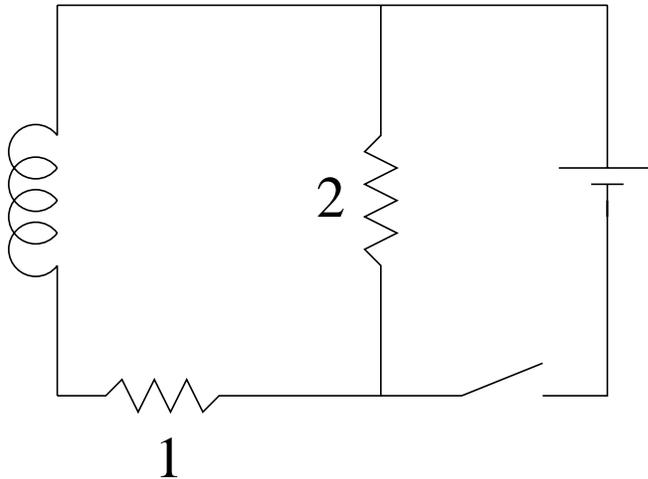


Solution to Exam 3

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1. Which of the following statements is true about the LR circuit shown?



- (a) Just after the switch is closed resistor 1 carries zero current.
- (b) Just after the switch is closed the two resistors carry identical current.
- (c) Just after the switch is closed the current in resistor 1 is greater than that in resistor 2.
- (d) A long time after the switch is closed resistor 1 carries zero current.
- (e) None of these statements are true.

Solution: The first statement is true. Before the switch is closed, no current flows only in any part of the circuit. As soon as the switch is closed, the branch with resistor 2 carries current immediately, while the current in the branch with resistor 1 starts at zero and then moves to its final value.

2. A magnet is held near a large block of material with certain magnetic properties. Which of the following statements are true?
- (a) If the material is paramagnetic, the magnetic field within the material is larger than it would be if the material were absent.
 - (b) If the material is diamagnetic, the magnetic field within the material is smaller than it would be if the material were absent.

- (c) If the material is paramagnetic, it would exert a (weak) attractive force on the magnet.

Solution: All of the statements are true. The first two statements follow from the definitions of paramagnetic and diamagnetic. The last statement follows from the fact that the magnetic field induced in the material is in the same direction as the applied field. Therefore the north pole of the inducing field is close to the south pole of the induced field.

3. An electromagnetic wave travelling parallel to the z -axis has intensity I_0 and is linearly polarized in the x -direction. It passes through 2 polaroid sheets both parallel to the xy plane. The polarization axis of the first sheet makes an angle of 60° with the x -axis and that of the second sheet is parallel to the y -axis. What is the intensity of the wave after it has passed both sheets?

Solution: The intensity after the first sheet is $I_1 = I_0 \cos^2 60^\circ = I_0/4$ and the axis of polarization is rotated by 60° from the x -direction. After it goes through the second sheet, the intensity is easily seen to be $I_2 = I_1 \cos^2 30^\circ = 3I_1/4 = 3I_0/16$.

4. Consider a series RLC circuit for which $R = 150\Omega$, $L = 20\text{mH}$, $V_{\text{rms}} = 2.0\text{V}$ and driving frequency $\omega = 5000 \text{ rad/sec}$. Determine the value of the capacitance (in μF) for which the average current is maximized, i.e., when the driving frequency equals the resonant frequency.

Solution: The current is maximized when the driving frequency is the same as the resonant frequency. The resonant frequency is given by $\omega_0 = 1/\sqrt{LC}$, thus C can be computed to be $2\mu\text{F}$.

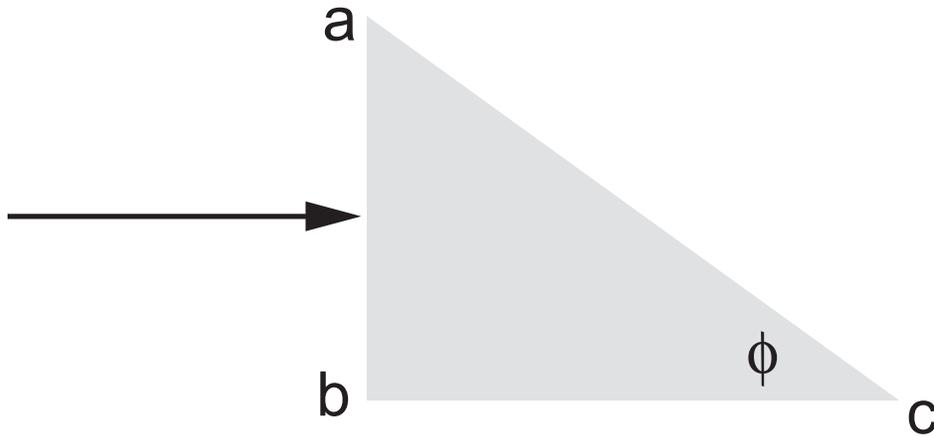
5. Consider the same series RLC circuit as in the previous problem. What is the average power in watts dissipated in the resistor R ?

Solution: At resonance, the power dissipated depends only on the resistance. The average power is given by $P_{\text{ave}} = V_{\text{rms}}^2/R = 4/150 \approx 0.027$ watts.

6. A circuit consists of a current source connected to an inductor with inductance 200 mH . The current in the conductor is 10 amps . What is the velocity (in m/s) of a baseball of mass 80 g whose kinetic energy is equal to the energy stored in the inductor?

Solution: The energy stored in the inductor is $U = \frac{1}{2}LI^2$, where L is the inductance and I is the current. Thus the equation we want to solve is $\frac{1}{2}mv^2 = \frac{1}{2}LI^2$, yielding $v = 10\sqrt{0.200/0.080} \approx 16 \text{ m/s}$.

7. In the figure shown, a ray of light is perpendicular to the face ab of a glass prism ($n = 1.65$) in the shape of a right triangle. Find the largest value for the angle ϕ (in degrees) so that the ray is totally reflected at face ac if the prism is in air.



Solution: The largest value of ϕ occurs when the ray hits ac at the critical angle. From the geometry of the problem, the critical angle occurs for $\cos \phi = 1/1.65$, or $\phi = 52.7^\circ$.

8. An elastic conducting material is stretched into a circular loop of radius 12cm. It is placed with its plane perpendicular to a uniform 0.8T magnetic field. When released, the radius of the loop starts to shrink at an instantaneous rate of 75 cm/s. What is the EMF (in volts) induced in the loop at that instant?

Solution: The generated EMF is given by $\mathcal{E} = -d\phi_B/dt$, where ϕ_B is the total magnetic field flux. Since the field is constant and the area is changing, we find that $\mathcal{E} = -d\phi_B/dt = 2B\pi r dr/dt$, where r is the radius of the loop. Substituting the appropriate quantities yields $\mathcal{E} = 0.452V$.

9. A small rocket of mass 10 kg. emits an intense light beam as a rocket exhaust. What must be the minimum total power (intensity times area) of the beam (in gigawatts) needed to lift the rocket against the force of gravity at the earth's surface?

Solution: The total power of the rocket is related to the total force by $F = P/c$, where c is the speed of light. Thus we require $P/c = mg$ to lift a rocket of mass m from the ground. This yields a power of $P = 10 \times (3 \times 10^8) \times 9.8 = 29$ gigawatts.

10. The speed of light in vacuum depends on which of the following wave properties?

- (a) None of these
- (b) Wavelength
- (c) Frequency
- (d) Intensity
- (e) Amplitude

Solution: The speed of light in vacuum is constant and independent of any wave property.

11. The induced magnetic field 9.0 mm from the central axis of a circular parallel-plate capacitor and between the plates is 2×10^{-7} T. The plates have radius 3.0 mm. At what rate dE/dt is the electric field between the plates changing in units of 10^{12} V/m/sec?

Solution: The magnetic field outside a solenoidal current is given by $B = \mu_0 I / 2\pi r$, where I is the current and r is the distance from the center of the solenoid. In this case, the current is a displacement current caused by a changing electric field, therefore $I = \epsilon_0 d\phi_E/dt$, where ϕ_E is the total electric flux inside the region where the magnetic field is formed. Putting it all together yields $B = \mu_0 \epsilon_0 d\phi_E/dt / 2\pi r$, where $\phi_E = E\pi R^2$, where E is the (changing) electric field strength and R is the (constant) radius of the capacitor. Solving for dE/dt yields $dE/dt = 36 \times 10^{12}$ V/m/sec.

12. A Gaussian surface in the shape of a right circular cylinder with end caps has a radius of 12.0 cm and a length of 80.0 cm. Through one end there is an inward magnetic flux of 25 mWb. At the other end there is a uniform magnetic field which gives rise to a flux of 72.4 mWb, directed outward. What is the net magnetic flux through the curved surface?

Solution: The net magnetic flux through any closed surface must be zero. Since the net flux from the two ends is 47.4 mWb directed outward, the flux through the rest of the surface (the curved part) must be 47.4 mWb directed *inward*.