Chapter 22: Electric Field
Concept Quiz

For which situations can we put a charge $q'$ on the left side of the two charges so that it is in equilibrium?

- (1) (a) only
- (2) (a) and (b)
- (3) (c) and (d)
- (4) (b) and (d)
- (5) (c) only

Should be near the smaller charge because if it is near the larger charge, there can be no total cancellation.
Concept Quiz

Which situation has the largest force acting on q?

All E fields are in the same direction in (3). Note that (4) has E = 0.
Electric Field of Single Point Charge

\[ q > 0 \quad \vec{E} = \frac{kq}{r^2} \hat{r} \]

\[ q < 0 \quad \vec{E} = \frac{kq}{r^2} \hat{r} \]
Example: Electric Field on Proton

➔ At surface of proton
  ◆ q = e = 1.6 \times 10^{-19} \text{ C}
  ◆ r = 10^{-15} \text{ m}

\[
E = \frac{kq}{r^2} = \frac{\left(9 \times 10^9\right) \left(1.6 \times 10^{-19}\right)}{\left(10^{-15}\right)^2} = 1.44 \times 10^{21} \text{ N/C}
\]

➔ E points radially outward for + charge
E Field of Two Equal, Positive Point Charges
E Field of Two Equal, Unlike Point Charges
Field Between Two Charged Parallel Plates

- Assume plates are much larger than separation
  - E is approx. constant between plates
  - E is zero outside the plates
  - This is a capacitor!

- E points from + plate to – plate

- We will calculate E in Chap. 23
  - Gauss’ law
  - Proportional to surface charge density
1. Rank magnitude of E at $P_1$, $P_2$, $P_3$. Assume charges on rings are $+q$ and $+q$. 
Answer to Question #1

- \( P_1 \) has \( E = 0 \) (equidistant from ring A and B and they are same sign)
- \( P_3 \) has largest \( E \) (contributions from ring A and B)
- \( P_2 \) has no contribution from ring B because it is at the center, thus it is only affected by ring A.
- So the order (smallest \( E \) to largest \( E \)) is \( P_1, P_2, P_3 \)
2. Which point has largest E?
Assume charges on rings are +q and –q
Answer to Question #2

$P_1$ has largest E field since it is equidistant from ring A and B and their E contributions add, rather than cancel, as in the first question.
Calculate E of Dipole (⊥ axis)

-> At point x, $E_x = 0$ and $E_y < 0$. Why?

$$r = \sqrt{x^2 + d^2 / 4}$$
$$\sin \theta = \frac{d / 2}{r}$$

$$E_y = \frac{-2kQ}{r^2} \sin \theta = 2 \left( \frac{-kQ}{x^2 + d^2 / 4} \right) \frac{d / 2}{\sqrt{x^2 + d^2 / 4}} = \frac{-kQd}{\left( x^2 + d^2 / 4 \right)^{3/2}}$$

$$E_y \approx -\frac{kp}{x^3} \quad x \gg d \quad p = Qd \text{ (dipole moment)}$$
Calculate E of Dipole (along axis)

At point x, \( E_x > 0 \) and \( E_y = 0 \). Why?

\[
E_x = \frac{kQ}{(x-d/2)^2} - \frac{kQ}{(x+d/2)^2} = \frac{2kQxd}{(x^2 - d^2/4)^2}
\]

\[
E_x \approx \frac{2kp}{x^3} \quad x \gg d \quad p = Qd \quad \text{(dipole moment)}
\]

Show this yourself (algebra)
Finding E Field from Charge Distribution

Perform integral over charge distribution
- Each component must be calculated separately (vector addition)

\[ dE_y = \frac{k dq}{r^2} (\sin \theta \text{ or } \cos \theta) \]

General helpful rules
- Use symmetry to see if any component must be zero
- Use symmetry to see if any component is doubled, etc.
- Express dq, r and trig functions in terms of "natural" variables defined by the problem
- Then we can integrate!
Find E at Center of Uniformly Charged Circle

+q on top half
−q on the bottom half

Symmetry:
$E_x = 0$: $E_x$ (right) = $-E_x$ (left)
$E_y$ doubled: $E_y$ (top) = $+E_y$ (bottom)

$\lambda = \frac{q}{\pi r} = \text{charge / unit length}$
$\ dq = \lambda \, ds = \lambda r \, d\theta$
$r = \text{const}$
Center of Uniformly Charged Circle

Find $E$

- Express $dq$, $r$, $\sin \theta$ in terms of $\theta$
- Top, bottom give same contribution

\[
dE_y = -\frac{k dq}{r^2} \sin \theta = -\frac{k \lambda rd \theta}{r^2} \sin \theta
\]

\[
E_y = 2 \times \int_0^\pi -\frac{k \lambda rd \theta}{r^2} \sin \theta
\]

\[
= \frac{-2k \lambda}{r} (-\cos \theta)_{0}^{\pi}
\]

\[
= \frac{-4k \lambda}{r}
\]

\[
\lambda = \frac{q}{\pi r}
\]

\[
E_y = -\frac{4k q}{\pi r^2}
\]