Direction of Induced Current

Bar magnet moves through coil

- ➢ Current induced in coil

Reverse pole

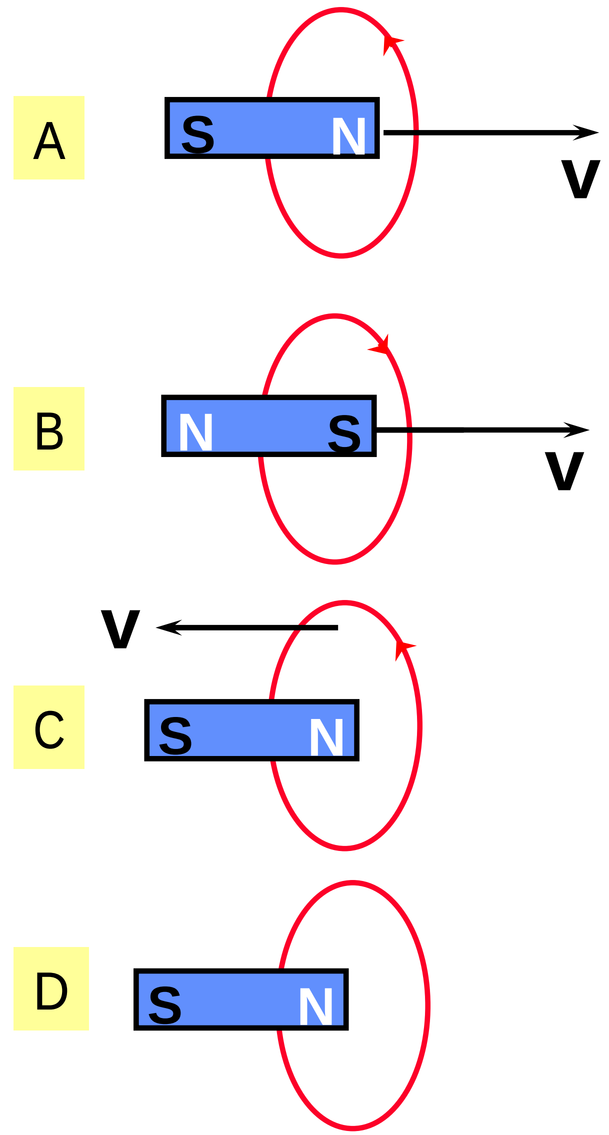
- ➢ Induced current changes sign

Coil moves past fixed bar magnet

- ➢ Current induced in coil as in (A)

Bar magnet stationary inside coil

- ➢ No current induced in coil

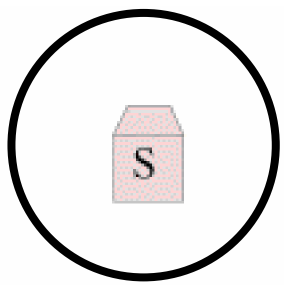


ConcepTest: Lenz's Law

➔ If a North pole moves towards the loop from above the page, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Must counter flux change in downward direction with upward B field

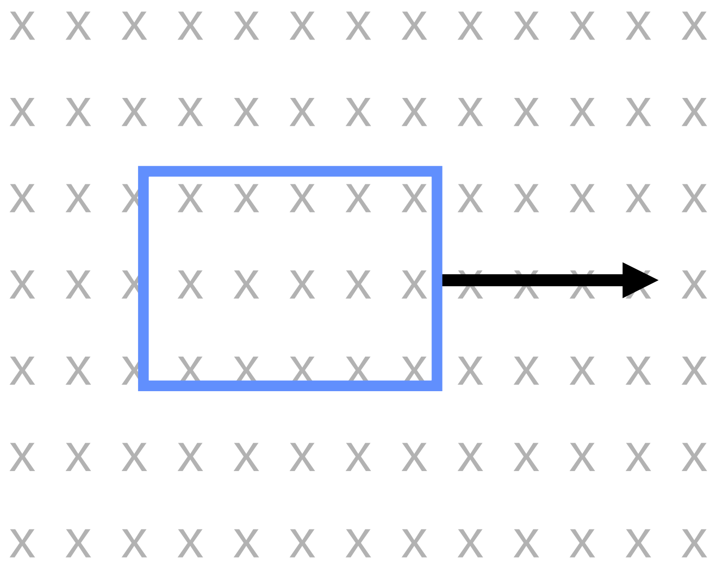


ConcepTest: Induced Currents

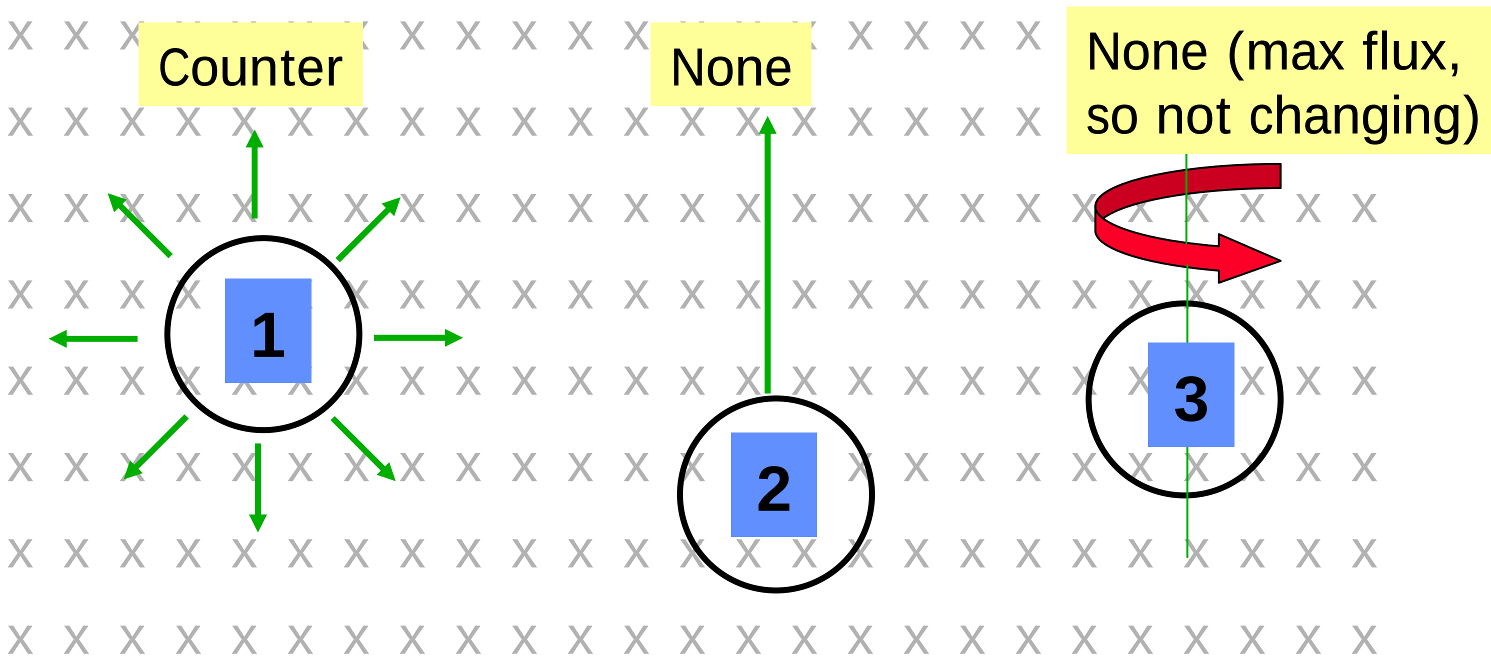
➔A wire loop is being pulled through a uniform magnetic field. What is the direction of the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

No change in flux, no induced current



ConcepTest: Induced Currents



In each of the 3 cases above, what is the direction of the induced current?

(Magnetic field is into the page and has no boundaries)

(a) clockwise

(b) counter-clockwise

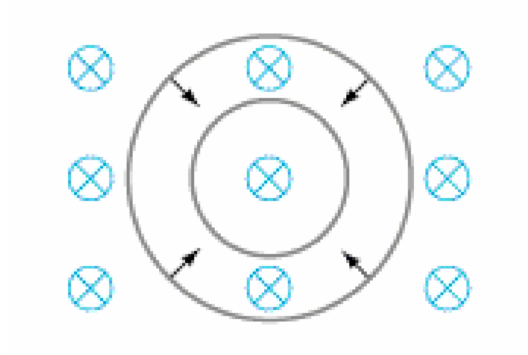
(c) no induced current

ConcepTest: Lenz's Law

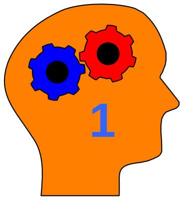
➔ If a coil is shrinking in a B field pointing into the page, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Downward flux is decreasing, so need to create downward B field



Induced currents



➔A circular loop in the plane of the paper lies in a 3.0 T magnetic field pointing into the paper. The loop's diameter changes from 100 cm to 60 cm in 0.5 s

- What is the magnitude of the average induced emf?
- What is the direction of the induced current?
- If the coil resistance is 0.05Ω , what is the average induced current?

$$|V| = \frac{d\Phi_B}{dt} = 3.0 \times \left| \frac{\pi\left(0.3^2 - 0.5^2\right)}{0.5} \right| = 3.016 \text{ Volts}$$

- Direction = clockwise (Lenz's law)

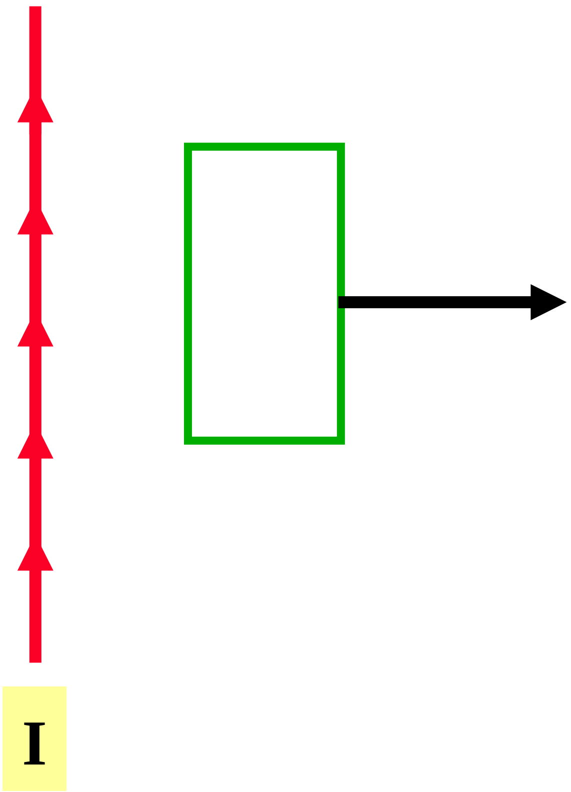
- Current = 3.016 / 0.05 = 60.3 A

ConcepTest: Induced Currents

➔A wire loop is pulled away from a current-carrying wire. What is the direction of the induced current in the loop?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Downward flux through loop decreases, so need to create downward field

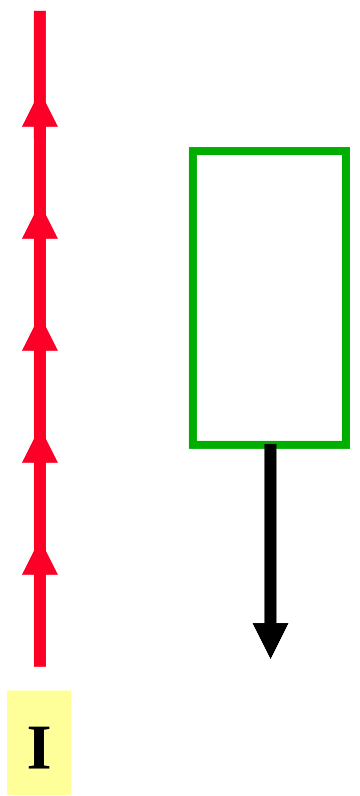


ConcepTest: Induced Currents

➔A wire loop is moved in the direction of the current. What is the direction of the induced current in the loop?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Flux does not change when moved along wire

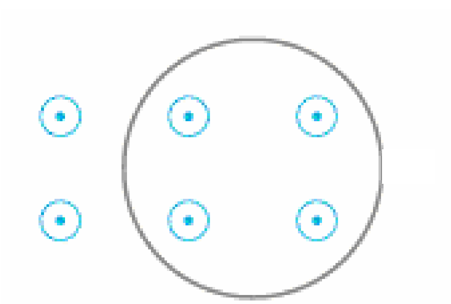


ConcepTest: Lenz's Law

➔ If the B field pointing out of the page suddenly drops to zero, in what direction is the induced current?

- ◆ (a) clockwise
- ◆ (b) counter-clockwise
- ◆ (c) no induced current

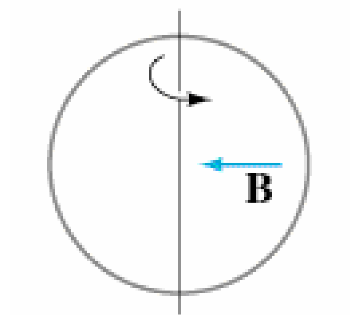
Upward flux through loop decreases, so need to create upward field



➔ If a coil is rotated as shown, in a B field pointing to the left, in what direction is the induced current?

- ◆ (a) clockwise
- ◆ (b) counter-clockwise
- ◆ (c) no induced current

Flux into loop is increasing, so need to create field out of loop



ConcepTest: Induced Currents

➔Wire #1 (length L) forms a one-turn loop, and a bar magnet is dropped through. Wire #2 (length 2L) forms a two-turn loop, and the same magnet is dropped through. Compare the magnitude of the induced currents in these two cases.

- (a) $I_1 = 2\ I_2$
- (b) $I_2 = 2\ I_1$
- (c) $I_1 = I_2 \neq 0$
- (d) $I_1 = I_2 = 0$
- (e) Depends on the strength of the magnetic field

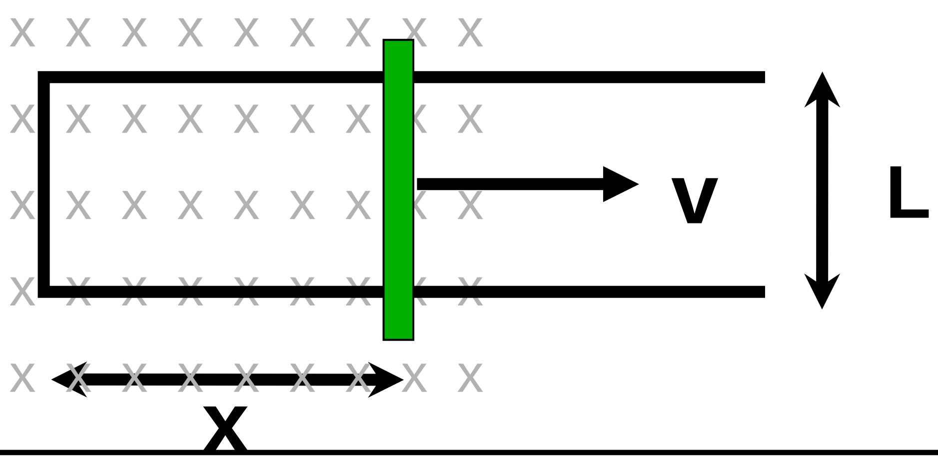
Voltage doubles, but R also doubles, leaving current the same

Motional EMF

➔Consider a conducting rod moving on metal rails in a uniform magnetic field:

$$\left|\mathcal{E}\right| = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{d(BLx)}{dt} = BL\frac{dx}{dt} = BLv$$

Current will flow counter-clockwise in this "circuit". Why?



$$\left|\mathcal{E}\right| = BLv$$

Force and Motional EMF

➔ Pull conducting rod out of B field

➔ Current is clockwise. Why?

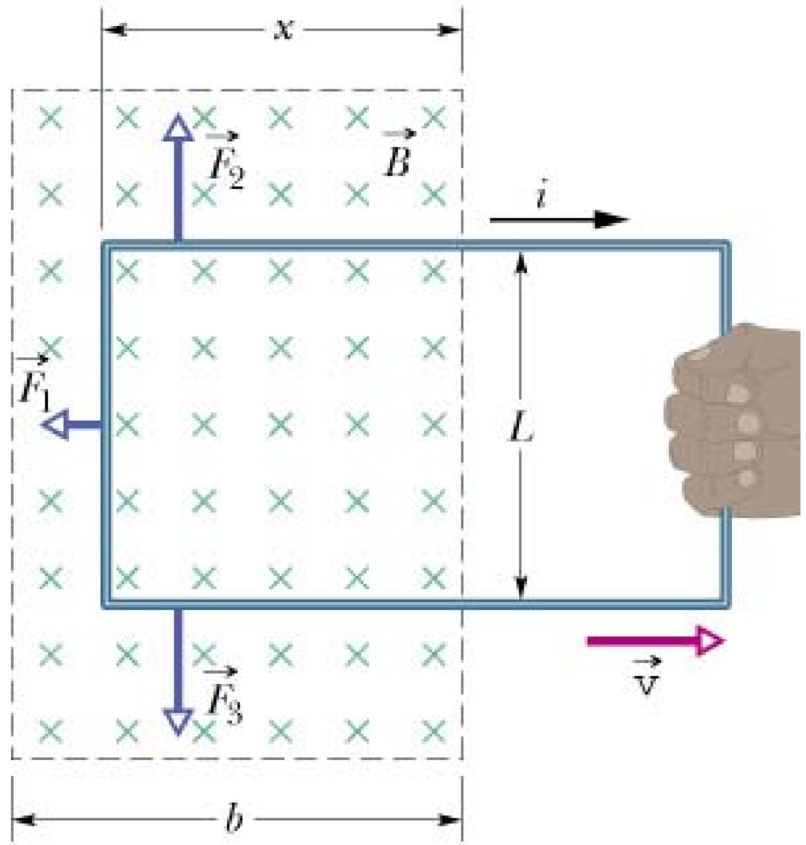
$$i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$$

➔ Current within B field causes force

$$F = iLB = \frac{B^2L^2v}{R}$$

- Force opposes pull (RHR)
- Also follows from Lenz's law

➔ We must pull with this force to maintain constant velocity



Power and Motional EMF

- ➔Force required to pull loop: $F = iLB = \dfrac{B^2 L^2 v}{R}$
- ➔Power required to pull loop: $P = Fv = \dfrac{B^2 L^2 v^2}{R}$
- ➔Energy dissipation through resistance

$$P = i^2 R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2 L^2 v^2}{R}$$

- ➔Same as pulling power! So power is dissipated as heat
  - ◆Kinetic energy is constant, so energy has to go somewhere
  - ◆Rod heats up as you pull it

Example

- ➔Pull a 30cm x 30cm conducting loop of aluminum through a 2T B field at 30cm/sec. Assume it is 1cm thick.
  - ◆Circumference = 120cm = 1.2m, cross sectional area = 10^{-4} m^2
  - ◆R = ρL/A = 2.75 x 10^{-8} * 1.2 / 10^{-4} = 3.3 x $10^{-4}\Omega$
- ➔EMF

$$\mathcal{E} = BLv = 2\times 0.3\times 0.3 = 0.18\,\mathrm{V}$$

- ➔Current

$$i = \mathcal{E}/R = 0.18/3.3\times 10^{-4} = 545\,\mathrm{A}$$

- ➔Force

$$F = iLB = 545\times 0.3\times 2 = 327\,\mathrm{N}$$

74 lbs!

- ➔Power

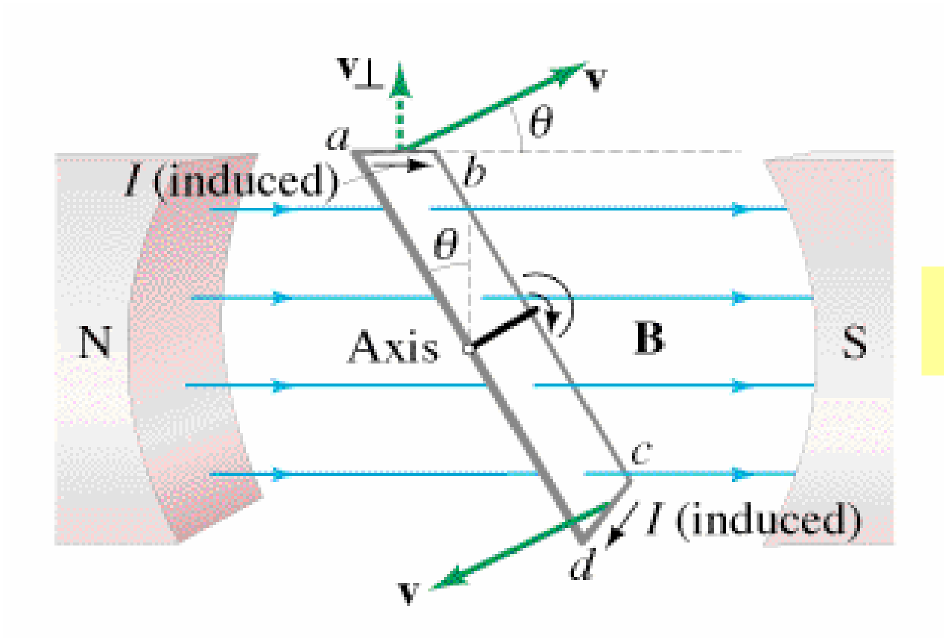
$$P = i^2 R = 98\,\mathrm{W}$$

➔ About 0.33^0 C per sec (from specific heat, density)

Electric Generators

➔Rotate a loop of wire in a uniform magnetic field:

- changing θ $\Rightarrow$ changing flux $\Rightarrow$ induced emf
- $\Phi_B = B\,A\cos\theta = B\,A\cos(\omega t)$



Rotation: $\theta = \omega t$

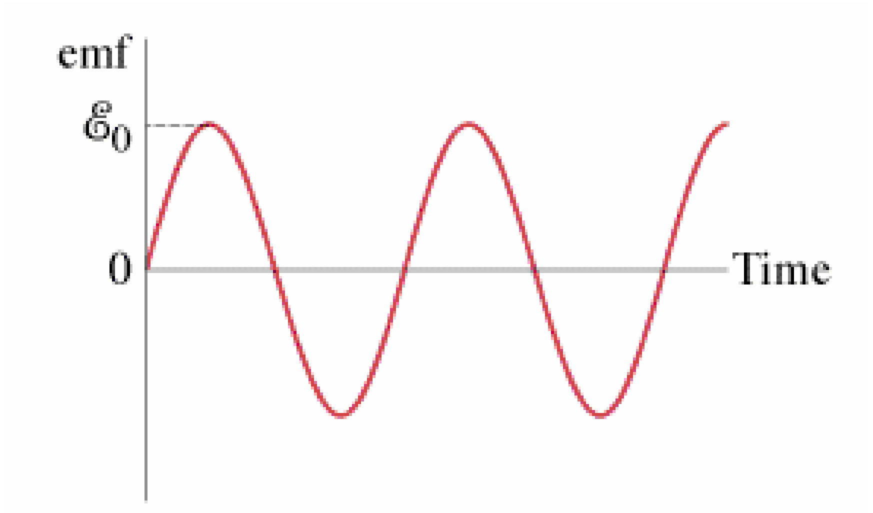
Electric Generators

➔Flux is changing in a sinusoidal manner

- Leads to an alternating emf (AC generator)

$$\left|\mathcal{E}\right| = N\frac{d\Phi_B}{dt} = NBA\frac{d\cos(\omega t)}{dt} = NBA\omega\sin(\omega t)$$

- This is how electricity is generated
- Water or steam (mechanical power) turns the blades of a turbine which rotates a loop
- Mechanical power converted to electrical power

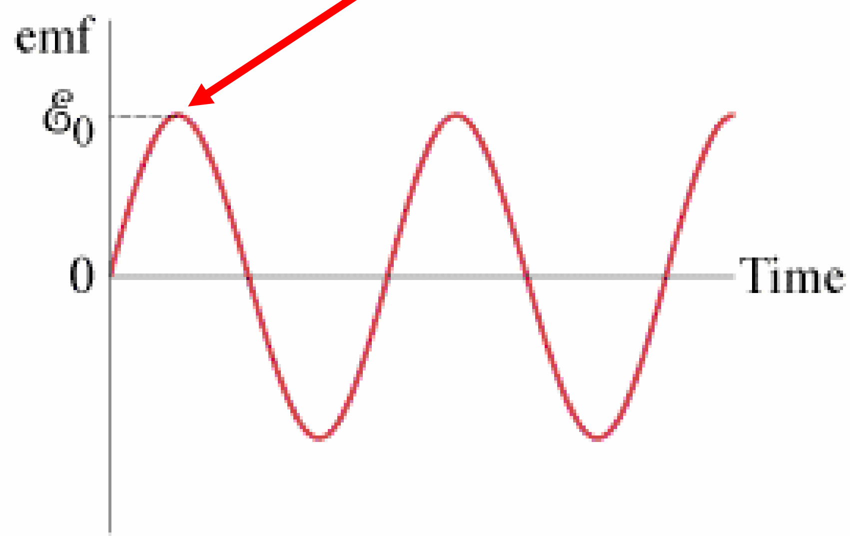


ConcepTest: Generators

➔A generator has a coil of wire rotating in a magnetic field. If the B field stays constant and the area of the coil remains constant, but the rotation rate increases, how is the maximum output voltage of the generator affected?

- (a) Increases
- (b) Decreases
- (c) Stays the same
- (d) Varies sinusoidally

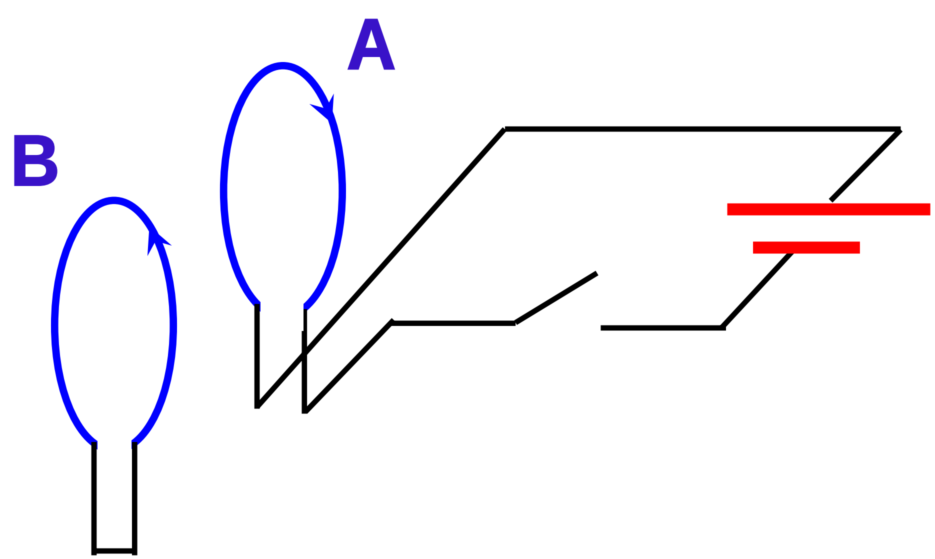
$$\left|\mathcal{E}\right| = NBA\omega \sin(\omega t)$$



Induction in Stationary Circuit

- ➔ Switch closed (or opened)
  - ◆ Current induced in coil B (directions as shown)

- ➔ Steady state current in coil A
  - ◆ No current induced in coil B



Inductance

- ➔Inductance in a coil of wire defined by $L = \frac{N\Phi_B}{i}$
- ➔Can also be written $Li = N\Phi_B$
- ➔From Faraday's law $\mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$
  - ◆This is a more useful way to understand inductance
  - ◆Calculate emf generated in coil from rate of change of current
- ➔Inductors play an important role in circuits when current is changing!

Self - Inductance

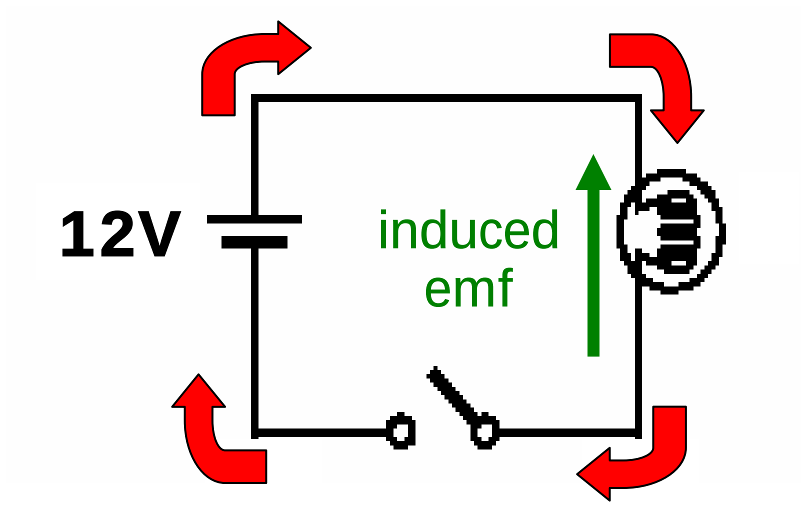
➔Consider a single isolated coil:

- Current (red) starts to flow clockwise due to the battery
- But the buildup of current leads to changing flux in loop
- Induced emf (green) opposes the change

This is a self-induced emf (also called "back" emf)

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -L\frac{di}{dt}$$

L is the self-inductance
units = "Henry (H)"



Inductance of Solenoid

➔Total flux (length *l*)

$$B = \mu_0 in$$

$$N\Phi_B = (nl)(BA) = \mu_0 n^2 Ali$$

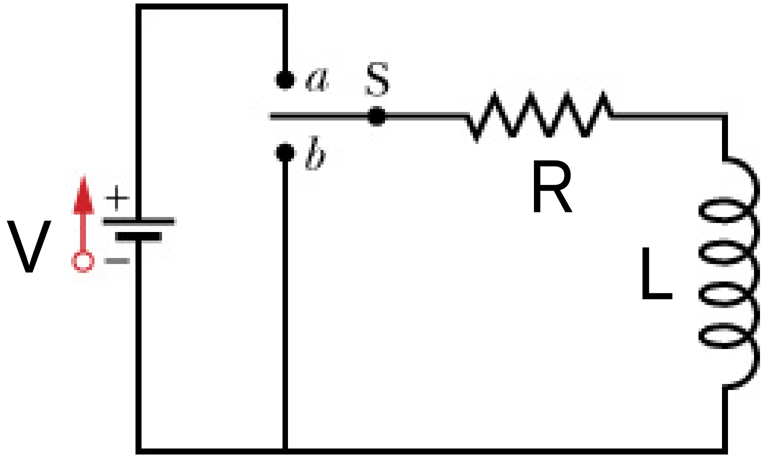
$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\mu_0 n^2 Al\frac{di}{dt} = -L\frac{di}{dt}$$

$$L = \mu_0 n^2 Al$$

To make large inductance:
- Lots of windings
- Big area
- Long

LR Circuits

- ➔Inductance and resistor in series with battery of EMF *V*
- ➔Start with no initial current in circuit
  - ◆Close switch at t = 0
  - ◆Current is initially 0 (initial increase causes voltage drop across inductor)
- ➔Find $i(t)$
  - ◆Resistor: $\Delta V = Ri$
  - ◆Inductor: $\Delta V = L\, di/dt$
  - ◆Apply loop rule



$$V - Ri - L\frac{di}{dt} = 0$$

Analysis of LR Circuit

➔Differential equation is $\dfrac{di}{dt}+i\left(\dfrac{R}{L}\right)=\dfrac{V}{R}$

➔General solution: $i=V/R+Ke^{-tR/L}$

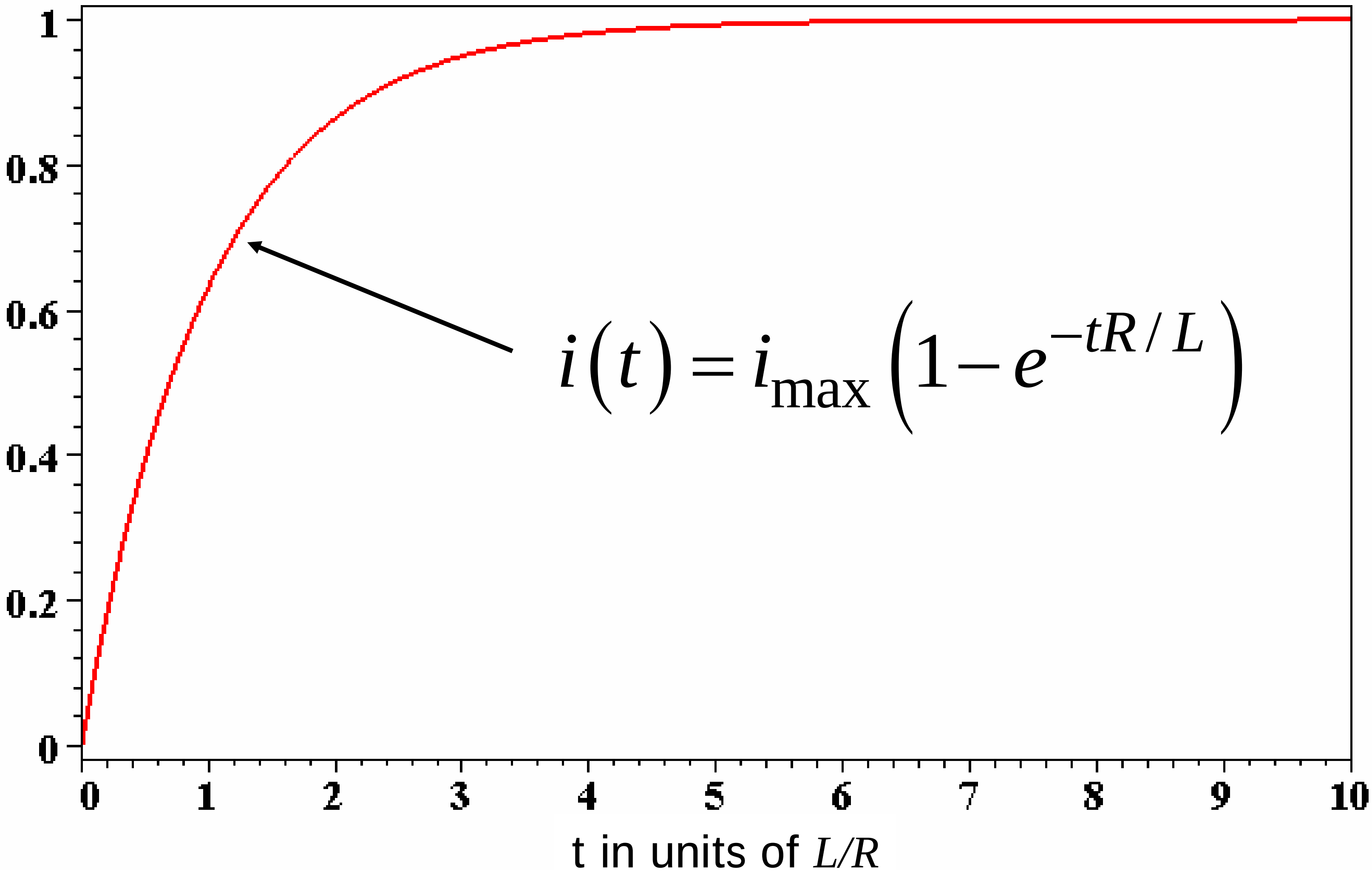
- ◆(Check and see!)
- ◆K = –V/R (necessary to make i = 0 at t = 0)

$$i=\frac{V}{R}\left(1-e^{-tR/L}\right)$$

← Rise from 0 with time constant τ = L / R

↑ Final current (maximum)

Current vs Time in RL Circuit (Initially Zero Current in Inductor)



$$i(t) = i_{\max}\left(1 - e^{-tR/L}\right)$$

L-R Circuits (2)

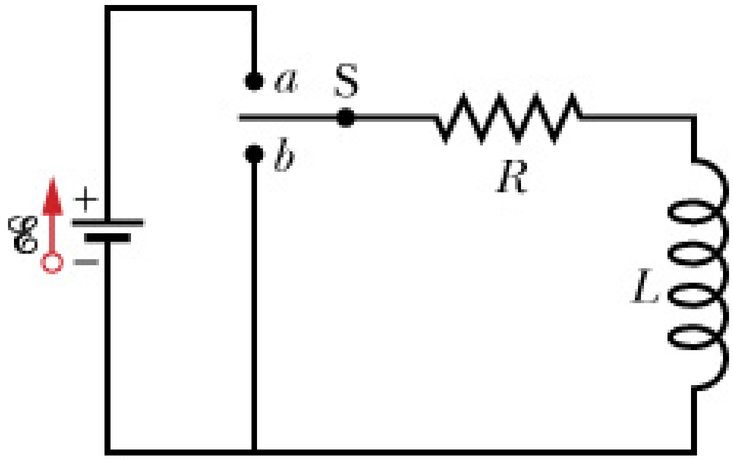
➔Switch off battery: Find $i(t)$ if current starts at i_0

$$0 = L\frac{di}{dt} + Ri$$

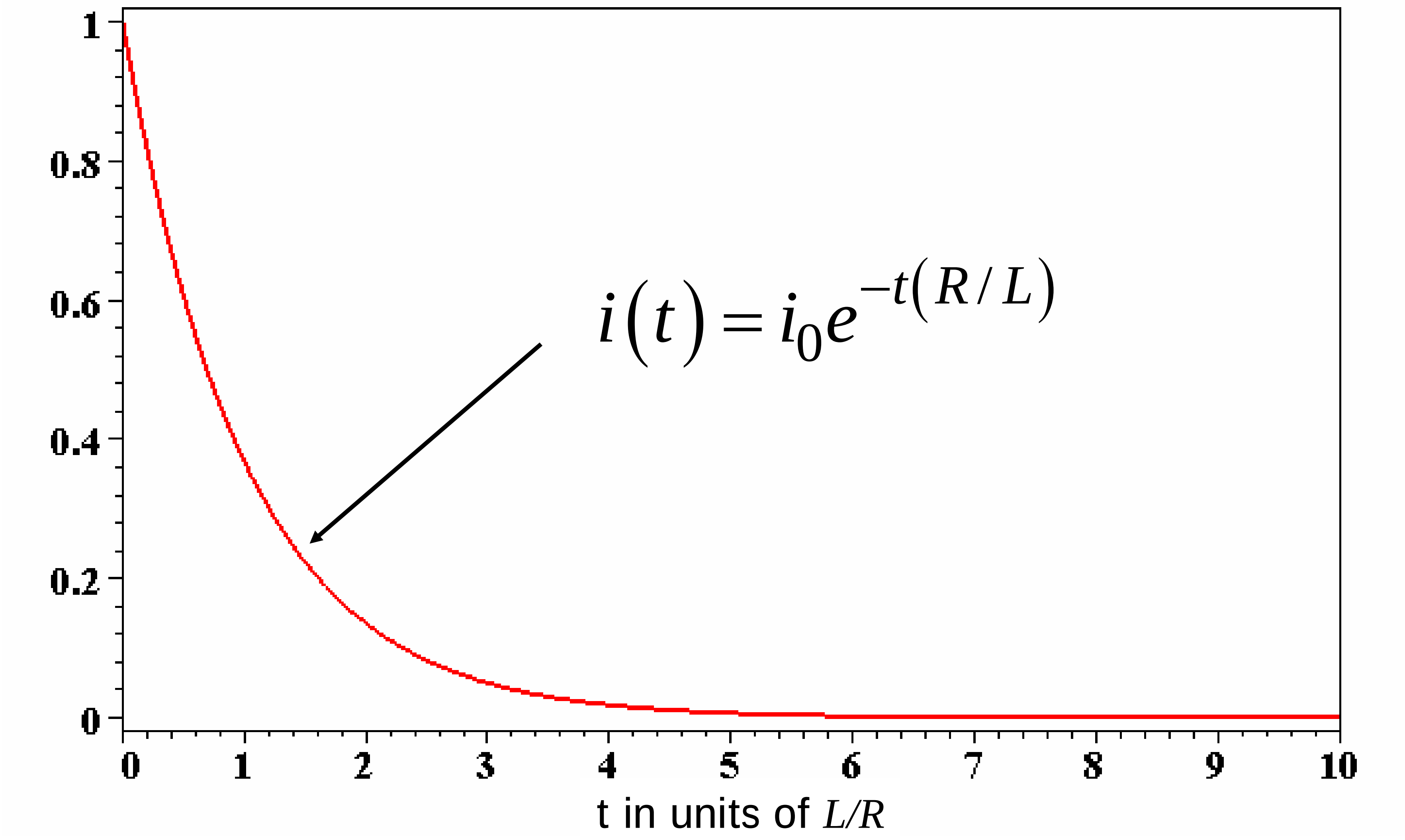
$$i = i_0 e^{-tR/L}$$

Exponential fall to 0 with time constant τ = L / R

Initial current (maximum)



Current vs Time in RL Circuit (For Initial Current i_{max} in Inductor)



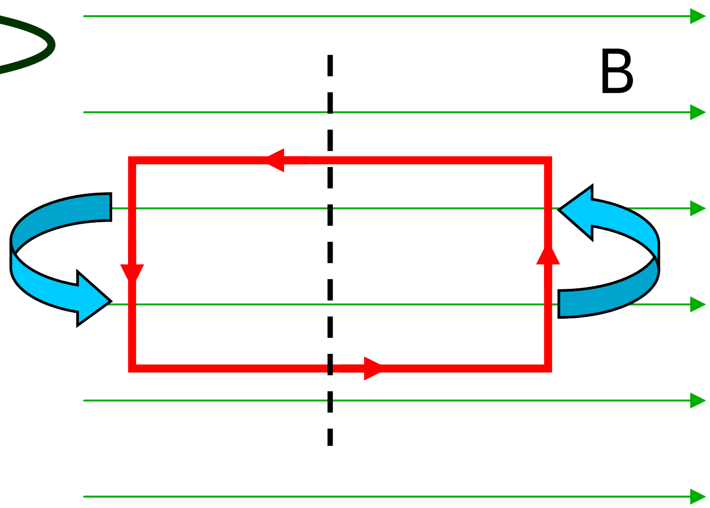
Exponential Behavior

- ➔ τ = L/R is the "characteristic time" of any RL circuit
  - ◆ Only t / τ is meaningful
- ➔ $t = \tau$
  - ◆ Current falls to 1/e = 37% of maximum value
  - ◆ Current rises to 63% of maximum value
- ➔ $t = 2\tau$
  - ◆ Current falls to $1/e^2$ = 13.5% of maximum value
  - ◆ Current rises to 86.5% of maximum value
- ➔ $t = 3\tau$
  - ◆ Current falls to $1/e^3$ = 5% of maximum value
  - ◆ Current rises to 95% of maximum value
- ➔ $t = 5\tau$
  - ◆ Current falls to $1/e^5$ = 0.7% of maximum value
  - ◆ Current rises to 99.3% of maximum value

ConcepTest: Generators and Motors

➔ A current begins to flow in a wire loop placed in a magnetic field as shown. What does the loop do?

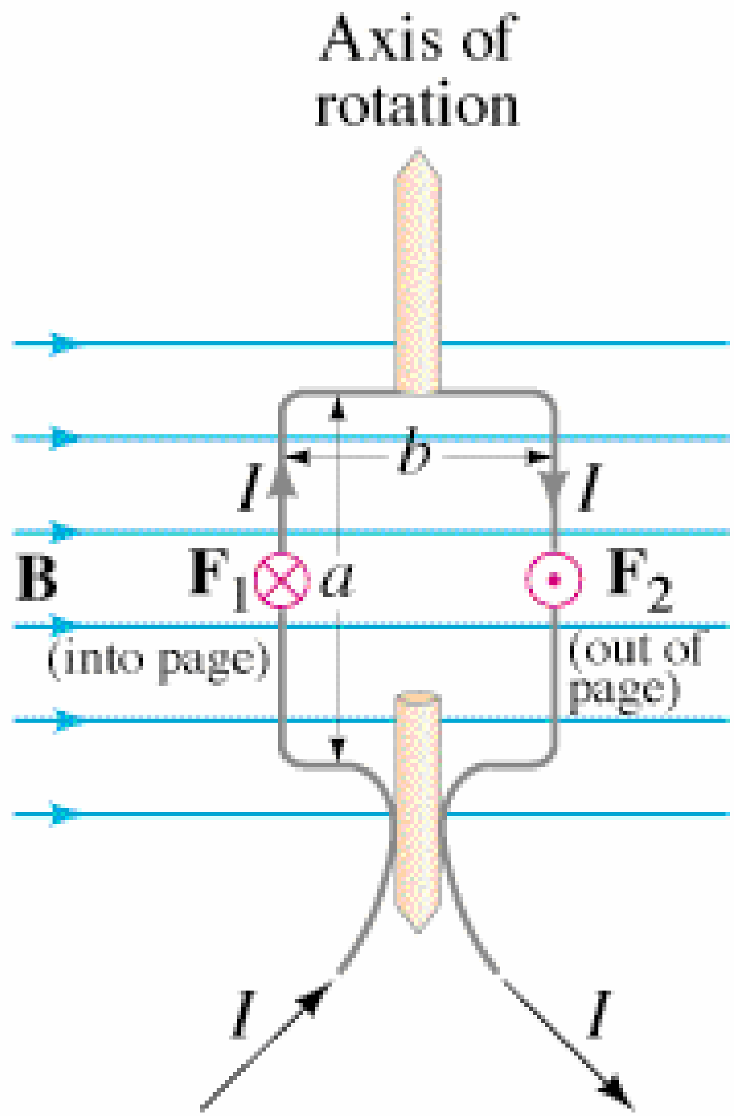
- (a) moves to the right
- (b) moves up
- (c) rotates around horizontal axis
- (d) rotates around vertical axis
- (e) moves out of the page



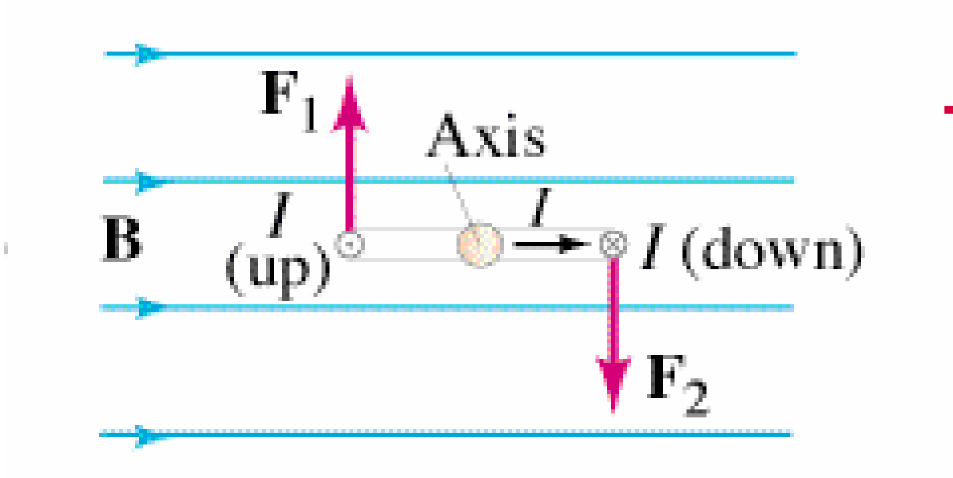
This is how a motor works !!

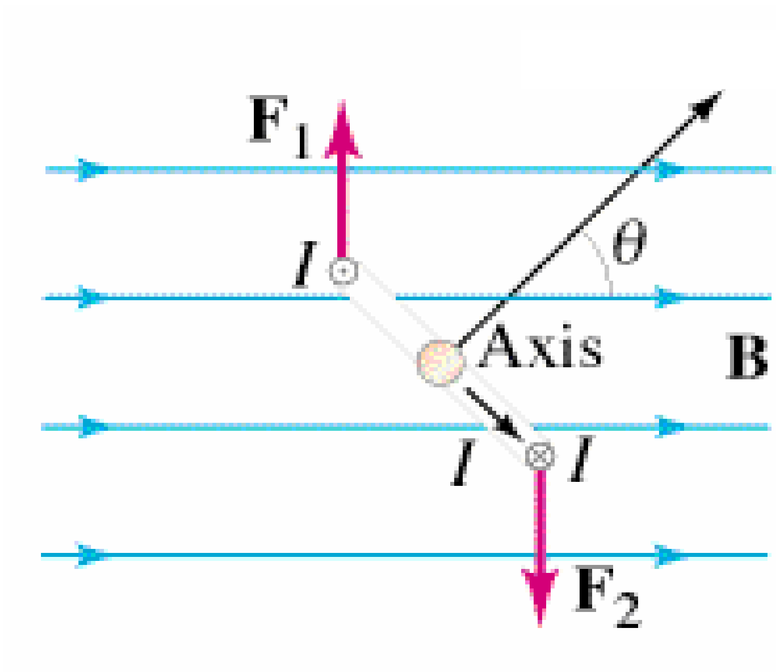
Electric Motors

- Current is supplied from an external source of emf (battery or power supply)
- Forces act to rotate the wire loop
- A motor is essentially a generator operated in reverse!



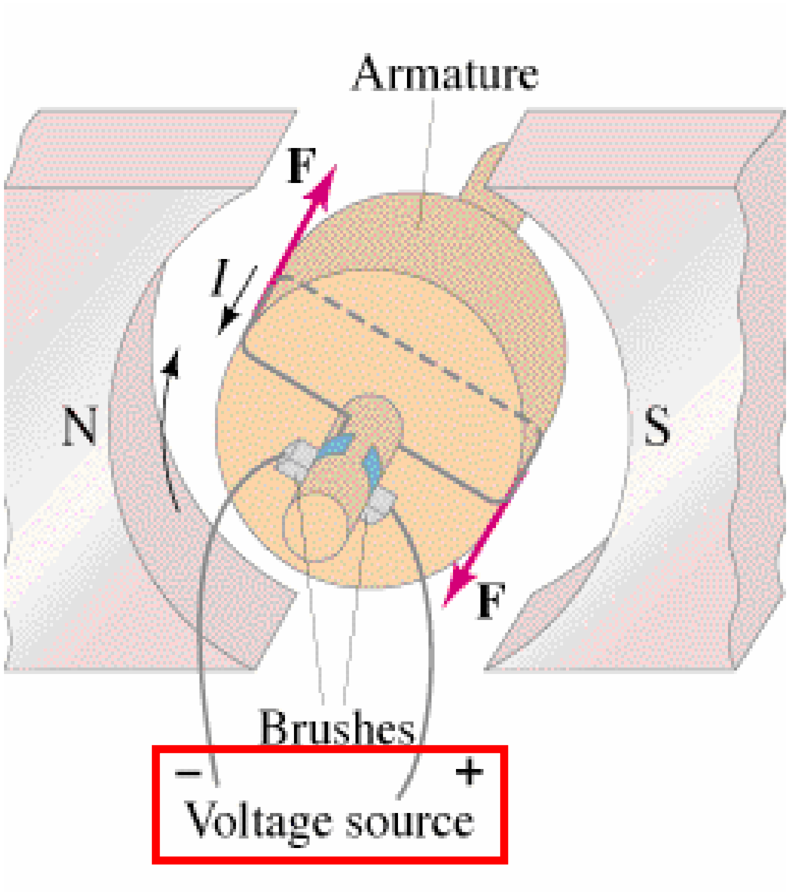
Motor





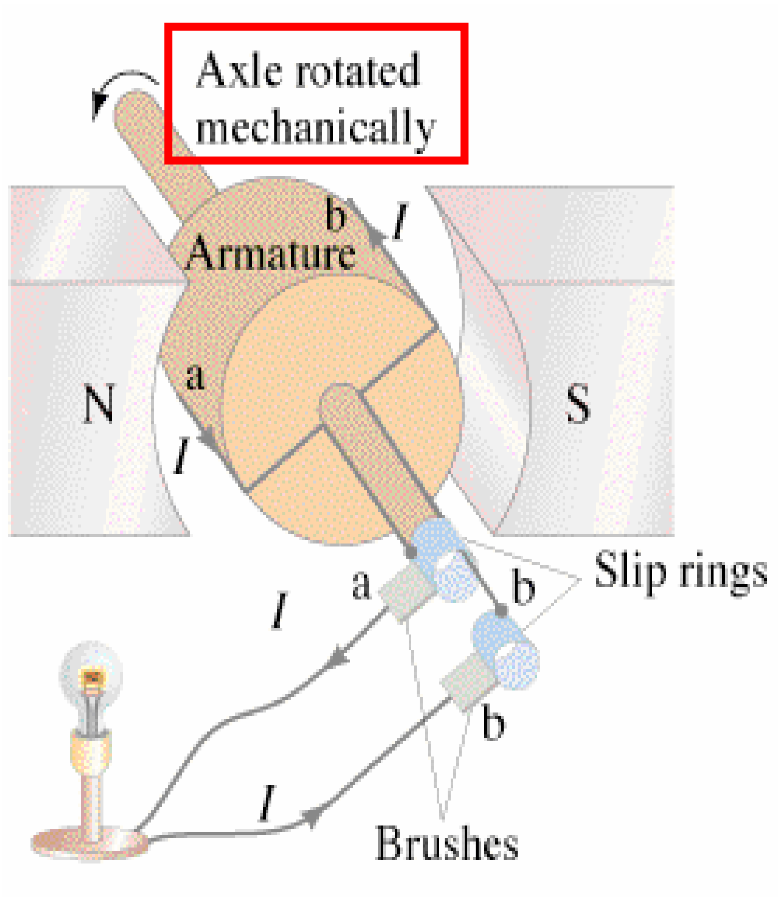
- ➔ Forces act to rotate the loop clockwise.
- ➔ When loop is vertical, current switches sign and the forces reverse, in order to keep the loop in rotation.
- ➔ This is why alternating current is normally used for motors.

Motors



Electrical ⇒ mechanical energy

Generators



Mechanical ⇒ electrical energy