Wave Interference and Diffraction

Part 1: Introduction, Double Slit

Paul Avery
University of Florida
http://www.phys.ufl.edu/~avery/
avery@phys.ufl.edu

PHY 2049
Physics 2 with Calculus
Need to Understand Light as Wave!

(You already have read this material)

Index of refraction

- Speed of EM wave in medium: \( c_n = \frac{c}{n} \)
- Wavelength of light: \( \lambda_n = \frac{\lambda}{n} \)

Propagation of light: Huygens principle (36-2)

- Explains reflection and refraction
- Explains interference (from superposition)
- Explains diffraction (spreading of light around barrier)
Interference as a Wave Phenomenon

- Interference of light waves
  - Caused by superposition of waves
  - Intensity can increase or decrease!
  - Contrast with particle model of light

- Effects and applications
  - Double slit
  - Single slit
  - Diffraction gratings
  - Anti-reflective coatings on lenses
  - Highly reflective coatings for mirrors
  - Iridescent coatings on insects
  - Colors on thin bubbles
  - Interferometry with multiple telescopes
Interference from Wave Superposition

Basic rule: Add displacement at every point
Constructive Interference

- Same wavelength, phase difference = 0°
- Amplitude larger: Higher intensity

\[ E(x) = \sin(kx) + 0.5\sin(kx) = 1.5\sin(kx) \]
Destructive Interference

- Same wavelength, phase difference = 180° (1/2 λ)
- Amplitude smaller: Lower intensity

\[ E(x) = \sin(kx) + 0.5\sin(kx + \pi) = 0.5\sin(kx) \]
Examples

→ Two waves, same $\lambda$, with amplitudes $2A$ and $A$
  ◆ Initial intensities $4I$ and $I$, respectively ($I \propto A^2$)

→ No interference
  ◆ Combined intensity: $I_{\text{new}} = 4I + I = 5I$

→ Maximum constructive interference ($\phi = 0$)
  ◆ New amplitude: $A_{\text{new}} = 2A + A = 3A$
  ◆ New intensity: $I_{\text{new}} = 9I$

→ Maximum destructive interference ($\phi = \pi$)
  ◆ New amplitude: $A_{\text{new}} = 2A - A = A$
  ◆ New intensity: $I_{\text{new}} = I$
General Treatment of Interference

Most interference is partial

- Amplitudes for 2 waves are generally different
- Phase difference: $0 < \phi < 180^\circ$

$$E(x, t) = E_1 \cos(kx - \omega t) + E_2 \cos(kx - \omega t + \phi)$$

Additional considerations

- Wavelengths can be different
- Multiple waves may interfere (e.g., diffraction grating)
- But easy to accommodate: just sum over all waves

$$E(x, t) = \sum_i E_i \cos(k_i x - \omega_i t + \phi_i)$$
Interference and Path Length

Two sources, spaced 3 wavelengths apart, emit waves with the same wavelength and phase. In how many places on the circle will the net intensity be a relative maximum?

Answer = 12
Can you see why?

Hint: Start at far right and move counterclockwise towards top, noting path length changes.

Key idea: Path difference leads to phase difference
Interference and Path Length

Two sources, separated by $4\lambda$, emit waves at same wavelength and phase. Find relative minima on $+x$ axis.

Solution: path difference must be a half-multiple of $\lambda$

$$\Delta L = \sqrt{x^2 + (4\lambda)^2} - x = (n + \frac{1}{2})\lambda$$

$$x = \frac{16 - \left(n + \frac{1}{2}\right)^2}{2n + 1}\lambda$$

4 values

<table>
<thead>
<tr>
<th>$n$</th>
<th>$x$</th>
<th>$\Delta L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$15.8\lambda$</td>
<td>$\lambda/2$</td>
</tr>
<tr>
<td>1</td>
<td>$4.58\lambda$</td>
<td>$3\lambda/2$</td>
</tr>
<tr>
<td>2</td>
<td>$1.95\lambda$</td>
<td>$5\lambda/2$</td>
</tr>
<tr>
<td>3</td>
<td>$0.54\lambda$</td>
<td>$7\lambda/2$</td>
</tr>
</tbody>
</table>
Double Slit Interference

** Incident light**
- Light waves strike 2 narrow slits close together
- Light goes through both slits, diffracts in all directions

** Interference**
- At certain angles, waves constructively interfere ⇒ brighter
- At other angles, waves destructively interfere ⇒ darker
Basic Requirements for Two Slit Setup

- Light beam strikes normal to slits
- Light beam illuminates both slits equally
- Light beam is in phase at both slits: coherent
  - Young used small slit in front of 2 slits to get coherence
  - Modern versions use laser for coherence (much brighter)
Two Slit Analysis

Path difference = $d \sin \theta$

$d \sin \theta = m\lambda$ \quad Maximum

$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$ \quad Minimum
Example 1: \( d = 5\lambda \)

Max \( \sin \theta = m \left( \frac{\lambda}{d} \right) = 0.2m \)

Min \( \sin \theta = \left( m + \frac{1}{2} \right) \left( \frac{\lambda}{d} \right) = 0.2 \left( m + \frac{1}{2} \right) \)

<table>
<thead>
<tr>
<th>( m )</th>
<th>( \sin \theta_{\text{max}} )</th>
<th>( \theta_{\text{max}} )</th>
<th>( \sin \theta_{\text{min}} )</th>
<th>( \theta_{\text{min}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>±0.1</td>
<td>±5.7</td>
</tr>
<tr>
<td>±1</td>
<td>±0.2</td>
<td>±11.5</td>
<td>±0.3</td>
<td>±17.5</td>
</tr>
<tr>
<td>±2</td>
<td>±0.4</td>
<td>±23.6</td>
<td>±0.5</td>
<td>±30</td>
</tr>
<tr>
<td>±3</td>
<td>±0.6</td>
<td>±36.9</td>
<td>±0.7</td>
<td>±44.4</td>
</tr>
<tr>
<td>±4</td>
<td>±0.8</td>
<td>±53.1</td>
<td>±0.9</td>
<td>±64.2</td>
</tr>
<tr>
<td>±5</td>
<td>±1.0</td>
<td>±90</td>
<td>±1.1</td>
<td>--</td>
</tr>
</tbody>
</table>
Intensity vs Angle for $d = 5\lambda$

Double slit, $a=0$, $\text{lambda}=0.2*d$
Example 2: $d' = 2.0 \, \mu m \, , \, \lambda = 550 \, nm$

How many bright fringes? Where are they?

$$ \sin \theta = m \left( \frac{\lambda}{d'} \right) = 0.275m $$

$m$ can equal 0, ±1, ±2, ±3 ⇒ 7 maxima

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\sin \theta$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0°</td>
</tr>
<tr>
<td>±1</td>
<td>0.275</td>
<td>16.0°</td>
</tr>
<tr>
<td>±2</td>
<td>0.55</td>
<td>33.4°</td>
</tr>
<tr>
<td>±3</td>
<td>0.825</td>
<td>55.6°</td>
</tr>
</tbody>
</table>
Intensity vs $\theta$ for $d = 2.0 \, \mu m$, $\lambda = 550 \, \text{nm}$
Calculating Double Slit Intensity

**Assumptions**
- Each slit acts as a source of waves
- Waves radiate equally in all directions

Path difference: \( d \sin \theta \)
Double Slit Intensity (2)

- Add amplitudes, include phase difference
  - Assume equal size slit widths
  - Phase difference from path difference: $2\pi \times \# \text{ wavelengths}$
  - We ignore $x$ dependence here (analysis does not depend on it)

$$E(t) = E\cos(\omega t) + E\cos(\omega t + \phi)$$

$$\phi = \frac{2\pi d \sin \theta}{\lambda}$$
Double Slit Intensity (3)

Intensity is time average of amplitude *squared*

- Consider single wave of amplitude $E = E_0 \cos \omega t$
- Intensity from time average of $E^2$: $I = K^2 E^2 \langle \cos^2 \omega t \rangle = \frac{1}{2} K^2 E^2$
- $\langle \ldots \rangle$ is time average over a period

\[
I_{\text{tot}} = K^2 \left\langle \left( E \cos(\omega t) + E \cos(\omega t + \phi) \right)^2 \right\rangle \\
= K^2 E^2 \left\langle \cos^2 \omega t \right\rangle + K^2 E^2 \left\langle \cos^2 (\omega t + \phi) \right\rangle + 2K^2 E^2 \left\langle \cos \omega t \cos(\omega t + \phi) \right\rangle
\]

*We work this out on next page*
Double Slit Intensity (4)

\[ \langle \cos^2 \omega t \rangle = \frac{1}{2} \]
\[ \langle \cos^2 (\omega t + \phi) \rangle = \frac{1}{2} \]
\[ \langle \cos \omega t \cos (\omega t + \phi) \rangle = \frac{1}{2} \cos \phi \]
\[ \phi = \frac{2\pi d \sin \theta}{\lambda} \]

\[ I_{\text{tot}} = K^2 E^2 (1 + \cos \phi) = 2I_0 (1 + \cos \phi) \]
\[ = 4I_0 \cos^2 \frac{1}{2} \phi \]
\[ = 4I_0 \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \]
Double Slit Intensity (5)

So the intensity is  \[ I = 4I_0 \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \]

Maxima occur when argument inside \( \cos() \) is \( n\pi \)

\[ d \sin \theta = n\lambda \]

Minima occur when argument inside \( \cos() \) is \( (n+1/2)\pi \)

\[ d \sin \theta = \left( n + \frac{1}{2} \right) \lambda \]