

Physics 2049 Exam 1 Solutions
Fall 2002

1. A metal ball is suspended by a string. A positively charged plastic ruler is placed near the ball, which is observed to be attracted to the ruler. What can we say for certain about the metal ball?

Answer. The metal ball is either electrically neutral or negatively charged.

Solution. If the ball is already negatively charged, then it will be attracted toward the ruler. There is another possibility, however—the ball could be neutral and charged ruler could *polarize* mobile charges in the ball, causing it to be attracted to the ruler.

2. A positively charged metal sphere A is brought into contact with an uncharged metal sphere B . As a result:

Answer. Both spheres are now positively charged.

Solution. Since both spheres are conductors, the charge on sphere A is able to flow onto B once they touch. Having charge flow from A to B occurs since the like charges repel and want to be as far apart from each other as possible.

3. A electron is fixed at the position $x = 0$, and a second charge q is fixed at $x = 4 \times 10^{-9}$ m (to the right). A proton is now placed between the two at $x = 1 \times 10^{-9}$ m. What must the charge q be (magnitude *and* sign) so that the proton is in equilibrium?

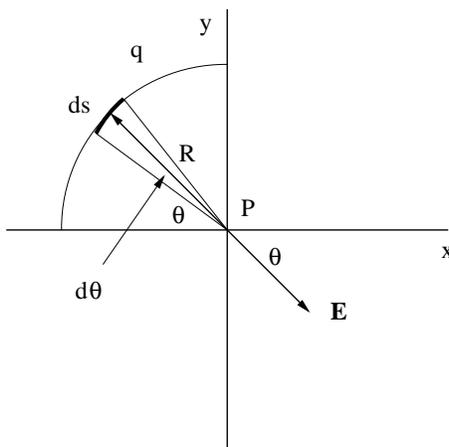
Answer. -1.44×10^{-18} C

Solution. In order that the net force on the proton is zero the charge q must be negative. If we call the distance between the electron and proton a , the distance between the proton and q is $3a$, so that the force on the proton is

$$F_x = -\frac{ke^2}{a^2} + \frac{k|q|e}{(3a)^2}. \quad (1)$$

In equilibrium $F_x = 0$; solving for $|q|$, we find $|q| = 9e$, or $q = -9e$. Using $e = 1.6 \times 10^{-19}$ C (something you should remember), then $q = -1.44 \times 10^{-18}$ C.

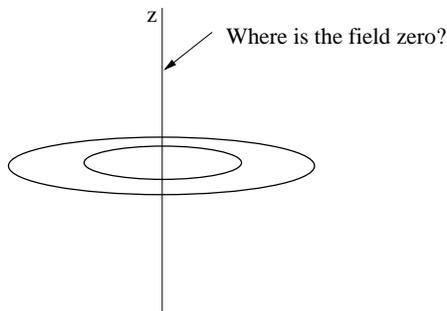
4. A charge q is distributed uniformly on a quarter circle of radius R as shown in the figure. What is the *magnitude* of the electric field at the center (the point P)?



Answer. $(2\sqrt{2}/\pi)(kq/R^2)$

Solution. The charge per unit length on the quarter circle is $\lambda = q/(\pi R/2) = 2q/\pi R$. The charge dq in the segment of length ds is $dq = \lambda ds = (2q/\pi R)ds$; in terms of $d\theta = ds/R$, we have $dq = \lambda ds = (2q/\pi) d\theta$. The magnitude of the electric field at the origin due to this segment is $dE = kdq/R^2 = (2kq/\pi R^2)d\theta$, with components $dE_x = (2kq/\pi R^2) \cos \theta d\theta$ and $dE_y = -(2kq/\pi R^2) \sin \theta d\theta$. Integrating from $\theta = 0$ to $\pi/2$, we find $E_x = 2kq/\pi R^2$ and $E_y = -2kq/\pi R^2$, so that the magnitude of the electric field is $E = \sqrt{E_x^2 + E_y^2} = 2\sqrt{2}kq/\pi R^2$.

5. Two concentric rings, one of radius R and total charge Q and the second of radius $2R$ and total charge $-\sqrt{8}Q$, are in the plane $z = 0$. The charge on each ring is distributed uniformly. Where on the positive z -axis is the electric field zero?



Answer. $\sqrt{2}R$

Solution. The electric field on the z -axis from a *single* ring of radius R and charge Q is

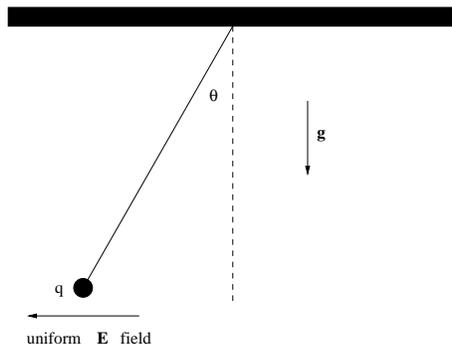
$$E_z = \frac{kQz}{(z^2 + R^2)^{3/2}}. \quad (2)$$

For both rings, we have

$$E_z = \frac{kQz}{(z^2 + R^2)^{3/2}} - \frac{k\sqrt{8}Qz}{[z^2 + (2R)^2]^{3/2}}. \quad (3)$$

Setting $E_z = 0$ and solving for z , we find $z = \sqrt{2}R$.

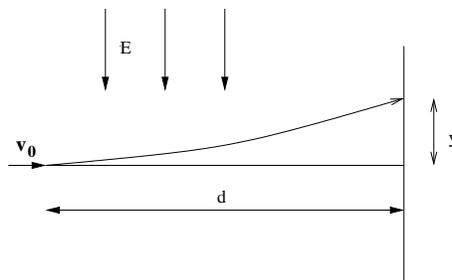
6. A ball of mass $m = 0.75$ grams is suspended from a thread of negligible mass. The ball carries a charge $q = +32 \times 10^{-6}$ C, and is placed in a uniform electric field of magnitude E which points to the left. If the angle $\theta = 28^\circ$, what is the magnitude of the electric field? (recall $g = 9.8$ m/s²).



Answer. 120 N/C

Solution. There are three forces acting upon the ball—the gravitational force $m\vec{g}$, the electrostatic force $q\vec{E}$, and the tension in the thread \vec{T} . By drawing a free-body diagram and setting the forces in the x and y directions equal to zero, we find that $T \sin \theta = qE$ and $T \cos \theta = mg$; dividing these two equations, we obtain $\tan \theta = qE/mg$. Finally, solving for E we find $E = (mg/q) \tan \theta = 120$ N/C.

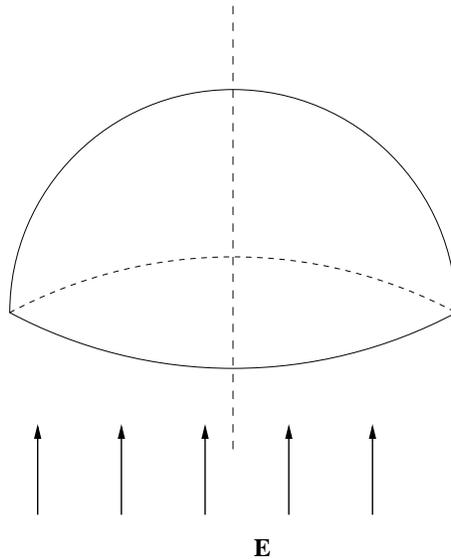
7. An ink droplet of mass m and charge q is injected horizontally with an initially velocity \vec{v}_0 into a region with an electric field \vec{E} which is perpendicular to \vec{v}_0 . If a piece of paper is positioned a distance d away from the injection point as shown in the figure, what is the vertical deflection y of the droplet?



Answer. $qEd^2/(2mv_0^2)$

Solution. The force on the ink droplet is $q\vec{E}$ in the y -direction, so that the acceleration in the y -direction is $a_y = qE/m$. The initial velocity in the y -direction is zero, and the droplet starts at $y = 0$, so integrating twice we obtain $y = (qE/2m)t^2$. There are no forces in the x -direction so that the velocity $v_x = v_0$ is constant. To travel a distance d in the x -direction requires a time $t = d/v_0$, so finally we have $y = qEd^2/(2mv_0^2)$.

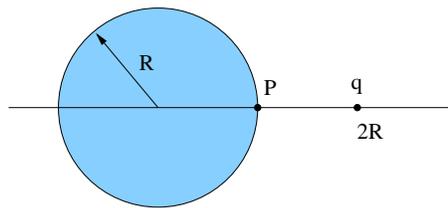
8. A hemispherical shell of radius R is placed in an electric field \vec{E} which is parallel to its axis. What is the flux Φ_E of the electric field through the shell?



Answer. $\Phi_E = \pi R^2 E$

Solution. If the shell had a bottom on it (i.e., if it were a closed surface), then from Gauss's Law the net flux through it must be zero since no charge is enclosed. Since the field is uniform and perpendicular to the circle at the bottom, the flux through that piece would be $\Phi_{\text{circle}} = -\pi R^2 E$ (negative since the field is *into* the interior of the surface). Therefore, the flux through the hemispherical shell itself is $\Phi_{\text{hemisphere}} = \pi R^2 E$.

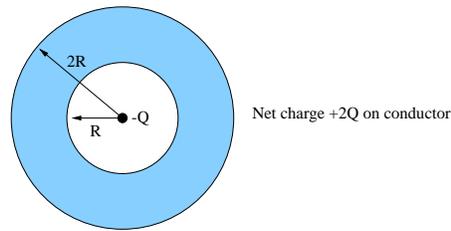
9. A sphere of radius R contains a total charge Q which is uniformly distributed throughout its volume. At a distance $2R$ from the center of the sphere we place a point charge of charge q . What is the value of q which makes the electric field at the point P on the surface zero?



Answer. Q

Solution. The electric field from the sphere of charge at its surface is kQ/R^2 (pointing outward from the sphere, assuming that the charge is positive). The electric field from q at the same point on the sphere is $-kq/R^2$ (pointing inward toward the center of the sphere, assuming that the point charge is positive). These two contributions will add to zero when $q = Q$.

10. A point charge of charge $-Q$ is placed at the center of a solid spherical conducting shell of inner radius R and outer radius $2R$. The shell is in static equilibrium and has a *net* charge $+2Q$. What is the total charge on the outer surface (at $r = 2R$) of the shell?



Answer. $+Q$

Solution. In order that the electric field in the conducting material (the gray/blue area) be zero, a net charge of $+Q$ must be present on the inner hollow surface of the conductor. Since the conductor itself has a *net* charge of $+2Q$, and excess charge can only reside on the surface of the conductor, there must be $+Q$ uniformly distributed on the outer surface.