop circuits
- ➔Multiloop circuits
- ➔Ammeters and voltmeters
- ➔RC circuits and time constant

Reading Quiz for Chapter 27

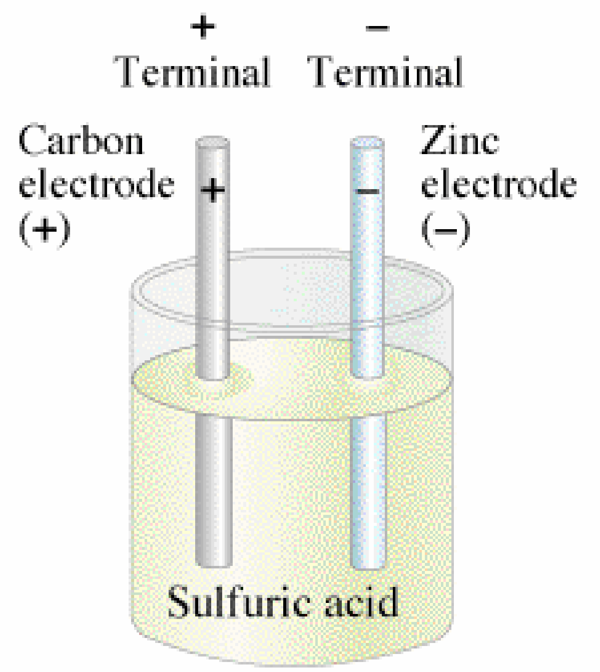
- ➔ The electric current is defined as:
  - ◆ (1) amount of charge per time
  - ◆ (2) amount of charge per area
  - ◆ (3) amount of charge per volume
- ➔ When resistors are connected in series
  - ◆ (1) the current in each resistor is different
  - ◆ (2) the current in each resistor is the same
  - ◆ (3) the voltage in each resistor is the same
- ➔ Which of the following is not related to Kirchhoff's Rules?
  - ◆ (1) conservation of charge
  - ◆ (2) conservation of energy
  - ◆ (3) conservation of momentum

EMF

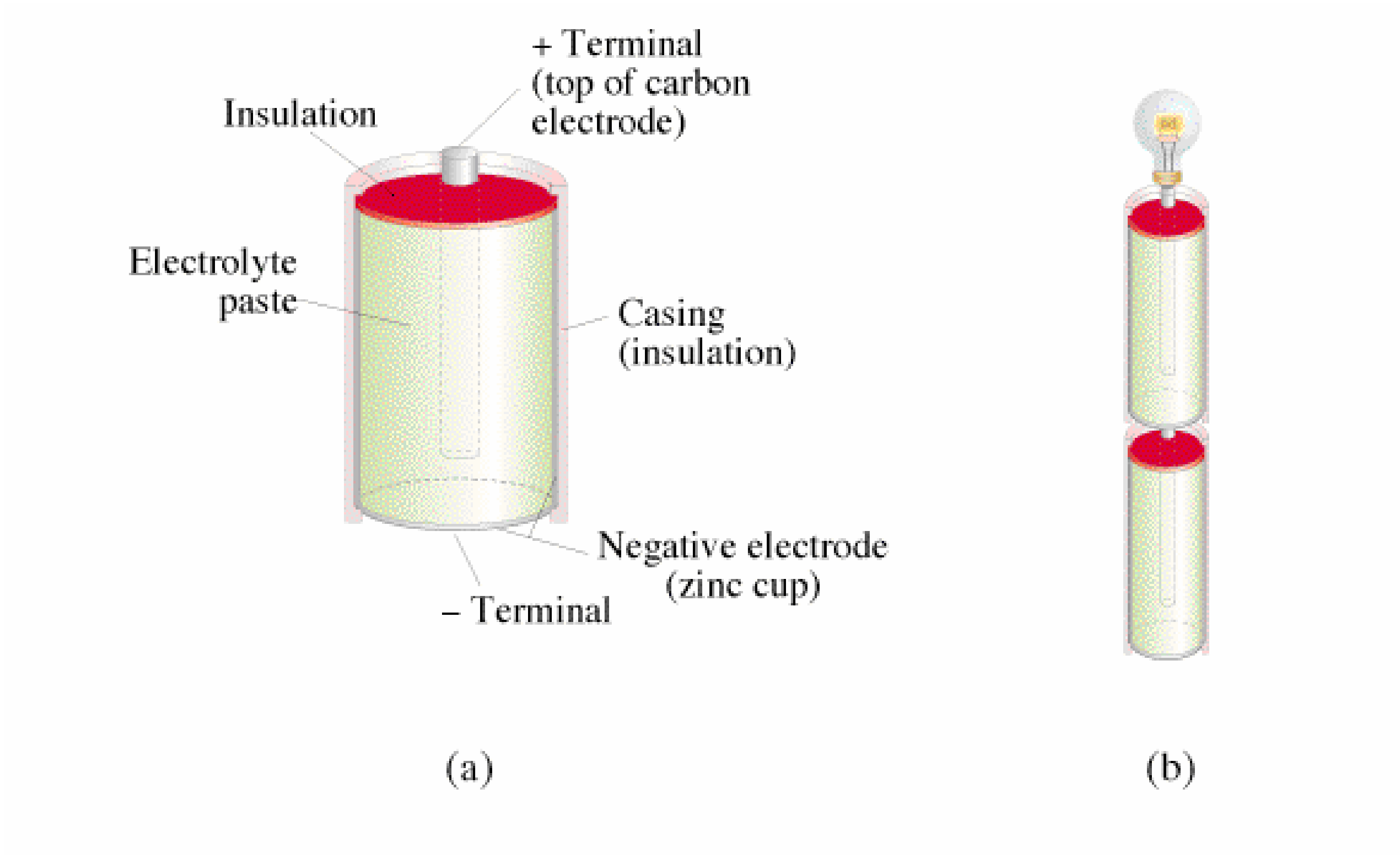
- ➔EMF device performs work on charge carriers
  - ◆Converts energy to electrical energy
  - ◆Moves carriers from low potential to high potential
  - ◆Maintains potential across terminals
- ➔Various types of EMF devices

◆Battery	Electrolytic reaction
◆Generator	Magnetic field
◆Fuel cell	Oxidation of fuel
◆Solar cell	Electromagnetic energy
◆Thermopile	Nuclear decay

- ➔Example: battery
  - ◆Two electrodes (different metals)
  - ◆Immersed in electrolyte (dilute acid)
  - ◆One electrode develops + charge, the other – charge



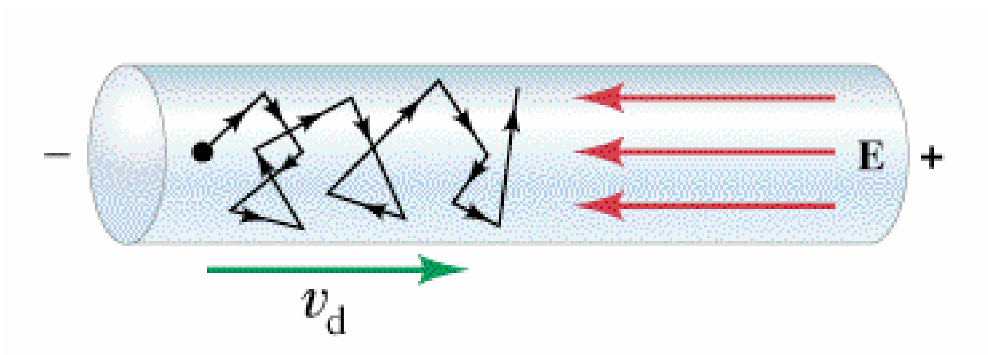
Common dry cell battery



Electrons in the wire

➔ If the electrons move so slowly through the wire, why does the light go on right away when we flip a switch?

- ◆ Household wires have almost no resistance
- ◆ The electric field inside the wire travels much faster
- ◆ Light switches do not involve currents
- ◆ None of the above



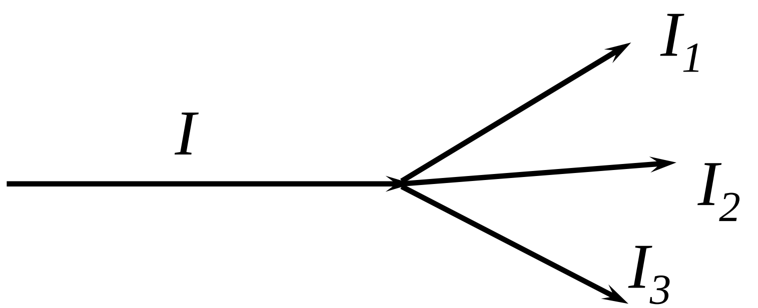
Electrons in the wire, part 2

➔ Okay, so the electric field in a wire travels quickly. But, didn't we just learn that E = 0 inside a conductor?

- True, it can't be the electric field after all!!
- The electric field travels along the outside of the conductor
- E = 0 inside the conductor applies only to static charges
- None of the above

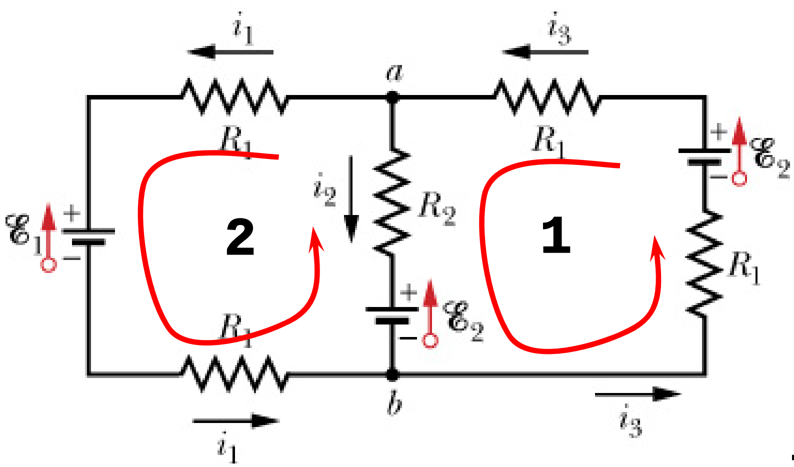
Kirchhoff's Rules

- Junction rule (conservation of charge)
  - Current into junction = sum of currents out of it



$$I = I_1 + I_2 + I_3$$

- Loop rule (conservation of energy)
  - Algebraic sum of voltages around a closed loop is 0

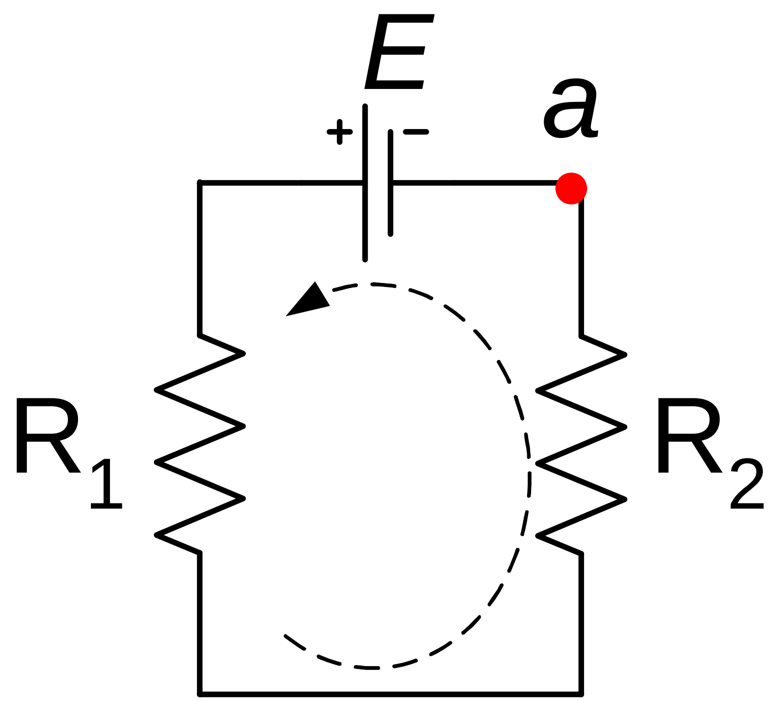


$$-I_3R_1 - I_2R_2 - E_2 - I_3R_1 + E_2 = 0$$

$$-I_1R_1 - E_1 - I_1R_1 + E_2 + I_2R_2 = 0$$

Finding Current in Series Circuit

- ➔Define current direction
- ➔Start at some position (a)
  - ◆ Define its potential to be 0
- ➔Move in direction of current, adding voltage sources & drops
  - ◆ Voltage source: +E
  - ◆ Resistor 1: $-iR_1$
  - ◆ Resistor 2: $-iR_2$
  - ◆ Total voltage change is 0



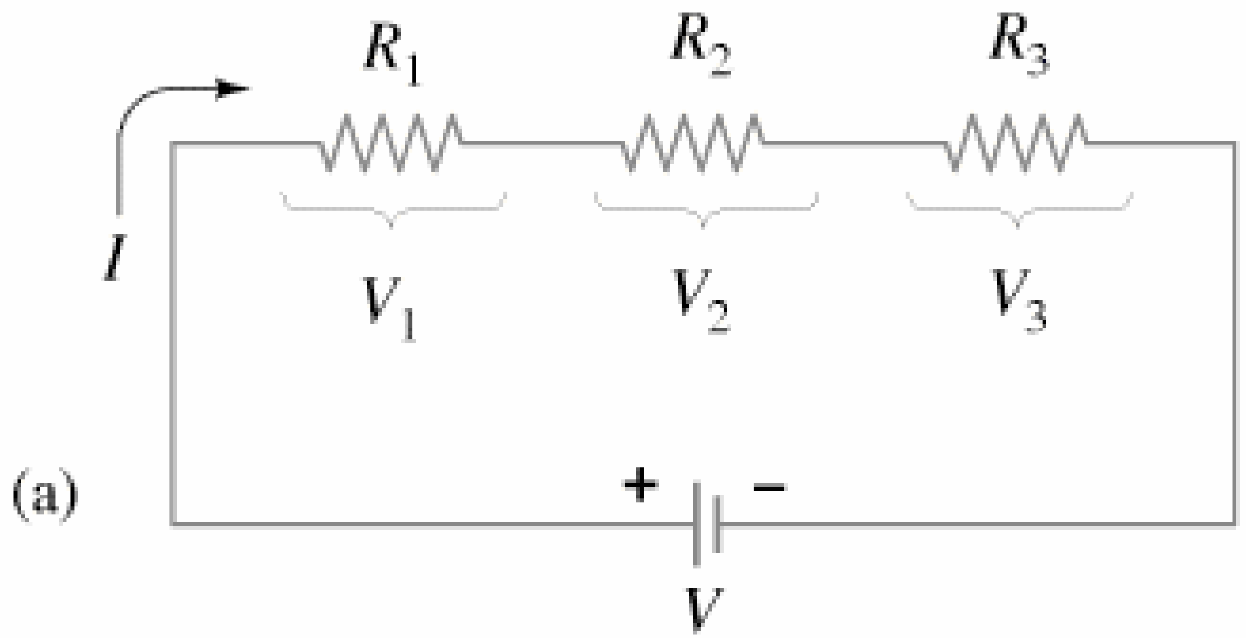
$$0 + E - iR_1 - iR_2 = 0$$

$$i = \frac{E}{R_1 + R_2}$$

Resistors in series

➔EMF of battery is 12 V, 3 identical resistors. What is the potential difference across each resistor?

- ◆12 V
- ◆0 V
- ◆3 V
- ◆4 V



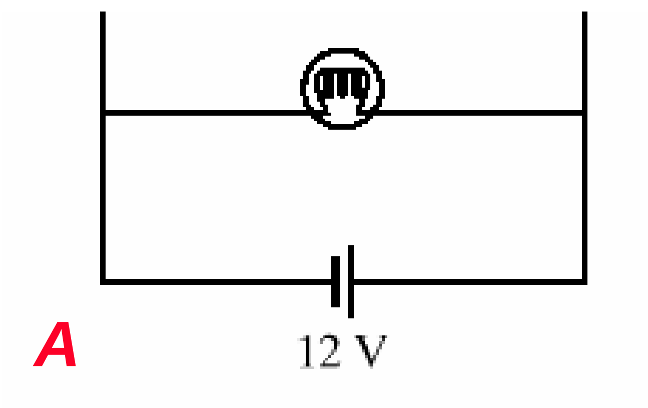
Resistors in series

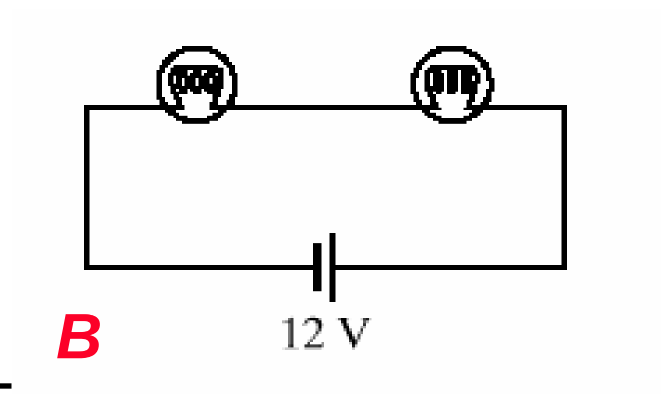
➔ If the light bulbs are all the same in each of these two circuits, which circuit has the higher current?

- circuit A
- circuit B
- both the same

➔ In which case is each light bulb brighter?

- circuit A
- circuit B
- both the same



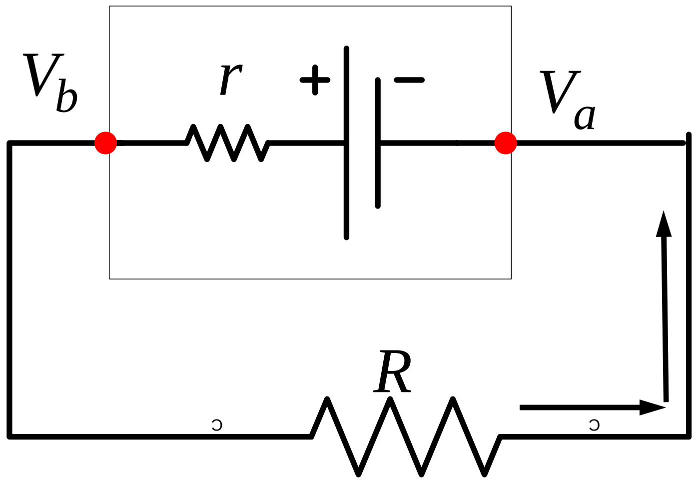


Real EMF Sources: Internal Resistance

➔ Real batteries have small internal resistance

- ◆ Lowers effective potential delivered to circuit

$$i = \frac{E}{r + R}$$



$$V_{\text{eff}} = V_b - V_a = E - ir$$

$$= E - \frac{E}{r + R} r$$

$$V_{\text{eff}} = \frac{E}{1 + r/R}$$

This is the voltage measured across the terminals!

Internal Resistance Example

➔Loss of voltage is highly dependent on load

➔E = 12V, r = 0.1Ω, R = 100Ω $V_{\text{eff}} = 12/1.001 = 11.99V$

- Loss of 0.01V

➔E = 12V, r = 0.1Ω, R = 10Ω $V_{\text{eff}} = 12/1.01 = 11.9V$

- Loss of 0.1V

➔E = 12V, r = 0.1Ω, R = 1Ω $V_{\text{eff}} = 12/1.1 = 10.9V$

- Loss of 1.1V

➔E = 12V, r = 0.1Ω, R = 0.5Ω $V_{\text{eff}} = 12/1.2 = 10.01V$

- Loss of 2.0V

Heating From Internal Resistance

- ➔ Heating of EMF source: $P = i^2 r$
  - ◆ Heating is *extremely* dependent on load

- ➔ $\mathcal{E} = 12\text{V}$, $r = 0.1\Omega$
  - ◆ $R = 100\Omega$ $V_{ba} = 11.99\text{V}$ $I = 0.12\text{ A}$ $P = 0.0014\text{ W}$
  - ◆ $R = 10\Omega$ $V_{ba} = 11.9\text{V}$ $I = 1.19\text{ A}$ $P = 0.14\text{ W}$
  - ◆ $R = 1.0\Omega$ $V_{ba} = 10.9\text{V}$ $I = 10.9\text{ A}$ $P = 11.9\text{ W}$
  - ◆ $R = 0.5\Omega$ $V_{ba} = 10.0\text{V}$ $I = 20\text{ A}$ $P = 40\text{ W}$

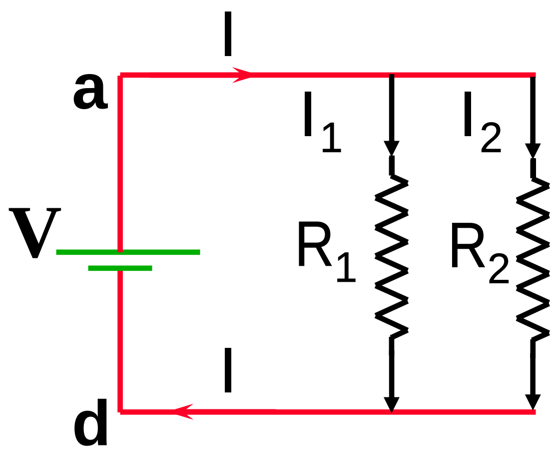
Resistors in Parallel

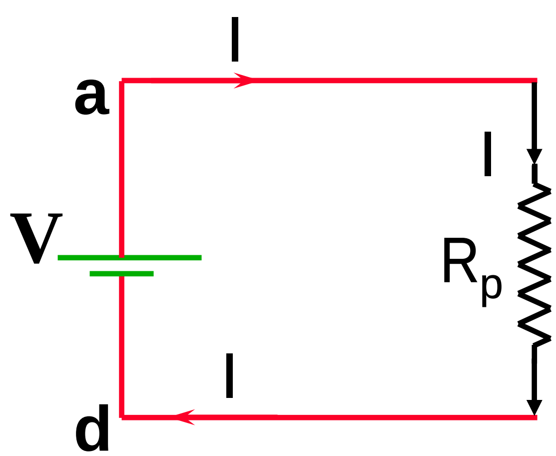
- ➔ Current splits into several branches. Total current is conserved
  - ◆ $I = I_1 + I_2$
- ➔ Potential difference is same across each resistor
  - ◆ $V = V_1 = V_2$

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

R_p = equivalent resistance

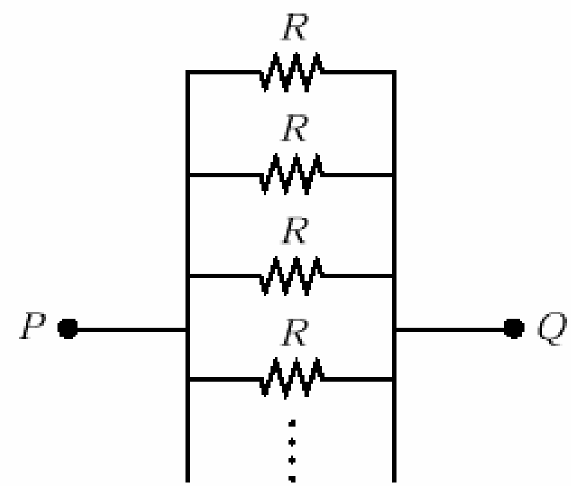




Resistors in Parallel

➔As more resistors R are added in parallel to the circuit, how does total resistance between points P and Q change?

- (a) increases
- (b) remains the same
- (c) decreases



➔If the voltage between P & Q is held constant, and more resistors are added, what happens to the current through each resistor?

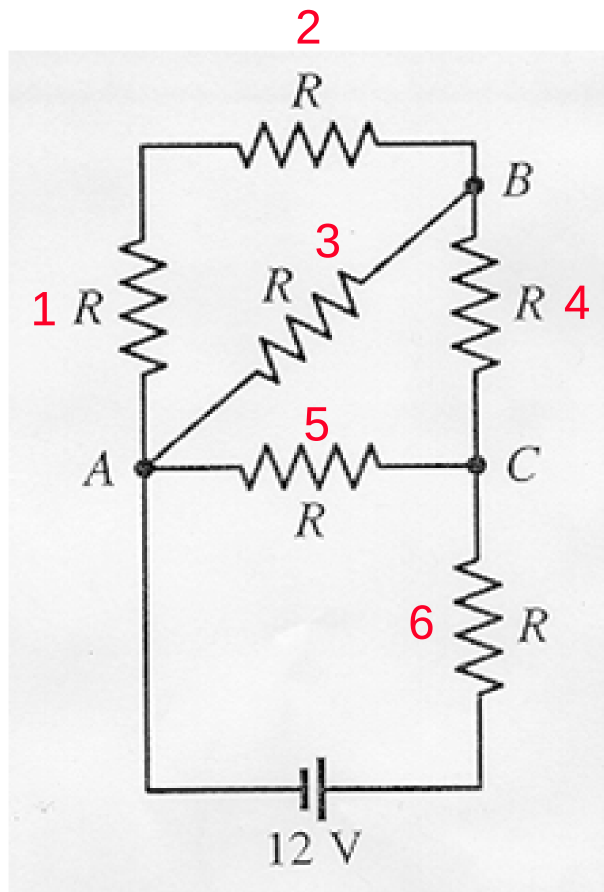
- (a) increases
- (b) remains the same
- (c) decreases

Overall current increases, but current through each branch is still V/R.

Resistance

➔What is the net resistance of the circuit connected to the battery? Each resistance has R = 3 kΩ

- 1&2 in series ⇒ 6kΩ
- 3 in parallel with 1&2 ⇒ 2kΩ
- 4 in series ⇒ 5kΩ
- 5 in parallel ⇒ 15/8kΩ = 1.875kΩ
- 6 in series ⇒ 4.875kΩ



Circuits

➔ If the light bulbs are all the same in each of these two circuits, which circuit has the higher current?

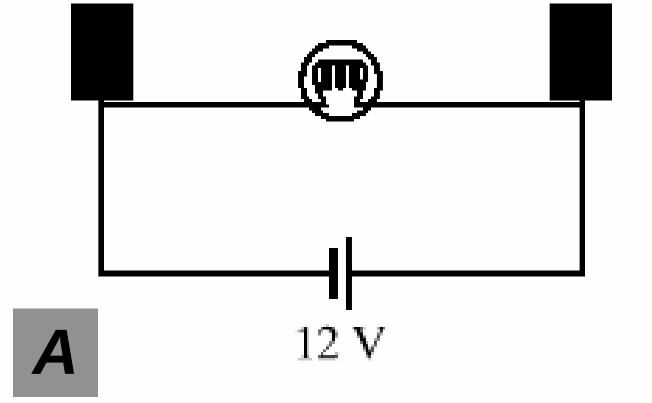
- ◆ (a) circuit A
- ◆ (b) circuit B
- ◆ (c) both the same

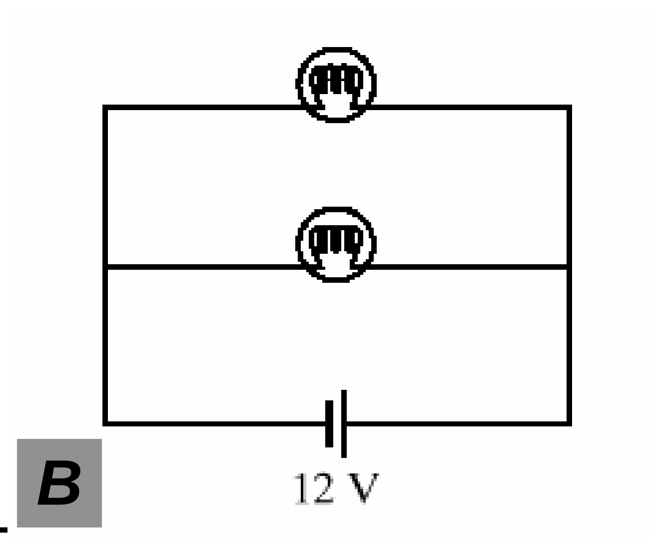
B draws twice the current as A

➔ In which case is each light bulb brighter?

- ◆ (a) circuit A
- ◆ (b) circuit B
- ◆ (c) both the same

Current through each branch is unchanged (V/R)





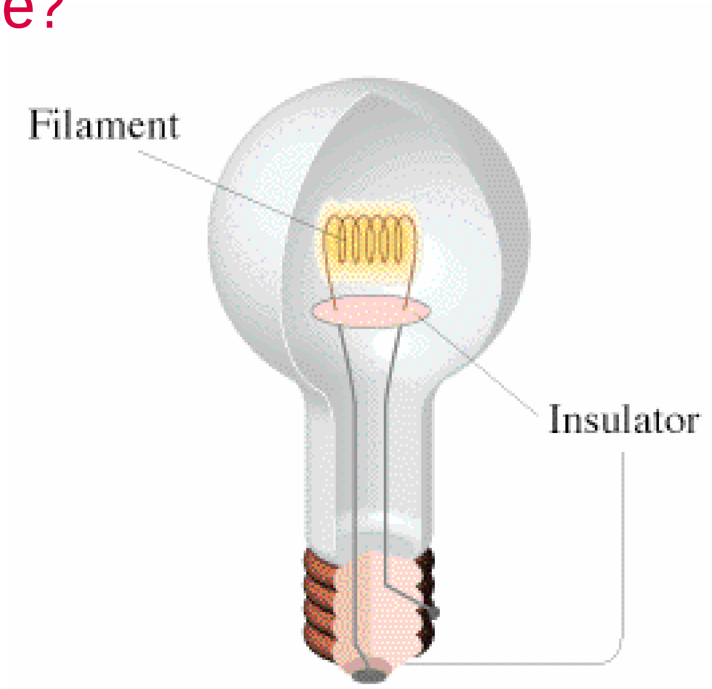
Light Bulb Problem

➔Two light bulbs operate at 120 V, one with a power rating of 25W and the other with a power rating of 100W. Which one has the greater resistance?

- ◆(a) the one with 25 W
- ◆(b) the one with 100 W
- ◆(c) both have the same resistance

➔Which carries the greater current?

- ◆(a) the one with 25 W
- ◆(b) the one with 100 W
- ◆(c) both have the same current



$P = I^2R = V^2/R$, so 100W bulb has ¼ the resistance of the 25W bulb and carries 4x the current.

Dimmer

➔When you rotate the knob of a light dimmer, what is being changed in the electric circuit?

- (a) the voltage
- (b) the resistance
- (c) the current
- (d) both (a) and (b)
- (e) both (b) and (c)

House voltage is always 110 – 120V. Turning the knob increases the circuit resistance and thus lowers the current.

Power lines

- ➔ At large distances, the resistance of power lines becomes significant. To transmit maximum power, is it better to transmit high V, low I or high I, low V?
  - ◆ (a) high V, low I
  - ◆ (b) low V, high I
  - ◆ (c) makes no difference

Power loss is I^2R, so want to minimize current.

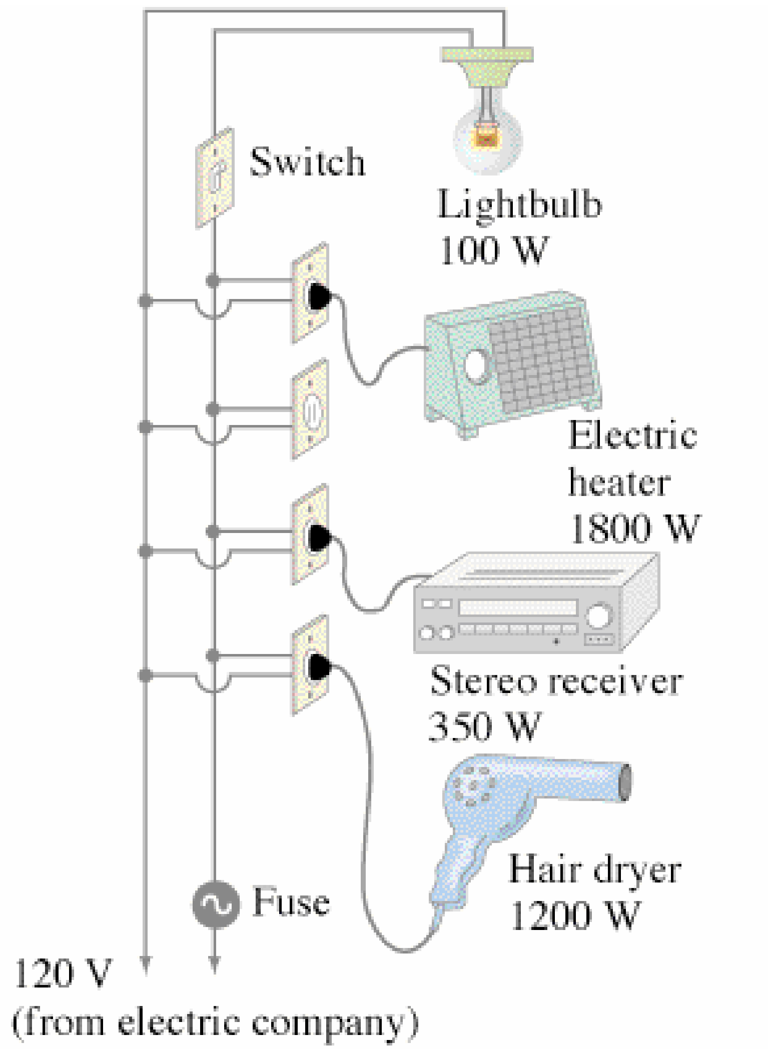
- ➔ Why do birds sitting on a high-voltage power line survive?
  - ◆ They are not touching high and low potential simultaneously to form a circuit that can conduct current

Household Circuits

➔ All devices are added in parallel.

➔ Overload: too many devices that require a lot of current can draw more current than wires can handle.

- ◆ Overheating of wires
- ◆ Fire hazard!

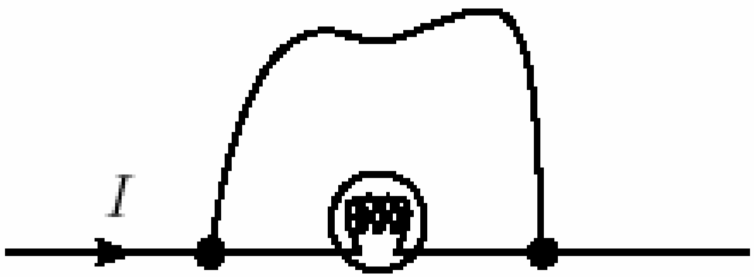


Resistors

➔ Current flows through a light bulb. If a wire is now connected across the bulb as shown, what happens?

- (a) All the current continues to flow through the bulb
- (b) Current splits 50-50 into wire and bulb
- (c) All the current flows through the wire
- (d) None of the above

The wire "shunt" has almost no resistance and it is in parallel with a bulb having resistance. Therefore all the current follows the zero (or extremely low) resistance path.

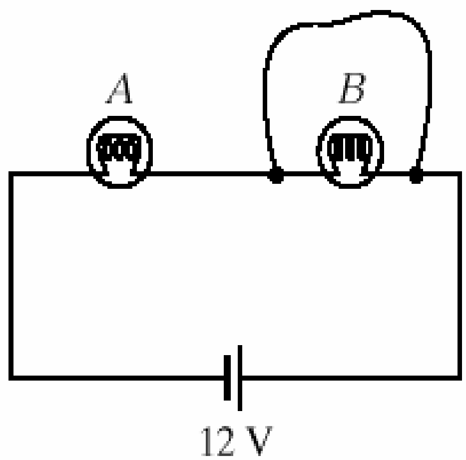


Circuits

➔ Two light bulbs A and B are connected in series to a constant voltage source. When a wire is connected across B, what will happen to bulb A?

- (a) burns more brightly than before
- (b) burns as brightly as before
- (c) burns more dimly than before
- (d) goes out

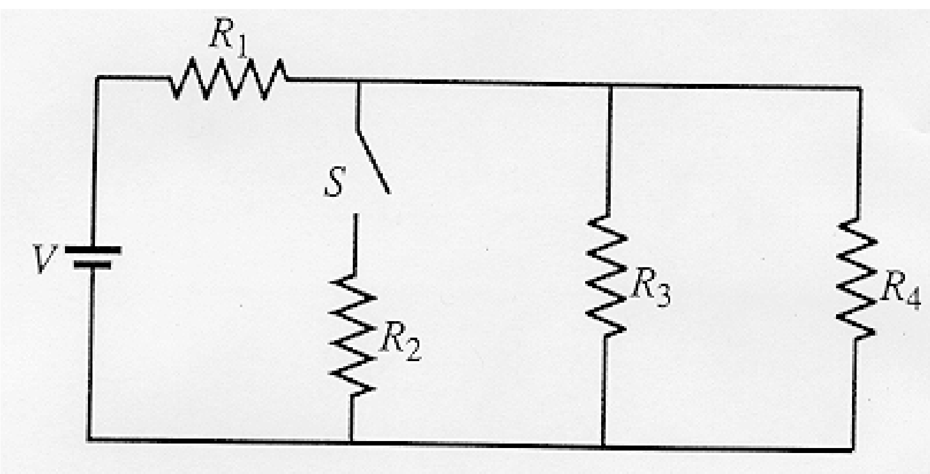
The wire shunt effectively eliminates the second resistance, hence increasing the current in the circuit by 2x. The first bulb burns 4x brighter ($I^2 R$).



Circuits

➔Consider the network of resistors shown below. When the switch S is closed, then:

- ◆What happens to the voltage across R_1, R_2, R_3, R_4? $\uparrow\uparrow\downarrow\downarrow$
- ◆What happens to the current through R_1, R_2, R_3, R_4? $\uparrow\uparrow\downarrow\downarrow$
- ◆What happens to the total power output of the battery? $\uparrow$
- ◆Let $R_1 = R_2 = R_3 = R_4 = 90\ \Omega$ and $V = 45$ V. Find the current through each resistor before and after closing the switch.



Before
- ➢$I_1 = 45/135 = 1/3$
- ➢$I_2 = 0$
- ➢$I_3 = I_4 = 15/90 = 1/6$

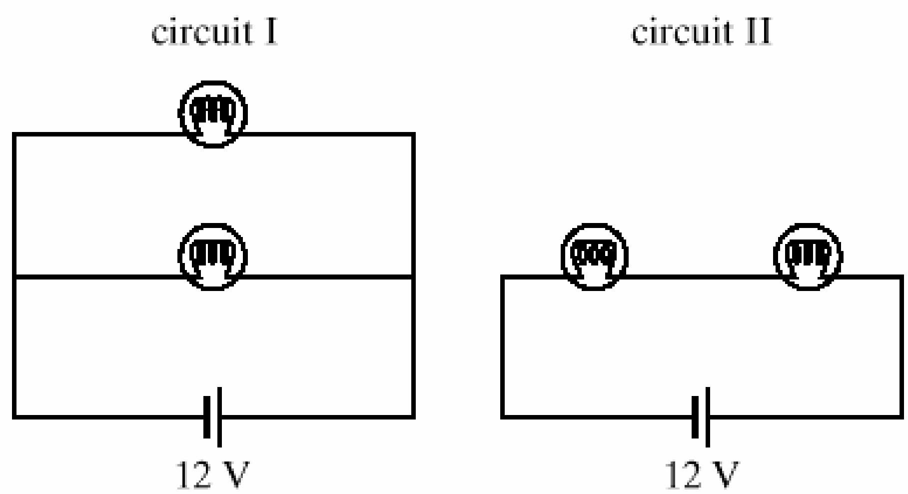
After
- ➢$I_1 = 45/120 = 3/8$
- ➢$I_2 = I_3 = I_4 = 1/8$

Circuits

➔ The light bulbs in the circuits below are identical. Which configuration produces more light?

- (a) circuit I
- (b) circuit II
- (c) both the same

Circuit II has ½ current of each branch of circuit I, so each bulb is ¼ as bright. The total light in circuit I is thus 4x that of circuit II.

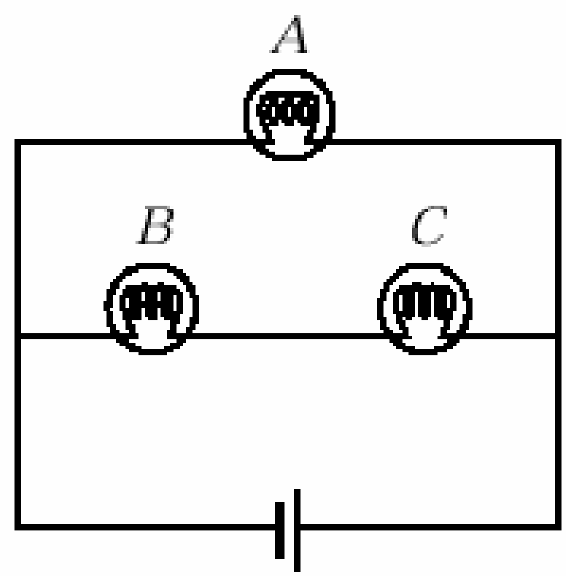


Circuits

➔ The three light bulbs in the circuit are identical. The current flowing through bulb B, compared to the current flowing through bulb A, is

- ◆ a) 4 times as much
- ◆ b) twice as much
- ◆ c) the same
- ◆ d) half as much
- ◆ e) 1/4 as much

Branch of circuit A has ½ resistance, thus it has 2x current.

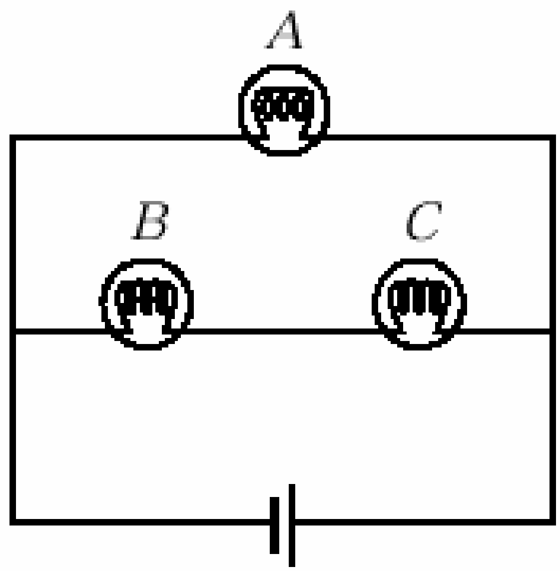


Circuits

➔The three light bulbs in the circuit are identical. What is the brightness of bulb B compared to bulb A?

- a) 4 times as much
- b) twice as much
- c) the same
- d) half as much
- e) 1/4 as much

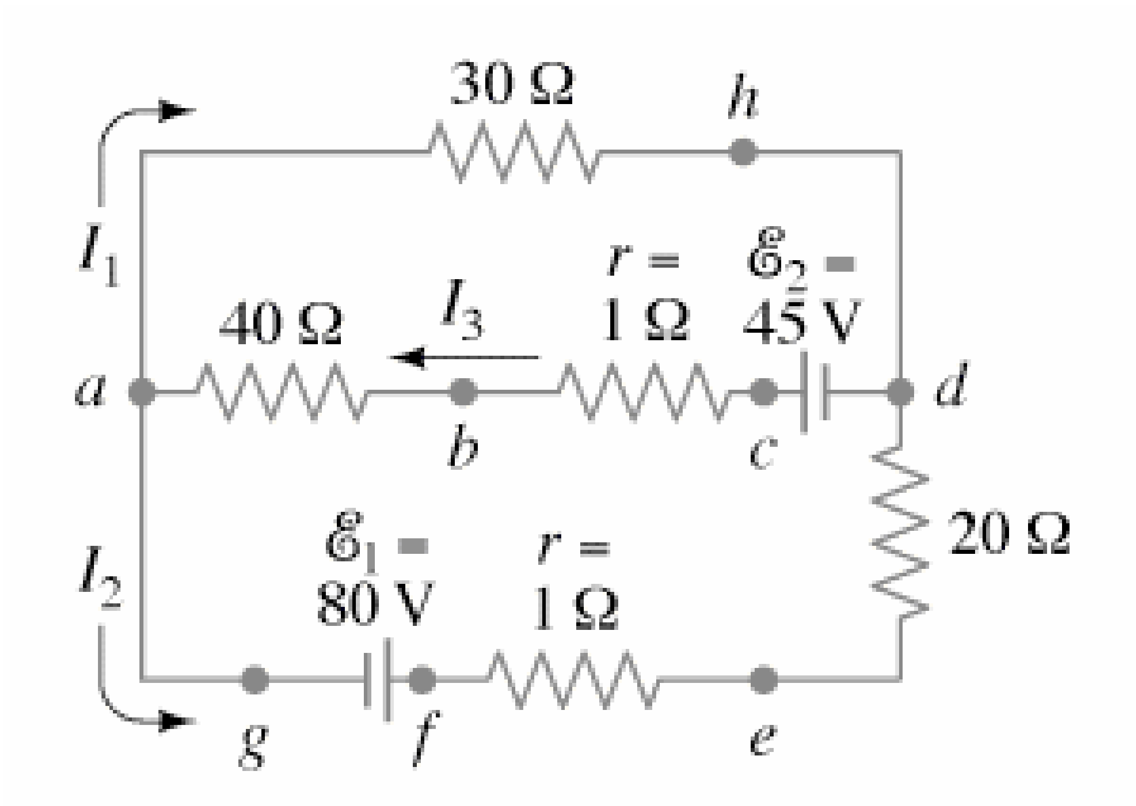
Use $P = I^2R$. Thus 2x current in A means it is 4x brighter.



More Complicated Circuits

➔Parallel and series rules are not enough!

➔Use Kirchoff's rules



Problem Solving Using Kirchhoff's Rules

➔ Label the current in each branch of the circuit
- Choice of direction is arbitrary
- Signs will work out in the end (if you are careful!!)
- Apply the junction rule at each junction
- Keep track of sign of currents entering and leaving

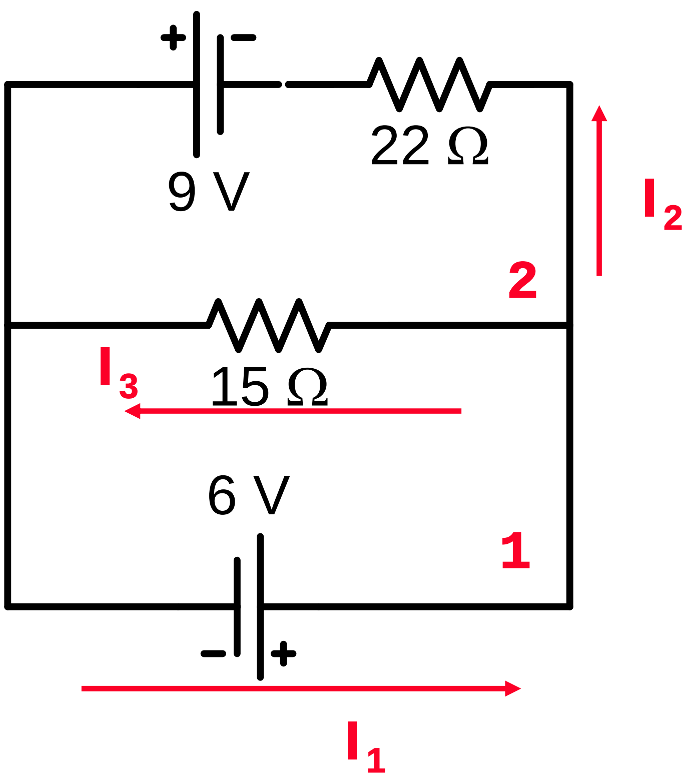
➔ Apply loop rule to each loop (follow in one direction only)
- Resistors: if loop direction matches current direction, voltage drop
- Batteries: if loop direction goes through battery in "normal" direction, voltage gain

➔ Solve equations simultaneously
- You need as many equations as you have unknowns

Kirchhoff's rules

➔ Determine the magnitudes and directions of the currents through the two resistors in the figure below.

- Take two loops, 1 and 2, as shown



Use $I_1 = I_2 + I_3$

1 $$+6 - 15 I_3 = 0$$

2 $$-22 I_2 + 9 + 15 I_3 = 0$$

1 $$I_3 = 6/15 = 0.40$$

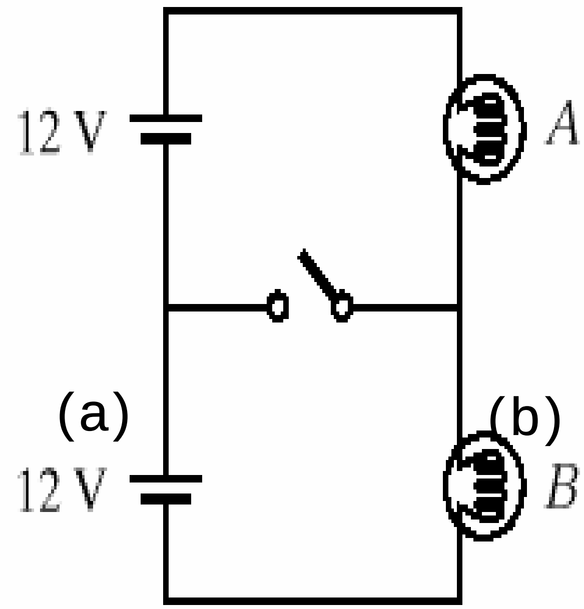
2 $$I_2 = 15/22 = 0.68$$

$$I_1 = I_2 + I_3 = 1.08$$

Circuit Problem (1)

➔The light bulbs in the circuit are identical. What happens when the switch is closed?

- a) both bulbs go out
- b) the intensity of both bulbs increases
- c) the intensity of both bulbs decreases
- d) nothing changes

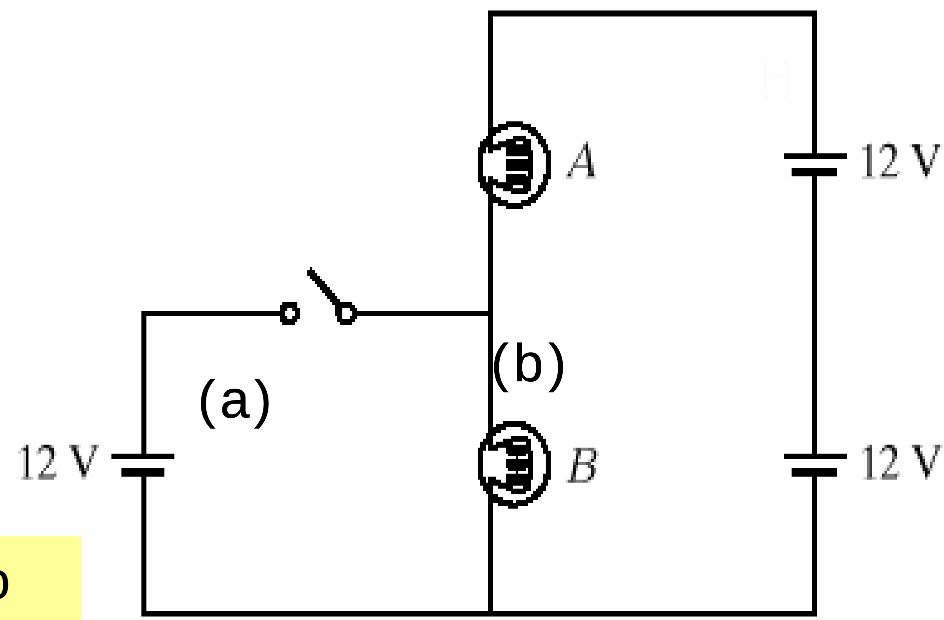


Before: Potential at (a) is 12V, but so is potential at (b) because equal resistance divides 24V in half. When the switch is closed, nothing will change since (a) and (b) are still at same potential.

Circuit Problem (2)

➔The light bulbs in the circuit shown below are identical. When the switch is closed, what happens to the intensity of the light bulbs?

- a) bulb A increases
- b) bulb A decreases
- c) bulb B increases
- d) bulb B decreases
- e) nothing changes

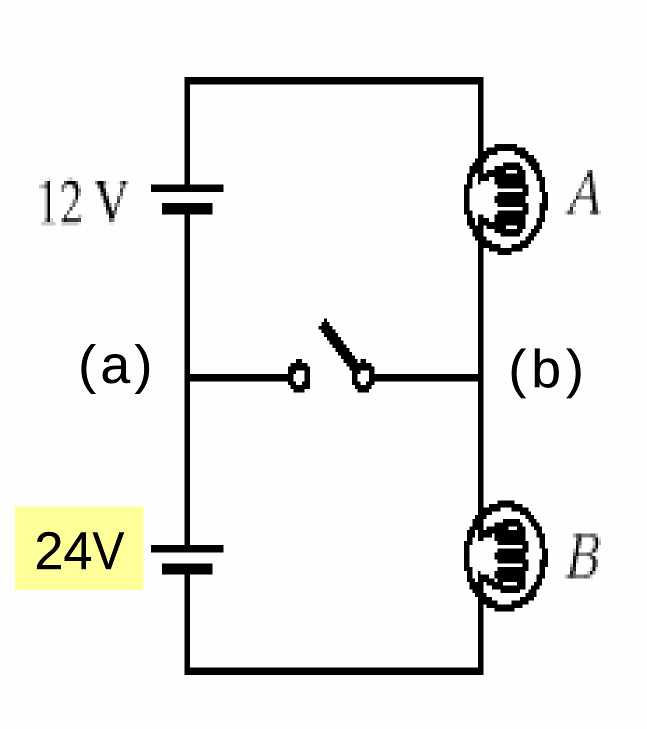


Before: Potential at (a) is 12V, but so is potential at (b) because equal resistance divides 24V in half. When the switch is closed, nothing will change since (a) and (b) are still at same potential.

Circuit Problem (3)

➔ The bulbs A and B have the same R. What happens when the switch is closed?

- a) nothing happens
- b) A gets brighter, B dimmer
- c) B gets brighter, A dimmer
- d) both go out



Before: Bulb A and bulb B both have 18V across them.

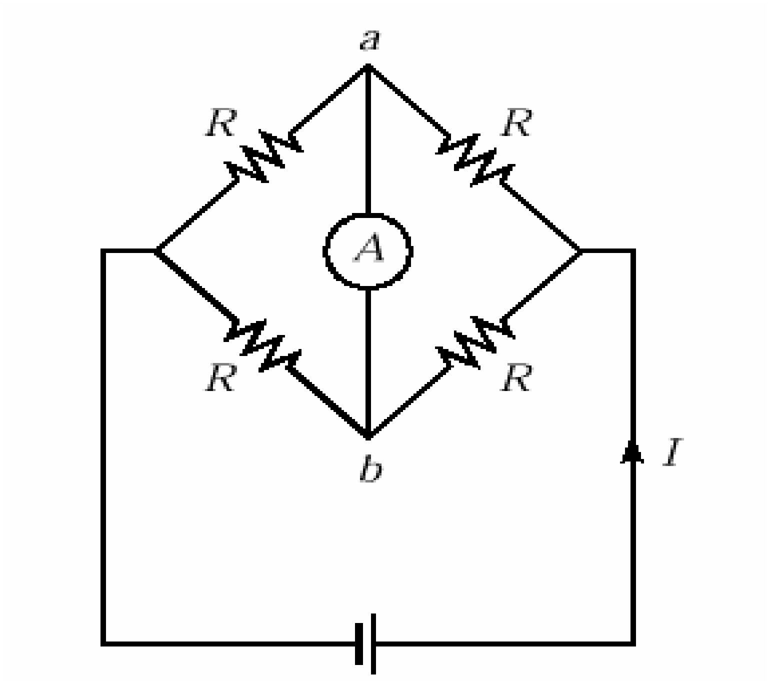
After: Bulb A has 12V across it and bulb B has 24V across it (these potentials are forced by the batteries).

Wheatstone Bridge

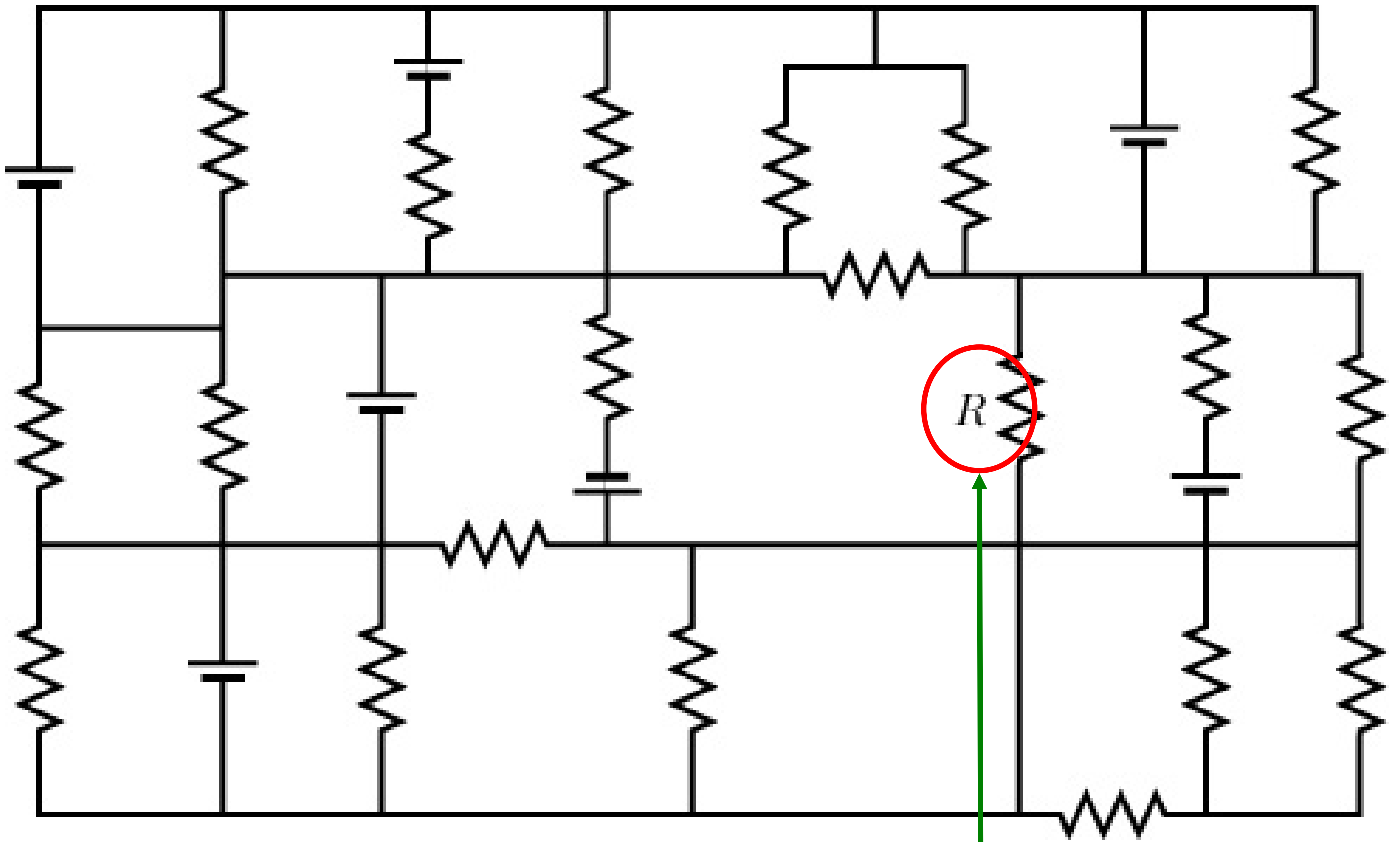
➔An ammeter A is connected between points a and b in the circuit below, in which the four resistors are identical. What is the current through the ammeter?

- a) I / 2
- b) I / 4
- c) zero
- d) need more information

The parallel branches have the same resistance, so equal currents flow in each branch. Thus (a) and (b) are at the same potential and there is no current flow across the ammeter.



Res-Monster Maze (p .725)

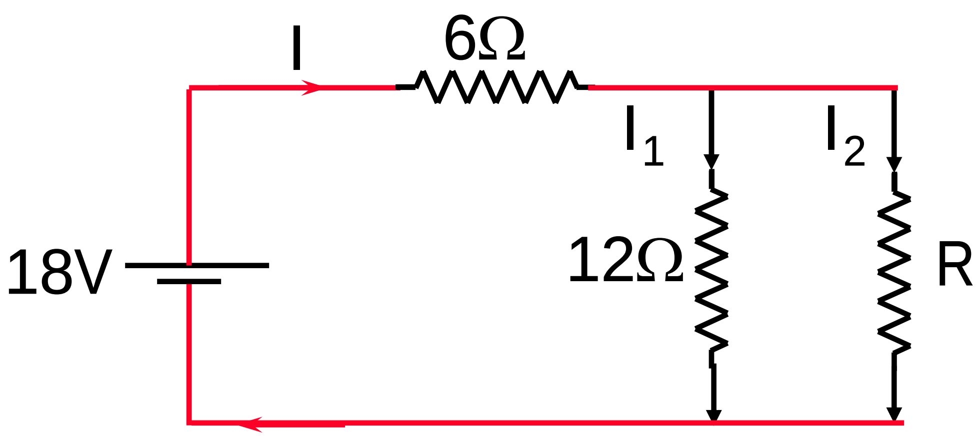


All batteries are 4V
All resistors are 4Ω

Find current in R

Problem solving

➔Find the value of R that maximizes power emitted by R.



$$P_2 = I_2^2 R$$

$$R_{\parallel} = \frac{12R}{12+R}$$

$$I_T = \frac{18}{6+\dfrac{12R}{12+R}} = \frac{12+R}{4+R}$$

$$\frac{I_2}{I_T} = \frac{\frac{1}{R}}{\frac{1}{R}+\frac{1}{12}} = \frac{12}{12+R} \Rightarrow I_2 = \frac{12}{4+R}$$

$$P_2 = I_2^2 R = \frac{144R}{(4+R)^2}$$

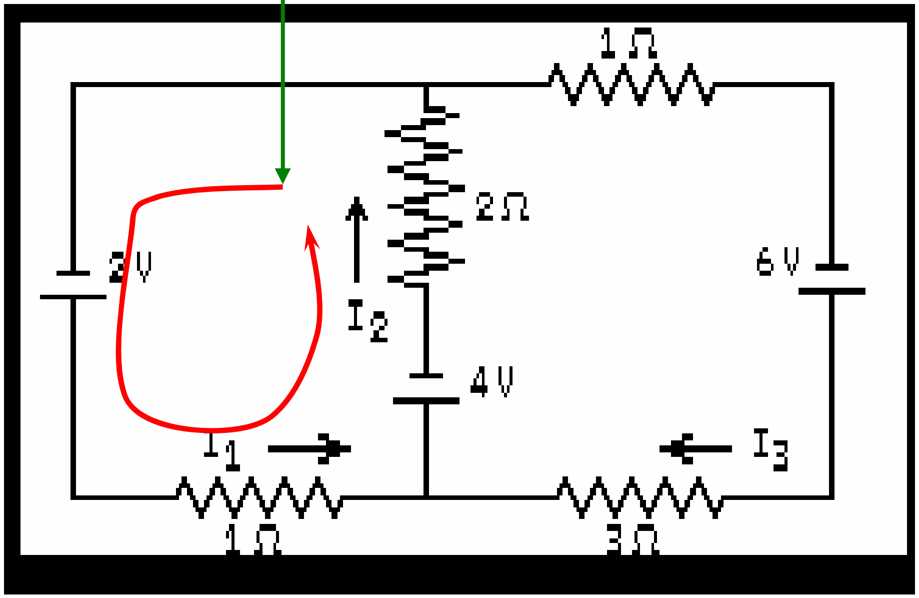
Maximize

$$\boxed{R = 4\Omega \quad P_2 = 9\,\mathrm{W}}$$

Circuits

➔ Which of the equations is valid for the circuit shown below?

- a) $2 - I_1 - 2I_2 = 0$
- b) $2 - 2I_1 - 2I_2 - 4I_3 = 0$
- c) $2 - I_1 - 4 - 2I_2 = 0$
- d) $I_3 - 2I_2 - 4I_3 = 0$
- e) $2 - 2I_1 - 2I_2 - 4I_3 = 0$



Light Bulbs

➔A three-way light bulb contains two filaments that can be connected to the 120 V either individually or in parallel.

- A three-way light bulb can produce 50 W, 100 W or 150 W, at the usual household voltage of 120 V.
- What are the resistances of the filaments that can give the three wattages quoted above?

Use $P = V^2/R$

- $R_1 = 120^2/50 \quad = 288\Omega$ (50W)
- $R_2 = 120^2/100 = 144\Omega$ (100W)

Problem

➔ What is the maximum number of 100 W light bulbs you can connect in parallel in a 100 V circuit without tripping a 20 A circuit breaker?

- (a) 1
- (b) 5
- (c) 10
- (d) 20
- (e) 100

Each bulb draws a current of 1A. Thus only 20 bulbs are allowed before the circuit breaker is tripped.

RC Circuits

➔Charging a capacitor takes time in a real circuit

- ◆Resistance allows only a certain amount of current to flow
- ◆Current takes time to charge a capacitor

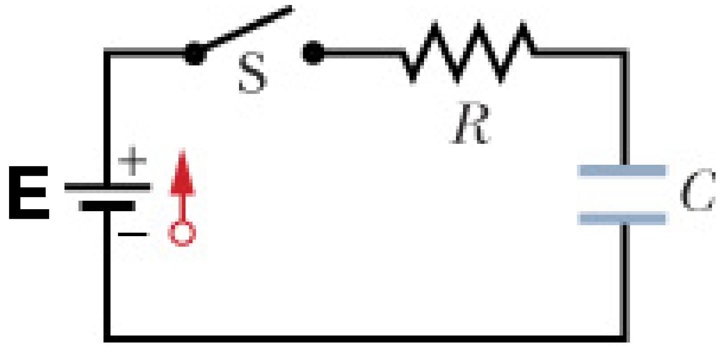
➔Assume uncharged capacitor initially

- ◆Close switch at t = 0
- ◆Initial current is $i = E/R$ (no charge on capacitor)

➔Current flows, charging capacitor

- ◆Generates capacitor potential of q/C

$$i = \frac{E - q/C}{R}$$



➔Current decreases continuously as capacitor charges!

- ◆Goes to 0 when fully charged

Analysis of RC Circuits

➔Current and charge are related

$$i = dq/dt$$

➔So can recast previous equation as “differential equation“

$$i = \frac{E - q/C}{R} \quad \Rightarrow \quad \frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R}$$

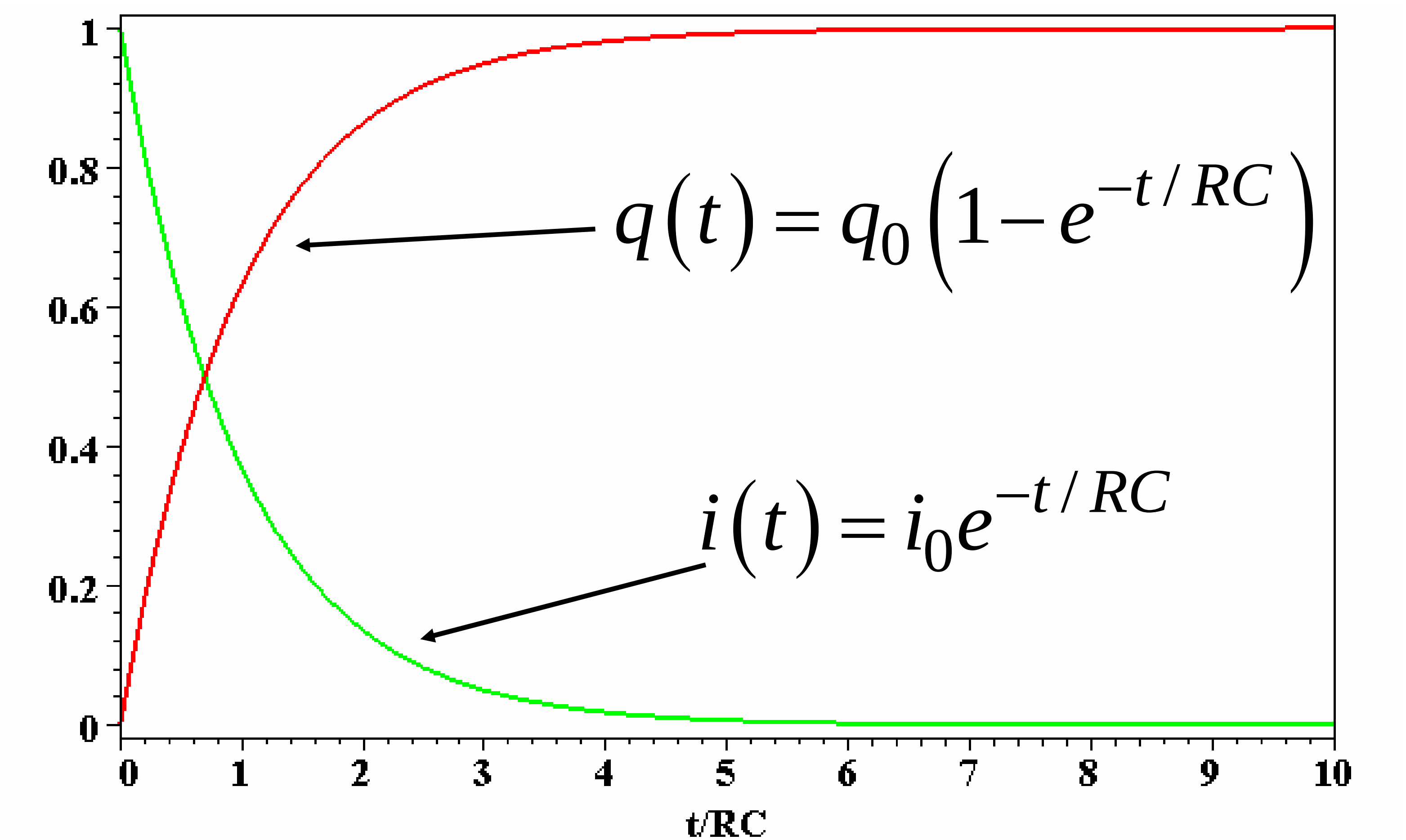
➔General solution is $q = EC + Ke^{-t/RC}$

- (Check and see!)
- K = –EC (necessary to make q = 0 at t = 0)

➔Solve for charge q and current i

$$q = EC\left(1 - e^{-t/RC}\right) \qquad i = \frac{dq}{dt} = \frac{E}{R}e^{-t/RC}$$

Charge and Current vs Time (For Initially Uncharged Capacitor)

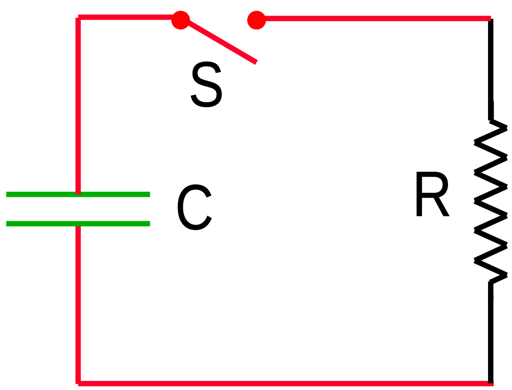


Exponential Behavior

- ➔ t = RC is the "characteristic time" of any RC circuit
  - ◆ Only t / RC is meaningful
- ➔ t = RC
  - ◆ Current falls to 37% of maximum value
  - ◆ Charge rises to 63% of maximum value
- ➔ t =2RC
  - ◆ Current falls to 13.5% of maximum value
  - ◆ Charge rises to 86.5% of maximum value
- ➔ t =3RC
  - ◆ Current falls to 5% of maximum value
  - ◆ Charge rises to 95% of maximum value
- ➔ t =5RC
  - ◆ Current falls to 0.7% of maximum value
  - ◆ Charge rises to 99.3% of maximum value

Discharging a Capacitor

➔Connect fully charged capacitor to a resistor at t = 0



$$-iR - \frac{q}{C} = 0$$

$$\frac{dq}{dt} + \frac{q}{RC} = 0$$

➔General solution is $q = Ke^{-t/RC}$

- ◆K = VC (necessary to make have full charge at t = 0)

➔Solve for charge q and current i

$$q = VCe^{-t/RC} \qquad i = \frac{dq}{dt} = -\frac{V}{R}e^{-t/RC}$$

Charge and Current vs Time
(For Initially Charged Capacitor)