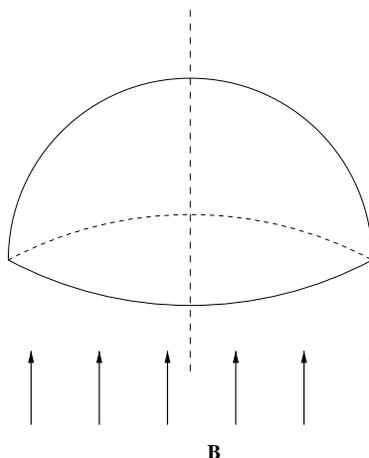


Physics 2049 Exam 4 Solutions
Fall 2002

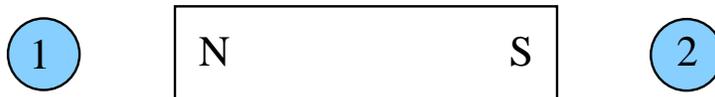
1. A closed surface consists of a hemispherical “bowl” of radius R along with a circular “cap”. This surface is placed in a uniform magnetic field of magnitude B which is parallel to its axis. What is the magnetic flux Φ_B through the closed surface?



Answer: 0

Solution: The second Maxwell equation (“Gauss’s law for magnetic fields”) tells us that the magnetic flux through any closed surface is zero. This is a mathematical statement of the experimental fact that there are no magnetic “monopoles”.

2. The figure shows two small diamagnetic spheres, one near each end of a bar magnet. Which of the following statements is true?



Answer: The forces on 1 and 2 are both repulsive.

Solution: Diamagnetic materials develop an induced magnetic moment which is opposed to the applied field. This will lead to a repulsive force on both spheres.

3. A parallel-plate capacitor of capacitance $C = 2 \times 10^{-3}$ F has circular plates with radius $R = 1.0$ cm. It is connected across a power supply which provides an emf which changes in time according to $\mathcal{E}(t) = at$, with $a = 150$ V/s. What is the magnitude of the magnetic field between the plates of the capacitor a distance $r = R$ from the center? [Recall that $\mu_0 = 4\pi \times 10^{-7}$ T m/A and $\epsilon_0 = 8.85 \times 10^{-12}$ C²/N m.]

Answer: 6.0×10^{-6} T

Solution: This problem requires using the fourth Maxwell equation; in the region between the capacitor plates there is no current, only “displacement current,” so this equation becomes

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}, \quad (1)$$

where Φ_E is the flux of the electric field. Using an amperian loop which is a circle concentric with the axis of the capacitor, we find that $\oint \vec{B} \cdot d\vec{s} = B(2\pi R)$. The electric flux through this loop is $\Phi_E = EA$, where E is the magnitude of the electric field and A is the area inside the loop. The electric field is uniform between the plates with a value $E = \mathcal{E}/d$, so that the flux is $\Phi_E = \mathcal{E}A/d$. Putting all of this together,

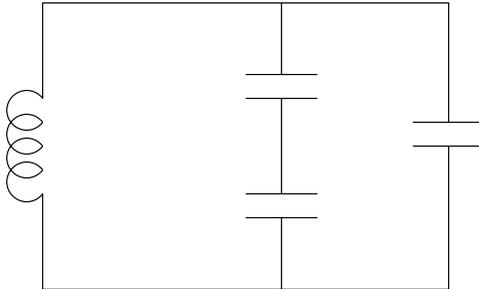
$$B(2\pi R) = \mu_0\epsilon_0(A/d)\frac{d\mathcal{E}}{dt}. \quad (2)$$

However, $\epsilon_0A/d = C$, the capacitance of the capacitor. Solving for B ,

$$B = \frac{\mu_0 C}{2\pi R} \frac{d\mathcal{E}}{dt} = \frac{\mu_0 C a}{2\pi R}. \quad (3)$$

Putting in the numbers, $B = 6.0 \times 10^{-6}$ T.

4. In the LC circuit shown the capacitors have the same capacitance C and the inductor has inductance L . Find the *period* of oscillations for this circuit.



Answer: $2\pi\sqrt{3LC/2}$

Solution: The angular frequency of oscillation is $\omega = 1/\sqrt{L_{\text{eff}}C_{\text{eff}}}$, where L_{eff} and C_{eff} are the effective (or equivalent) values of the inductance and capacitance. The inductance is simply $L_{\text{eff}} = L$; combining the capacitors we find that $C_{\text{eff}} = 3C/2$. The period of oscillation is $T = 2\pi/\omega = 2\pi\sqrt{3LC/2}$.

5. Consider the average power $P(\omega_d)$ dissipated in the resistor of a series RLC circuit driven at angular frequency ω_d ; as a function of ω_d , $P(\omega_d)$ has a maximum at the resonant frequency $\omega = 1/\sqrt{LC}$. For $R = 1.0 \Omega$, $L = 1.0$ H, and $C = 1.0 \mu\text{F}$, what are the angular frequencies for which P is half this maximum value?

Answer: (1000 ± 0.5) rad/s

Solution: The average power dissipated is $P = I^2 R = \mathcal{E}^2 R/Z^2$ (where we have used $I = \mathcal{E}/Z$, where Z is the impedance). The power is therefore

$$P(\omega_d) = \frac{\mathcal{E}^2 R}{[\omega_d L - 1/(\omega_d C)]^2 + R^2}. \quad (4)$$

At resonance, the power is $P = \mathcal{E}^2/R$. We want to find the frequencies at which the power is half this maximum value:

$$\frac{\mathcal{E}^2 R}{[\omega_d L - 1/(\omega_d C)]^2 + R^2} = \frac{\mathcal{E}^2}{2R}. \quad (5)$$

The factor of \mathcal{E}^2 drops out, and we have

$$\left[\omega_d L - \frac{1}{\omega_d C}\right]^2 + R^2 = 2R^2. \quad (6)$$

Subtracting R^2 from both sides, and taking a square root, we find

$$\omega_d L - \frac{1}{\omega_d C} = \pm R. \quad (7)$$

Finally, we multiply through by ω_d to obtain a quadratic equation for ω_d . Substituting in the numbers, and finding the roots, we obtain (1000 ± 0.5) rad/s. You can see that this a very sharp maximum.

6. In an oscillating LC circuit, the total stored energy is U and the maximum charge on the capacitor is Q . At some instant of time the charge on the capacitor is $Q/2$; what is the energy stored in the inductor at this time?

Answer: $3U/4$

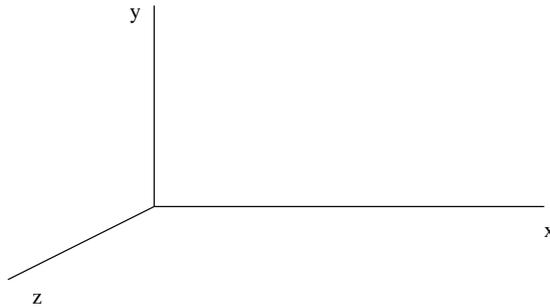
Solution: When the capacitor has its maximum charge there is no current in the circuit and the energy stored in the inductor is zero; therefore, $U = U_C = Q^2/2C$. If the charge on the capacitor is now halved, then $U_C = (Q/2)^2/2C = U/4$. Since the total energy $U = U_C + U_L$ is conserved, the energy stored in the inductor at this instant is $U_L = U - U_C = 3U/4$.

7. The primary of an ideal transformer has 100 turns and the secondary has 600 turns. Which of the following statements is correct?

Answer: The primary current is six times the secondary current.

Solution: This is “step-up” transformer, which steps up the potential difference by a factor of six, but correspondingly steps down the current by a factor of six; i.e., the current in the primary is six times the current in the secondary.

8. The electric field of a plane-polarized electromagnetic wave is given by $E_z(x, t) = E_0 \sin(kx - \omega t)$. What is the corresponding magnetic field?



Answer: $B_y(x, t) = -(E_0/c) \sin(kx - \omega t)$

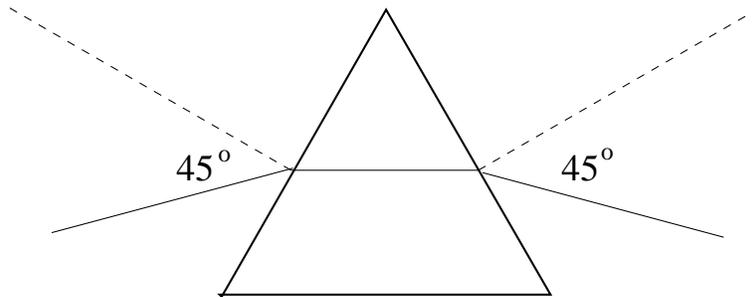
Solution: The wave is propagating in the positive x -direction, so using the right-hand rule we see that if the electric field is in the z -direction, then the magnetic field must be in the $-y$ direction. It has a magnitude of $B_0 = E_0/c$, and the same functional form as the electric field, so that $B_y = -(E_0/c) \sin(kx - \omega t)$.

9. A horizontal beam of vertically polarized light of intensity I_0 is sent through two polarizing sheets. The polarizing direction of the first is at a 70° to the vertical, and that of the second is horizontal. What is the intensity of the light transmitted by the pair of sheets?

Answer: $0.103I_0$

Solution: The transmitted intensity through the first sheet is $I_1 = I_0 \cos^2 70^\circ = 0.117I_0$. The transmitted intensity through the second sheet is $I_2 = I_1 \cos^2 20^\circ = 0.883I_1 = 0.103I_0$.

10. Light is incident upon the left face of an equilateral triangle at an angle of 45° with respect to the normal (indicated by the dashed lines in the figure). What is the index of refraction of the glass if the incoming and outgoing angles are equal? The index of refraction of the surrounding air is $n_{\text{air}} = 1$.



Answer: 1.41

Solution: By doing a bit of geometry you can see that the angle of refraction for the ray incident on the left face of the triangle is 30° ; Snell's law then tells us that $n_{\text{air}} \sin 45^\circ = n_{\text{glass}} \sin 30^\circ$; solving for n_{glass} we find $n_{\text{glass}} = \sqrt{2} = 1.41$.