Solution to Exam 1

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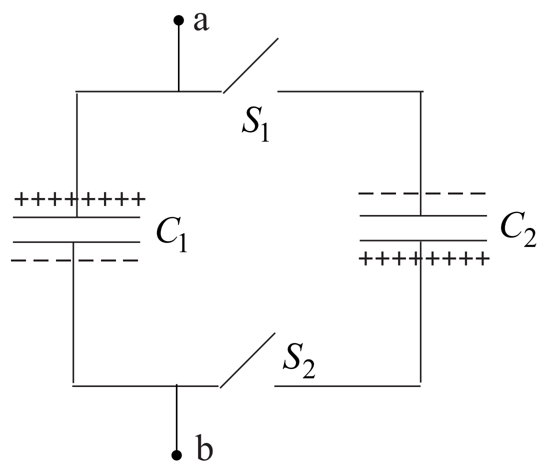
PHY2049, Spring 2003
Feb. 5, 2003

1. In the figure shown, each capacitor is charged to a potential difference of 100 V. After switches S_1 and S_2 are closed, what is the final potential difference $V_b - V_a$ in volts between point a and point b. Note that $C_1 = 1\mu$F and $C_2 = 3\mu$F.



Solution: Initially, the absolute value of the charges on the capacitors are $q_1 = C_1 \times 100 = 10^{-4}$ C and $q_1 = C_2 \times 100 = 3 \times 10^{-4}$ C. When the switches close, a charge Q moves from the positive plate of C_2 to the negative plate of C_1 to make the potential across each capacitor equal. This condition is $V_b - V_a = (q_2 - Q)/C_2 = (-q_1 + Q)/C_1$. Solving for Q yields $Q = 150\mu$C. Thus $V_b - V_a = (q_2 - Q)/C_2 = 50$V.

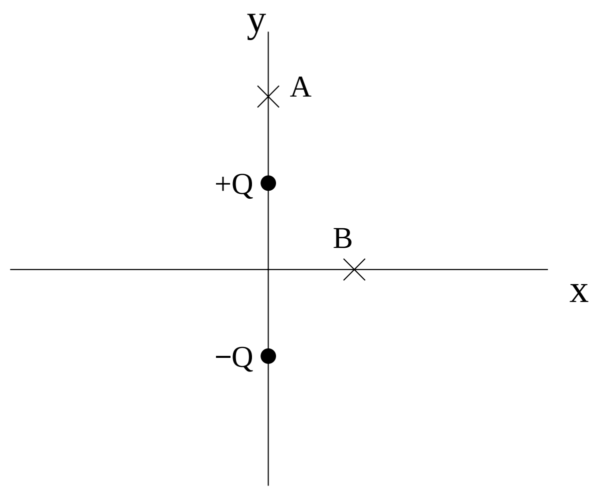
2. Under the influence of the electric force of attraction, the electron in a hydrogen atom orbits around the proton on a circle of radius $r = 0.53 \times 10^{-10}$ m. What is the orbital speed of the electron in m/s?

Solution: The centripetal force holding the electron in its orbit is supplied by the electric field. Let r be the radius of the orbit, e be the magnitude of the proton and electron charge and m_e be the mass of the electron. Then $m_e v^2/r = ke^2/r^2$ or $v = \sqrt{2ke^2/rm_e} = 2.18 \times 10^6$ m/s.

3. Three charges q_1, q_2, and q_3 are placed on the x-axis at $x = 0, a, 2a$ respectively. The electric flux through a sphere of radius $1.5a$, centered on the origin, is found to be $\Phi = 10$ N$\cdot$m^2/C. On the basis of only this information which of the following *must* be true?

Solution: Since only q_1 and q_2 lie inside the sphere, only they contribute to the total flux. By Gauss' law, $\int_S \vec{E} \cdot d\vec{s} = 10 = (q_1 + q_2)/\epsilon_0$. Thus $q_2 = -q_1 + 10\epsilon_0$

4. A positive charge Q is placed on the y-axis at $y = a$ and a charge $-Q$ is placed on the y-axis at $y = -a$. What is the potential difference $V_A - V_B$ between the points $A = (x = 0, y = 2a)$ and $B = (x = a, y = 0)$?



Solution: We calculate the potentials $V_A = kQ/a - kQ/3a = 2kQ/3a$ and $V_B = kQ/\sqrt{2}a - kQ/\sqrt{2}a = 0$. Thus $V_A - V_B = 2kQ/3a$.

5. Neglecting fringe effects, how much energy (in joules) is stored in the field of a parallel plate capacitor charged to 100 volts, where the plates have area $A = 100$ cm^2 and separation $d = 1$ mm and are separated by a dielectric material, with $\kappa = 2$, which completely fills the space between the plates?

Solution: The capacitance is given by $C = \kappa\epsilon_0 A/d = 2 \times 8.85 \times 10^{-12} \times (100 \times 10^{-4})/10^{-3} = 1.77 \times 10^{-10}$F. The total energy is $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 1.77 \times 10^{-10} \times 100^2 = 8.85 \times 10^{-7}$ J.

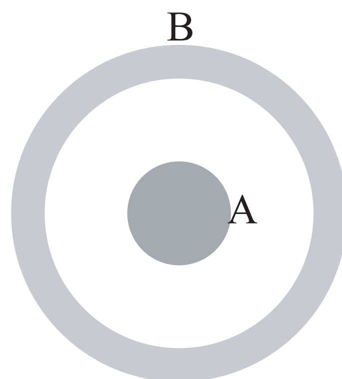
6. The water molecule, H_2O, has a permanent electric dipole moment of 6×10^{-30} C·m. Calculate the maximum torque in N·m on a water molecule in a uniform electric field of 10^5 N/C.

Solution: The maximum torque is $\tau_{\max} = pE$, where p is the dipole moment and E is the electric field strength. This yields $\tau_{\max} = 6 \times 10^{-30} \times 10^5 = 6 \times 10^{-25}$ N·m.

7. Two spherical conductors of radii r_1 and r_2 are connected by a conducting wire. The spheres are separated by a distance much larger than the radius of either sphere. If the charges on the spheres in equilibrium are q_1 and q_2, respectively, find the ratio of the field strengths at the surfaces of the spheres.

Solution: Since the spheres are connected by a conducting wire, they are at the same potential. Thus $kq_1/r_1 = kq_2/r_2$ or $q_1/q_2 = r_1/r_2$. The electric field at the surface of each shell is kq/r^2, thus the ratio of electric fields is $E_1/E_2 = (q_1/q_2)(r_2^2/r_1^2)$. From the previous equality, $E_1/E_2 = r_2/r_1$.

8. The system shown in the figure represents a conducting shell B (with inner and outer radii of 2.0 m and 2.5 m, respectively) surrounding a charged ball A of radius 0.5 m in the center. A and B have charges +10 C and +11 C, respectively. What is the surface charge density in C/m^2 on the inner surface of B?



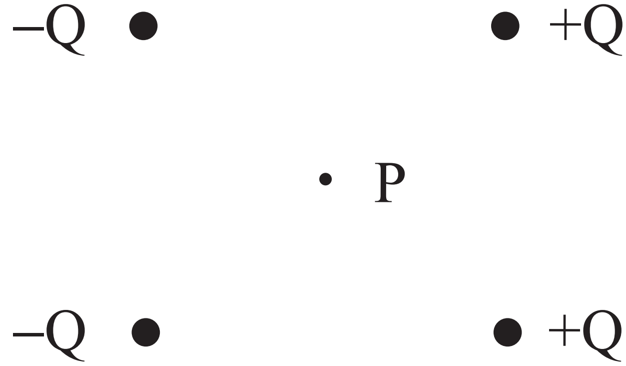
Solution: To get zero electric field inside the conducting spherical shell, the net charge inside the shell (including the inner surface) must be zero. Thus the charge on the inside of the shell is -10C and since the area of the inner shell is $A = 4\pi r^2 = 16\pi$, the surface charge density is $-5/8\pi$ C/m^2.

9. An equipotential surface is *always perpendicular to the electric field*. Potential does not change when a charge is moved perpendicular to an electric field. Remember that $dV = -\vec{E} \cdot d\vec{x}$.

10. A proton and an electron are each accelerated across the same region of constant E field. Which has the larger magnitude of acceleration?

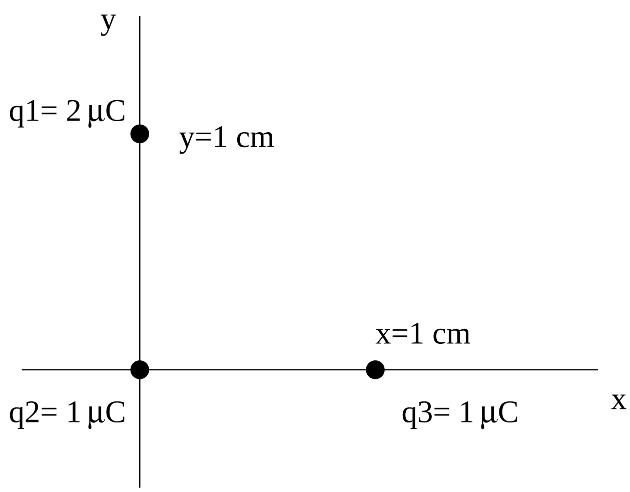
    Solution: The *electron* accelerates more quicly because the force is the same and its mass is less than that of the proton.

11. Consider the 4 charges arranged at the corners of a square as shown in the figure. Which statement is true about the electric field E and potential V at the center of the square?



Solution: The correct choice is $E \neq 0$ and $V = 0$. The potential clearly cancels since the point P is equidistant to all the charges and the charges cancel in pairs. The electric fields add up to give a component along the $-x$ direction.

12. Three charges are placed in the configuration shown with $q_1 = 2\mu$C, $q_2 = 1\mu$C and $q_3 = 1\mu$C. What is the force vector on charge q_3 in newtons?



Solution: The magnitude of the force F_1 from charge 1 acting on charge 3 is $F_1 = kq_1q_3/r_{13}^2 = 90$ N. The magnitude of the force from charge 2 acting on charge 3 is $F_2 = kq_2q_3/r_{23}^2 = 90$ N. Note that $F_{x1} = 90\cos 45 = +63.6, F_{y1} = -90\cos 45 = -63.6, F_{x2} = 90, F_{y2} = 0$. Adding up the components yields a total force of $F_x = 153.6$ and $F_y = -63.6$, or $\mathbf{F} = 153.6\mathbf{i} - 63.6\mathbf{j}$.