

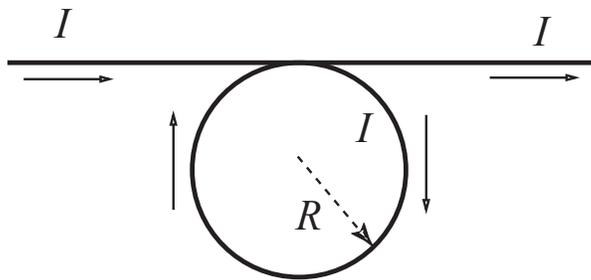
# Solution to Exam 2

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1. Two 12 V batteries, A and B, are connected in series. Battery A has an internal resistance of  $1.0\Omega$  and battery B has an internal resistance of  $0.5\Omega$ . What is the effective voltage and internal resistance of the combination?

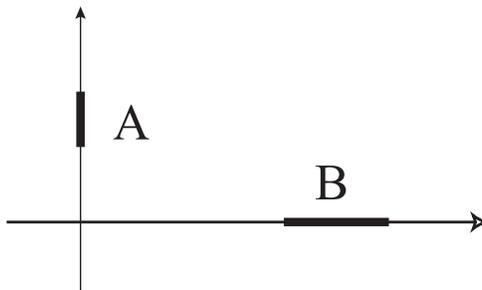
**Solution:** When batteries are put in series, the equivalent EMF is the sum of the voltages and the total resistance is the sum of the individual resistances. Thus the effective EMF is 24V and the internal resistance is  $1.5\Omega$ .

2. An infinitely long insulated wire carrying a current  $I$  is bent into the shape shown (straight line plus circle of radius  $R$  with the currents in the direction shown). The magnitude of the field  $B$  at the center of the circle is:



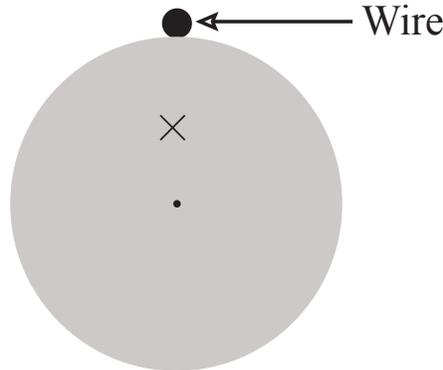
**Solution:** The total field is the sum of the field from the infinite wire and the circle. Taking into account the fact that the fields are in the same direction, we obtain  $B = \mu_0 I / 2\pi R + \mu_0 I / 2R = \mu_0 I (\pi + 1) / 2\pi R$ .

3. Two currents, flow at right angles as shown in the figure. What is the direction of the force on wire segment B due to wire segment A?



**Solution:** Applying the right hand rule for the field from A, we see that it is going into the page at location B. Using the force right hand rule for the current at B, this gives a force acting in the upwards direction.

4. A solid circular cylinder of radius  $R$  carries a uniformly distributed current  $I$ . A wire runs parallel to the cylinder along the surface (see figure). Find the magnitude and direction (same or opposite to the current in the cylinder) of the current in the wire such that the resultant magnetic field at a point (marked with a cross) halfway between the wire and the center is zero.



**Solution:** From Ampere's law, the effective current flowing inside the circle is  $I/4$ . Since the point marked with the cross is the same distance from the center of the cylinder as it is from the wire, the current needed in the wire is also  $I/4$  to create a magnetic field of the same magnitude. The current in the wire must run in the *same* direction as the current in the cylinder to cancel the  $B$  field.

5. A lightbulb gives a power output of 100 Watts when connected across a standard voltage source. Keeping the volume of the filament (the thin wire in the middle of the bulb) constant, by what factor must its length be multiplied to maintain the same power output if the EMF of the voltage source is doubled?

**Solution:** The power output for fixed voltage is  $V^2/R$ . Thus if the EMF of the voltage source is doubled, the resistance of the filament must be made 4 times larger. By stretching the filament to twice its length, the cross-sectional area is twice as small, leading to 4 times the resistance. Thus the answer is *2 times* the original length.

6. Two parallel wires 100m long carry 10A currents in opposite directions. If the magnitude of the force between the wires is 1N, what is the distance in meters between the wires and the direction of the force?

**Solution:** The total force between the wires is  $F = \mu_0 I_1 I_2 L / 2\pi d$ , where  $d$  is the distance between the wires and  $L$  is their mutual length. Substituting yields  $F = 0.002\text{N}$ , and the force is repulsive since the currents are anti-parallel.

7. A particle of charge  $2\mu\text{C}$  is accelerated from rest across a potential difference of  $1.0\text{kV}$ . It proceeds to enter a region of uniform magnetic field of magnitude  $B = 2\text{T}$  directed perpendicular to the particle's velocity. If the particle then moves in a circle of radius  $1.25\text{mm}$ , what is the mass of the particle (in kg)?

**Solution:** The kinetic energy acquired from the acceleration is  $\frac{1}{2}mv^2 = qV$ , where  $V$  is the potential. The momentum and radius of the circle in the  $B$  field are related by  $mv = qBr$ . Eliminating the velocity  $v$  yields for the mass  $m = qB^2r^2/2V = 6.25 \times 10^{-15}\text{kg}$ .

8. You have a bag of  $100\Omega$  resistors from which you have to create a total resistance of  $225\Omega$  in a circuit. What is the minimum number of resistors needed?

**Solution:** To get a multiple of  $25\Omega$ , you must have 4 resistors in parallel. Then add that combination in series with two other resistors to yield a total of 6 resistors.

9. A charge of  $1.5\mu\text{C}$  is stored in a  $1.0\mu\text{F}$  capacitor. At time  $t = 0$  a  $10\Omega$  resistor is connected across the capacitor. Find the instantaneous voltage (in volts) across the resistor at  $t = 2RC$ , i.e., two time constants later.

**Solution:** When the resistor is connected, the charge  $Q$  changes according to  $Q = Q_0e^{-t/RC}$ , where  $Q_0$  is the initial charge. Differentiating yields the current  $I = Q_0/RCe^{-t/RC}$ , thus the voltage across the resistor is  $V = IR = Q_0/Ce^{-t/RC}$ . When  $t = 2RC$ , the voltage is calculated as  $0.20\text{V}$ .

10. Three identical current loops, A, B and C, are stacked on top of one another a short distance apart, with B in the middle. The current in B is counterclockwise as seen from above. In what directions (as seen from above) should the currents in A and C flow to minimize the total energy of the configuration? Note that cw = clockwise and ccw = counterclockwise.

**Solution:** The lowest energy solution is the one where the  $B$  fields from the loops are lined up. Thus if one loop has a counterclockwise current, all currents must be counterclockwise.