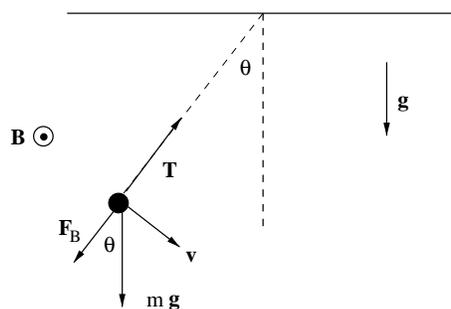


Physics 2049 Exam 3 Solutions
Fall 2002

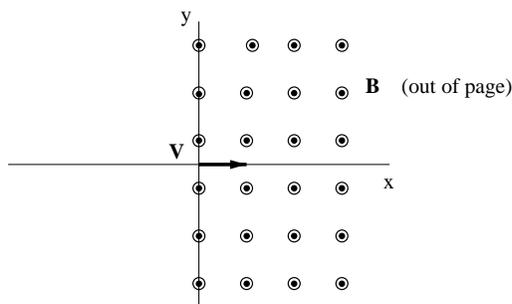
1. A pendulum made of a rigid massless rod of length l is attached to a ball of mass m and charge q ; there is a uniform, horizontal magnetic field \vec{B} out of the plane of the page, as shown in the figure. If the pendulum is set swinging, what is the period T of small amplitude oscillations (i.e., for small θ)?



Answer: $2\pi\sqrt{l/g}$

Solution: The magnetic field does no work on the moving charge, so it can not change the period of oscillation, which for small amplitude oscillations is $T = 2\pi\sqrt{l/g}$. Another way of seeing this is to draw a free-body diagram; if the charge on the ball is positive, then when the ball is descending, as shown in the figure, the magnetic force \vec{F}_B is perpendicular to the path of the ball. This changes the tension \vec{T} in the rod, but not the angular acceleration of the ball.

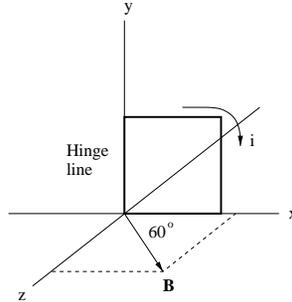
2. The region of space $x > 0$ is permeated by a uniform magnetic field $\vec{B} = B\hat{k}$ pointing in the z direction; the field is zero in the region $x < 0$. A particle of charge q and velocity $\vec{v} = v\hat{i}$ enters the region of non-zero field at the point with (x, y, z) coordinates $(0, 0, 0)$. What are the coordinates of the point where the particle exits the region of nonzero field?



Answer: $(0, -2mv/(qB), 0)$

Solution: Once the particle enters the region $x > 0$ it experiences the Lorentz force, which leads to a centripetal acceleration. Using Newton's Second Law, $mv^2/r = qvB$, so that the radius of the circle is $r = mv/(qB)$ and the diameter is $d = 2mv/(qB)$. If we assume for the moment that $q > 0$, then the particle moves in a clockwise sense and will exit the field at the point $(0, -2mv/(qB), 0)$.

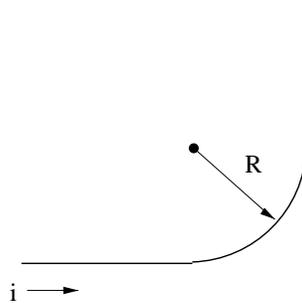
3. The figure shows a square ($10\text{ cm} \times 10\text{ cm}$), 10-turn coil of wire, which carries a current of 1.0 A . The coil is hinged along one side, as shown in the figure, and is mounted in the $x - y$ plane, at 60° to the direction of a uniform magnetic field of magnitude 2.0 T . What is the magnitude of the torque acting on the coil (about the hinge line)?



Answer: $0.10\text{ N} \cdot \text{m}$

Solution: The coil has a magnetic dipole moment $\vec{\mu}$ which points along the negative- z axis, with a magnitude $\mu = NiA$, with N the number of turns, A the area of the loop, and i the current. The magnetic field exerts a torque $\vec{\tau} = \vec{\mu} \times \vec{B}$, which points in the negative- y direction and has the magnitude $\tau = \mu B \sin \theta$, with $\theta = 150^\circ$ the angle between $\vec{\mu}$ and \vec{B} . Putting all of this together, we find $\tau = 0.10\text{ N} \cdot \text{m}$.

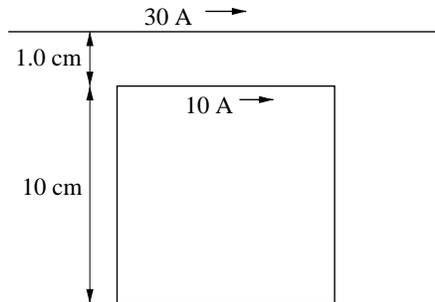
4. A conducting wire is formed into two long, semi-infinite straight sections connected by a quarter circle of radius R , as shown in the figure. What is the magnitude and direction of the magnetic field at the center of the quarter circle?



Answer: $(\mu_0 i / 4\pi R)(2 + \pi/2)$, out of the page

Solution: Using the right hand rule we see that all three segments (semi-infinite line, quarter circle, semi-infinite line) produce a field at the center of the quarter circle which is out of the plane of the page. The magnitude of the field at this point is then obtained by superposition: the two semi-infinite lines each contribute a field $\mu_0 i / (4\pi R)$, while the quarter circle contributes $[\mu_0 i / (4\pi R)](\pi/2)$. Adding these, we obtain the answer above.

5. As shown in the figure, a long straight wire carries a current of 30 A and is coplanar with a square loop of side 10 cm which carries a current of 10 A. If the distance between the wire and the nearest side of the loop is 1.0 cm, what is the magnitude and direction of the net force on the loop? Recall that $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$.



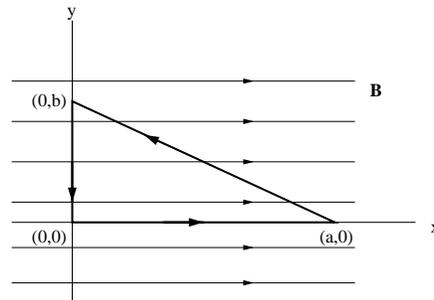
Answer: $5.4 \times 10^{-4} \text{ N}$, up

Solution: The force on the top segment of the loop is upward, with a magnitude $F = \mu_0 i_1 i_2 L / (2\pi d_1)$, with L the length of one side of the loop and d_1 the distance between the infinite wire and the nearest side of the loop. Likewise, the force on the bottom segment of the loop is repulsive, with magnitude $F = \mu_0 i_1 i_2 L / (2\pi d_2)$, with d_2 the distance between the infinite wire and the far side of the loop. The net force is attractive (so that the force is up), with a magnitude

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi} \left(\frac{1}{d_1} - \frac{1}{d_2} \right). \quad (1)$$

Using $d_1 = 0.010 \text{ m}$, $d_2 = 0.11 \text{ m}$, and $L = 0.10 \text{ m}$, we obtain the answer above.

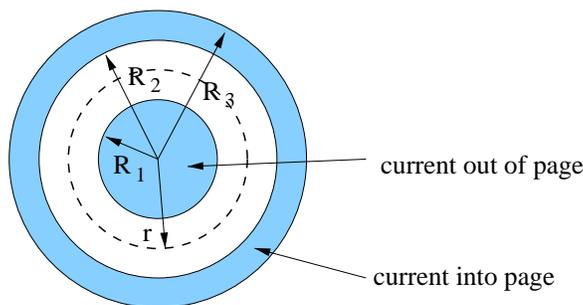
6. A uniform magnetic field $\vec{B} = B_0 \hat{i}$ points in the x -direction. What is the value of the integral $\oint \vec{B} \cdot d\vec{s}$ around the closed triangular path shown in the figure?



Answer: 0

Solution: The path encloses zero current, so by Ampère's Law the integral is zero. This can also be obtained by carrying out the integral explicitly.

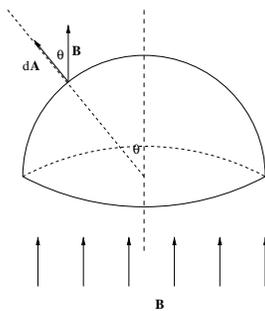
7. A long, straight coaxial cable consists of an inner conductor of radius R_1 , surrounded by an outer “tube” of inner radius R_2 and outer radius R_3 , as shown in the figure. The inner conductor carries a net current i (out of the page) and the outer conductor carries a net current $-i$ (into the page); the current is uniformly distributed in both conductors. What is the magnitude of the magnetic field at a distance r from the axis of the inner conductor, in the region between the conductors (i.e., for $R_1 < r < R_2$)?



Answer: $\mu_0 i / (2\pi r)$

Solution: Draw an Amperian loop of radius r as shown in the figure. This loop encloses a total current i (the return current in the outer conductor is outside the loop and doesn't enter the calculation), so that the field in this region is the same as that produced by a thin wire at the origin which carries a current i ; i.e., $B = \mu_0 i / (2\pi r)$.

8. A hollow hemispherical “bowl” of radius R is placed in a *uniform* magnetic field of magnitude B which is parallel to its axis. What is the magnetic flux Φ_B through the bowl?

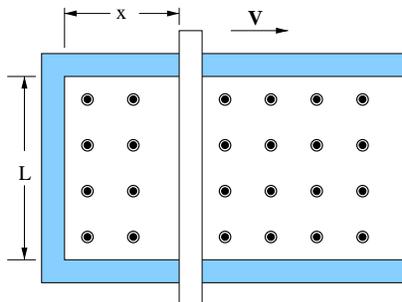


Answer: $\pi R^2 B$

Solution: This is the same problem which appeared on Exam 1 (Problem 8), but for the magnetic field rather than the electric field. The flux through the hemispherical bowl is the same as the flux through the the circle of radius R , so the result is $\Phi_B = \pi R^2 B$. It is also possible to obtain this result by explicit integration. The quantity $\vec{B} \cdot d\vec{A} = B dA \cos \theta$. In spherical coordinates, the area element is $dA = R^2 \sin \theta d\theta d\phi$, with ϕ the azimuthal angle (the angle about the axis of the bowl). We need to integrate this from $\phi = 0$ to 2π , and $\theta = 0$ to $\pi/2$:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin \theta d\theta (BR^2 \cos \theta) = \pi R^2 B. \quad (2)$$

9. A rod of resistance R lies across frictionless, conducting rails in a constant, uniform magnetic field \vec{B} which points out of the plane of the page, as shown in the figure. Assume that the rails have negligible electrical resistance. What is the magnitude of the force which must be applied to the rod to move it to the right with a speed v ?



Answer: B^2L^2v/R

Solution: The flux through the loop at a time t will be $\Phi_B = BLx$, so that $d\Phi_B/dt = (BL)dx/dt = BLv$. This gives the emf in the circuit; since the rod has a resistance R , there is a current in the circuit $i = \mathcal{E}/R = BLv/R$. The rod then experiences a Lorentz force of magnitude $F = iBL = B^2L^2v/R$.

10. What is the emf induced in a solenoid with an inductance of 0.25 H if the current is uniformly reduced from 2.0 A to 0 A in 1/16 of a second?

Answer: 8.0 V

Solution: The emf is Ldi/dt ; in this case $di/dt = (2.0 \text{ A})/(1/16 \text{ s}) = 32 \text{ A/s}$, so the emf is 8.0 V.