

Solution to Exam 3

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1. A resistance R and an inductance L are hooked up to a battery. The current in the circuit builds up to one half of its steady-state value in 6.0 s. Find the inductive time constant $\tau_L \equiv L/R$ (in sec).

Solution: The current in this RL circuit varies according to $i = i_0(1 - e^{-t/\tau})$, where $\tau = R/L$ is the time constant. We get $e^{-t/\tau} = 1/2$, or $\tau = t/\ln 2$, yielding $\tau = 8.66$ sec.

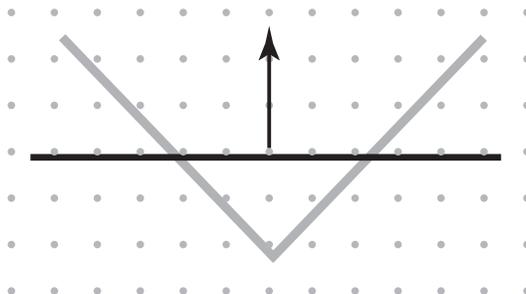
2. Assume the average value of the vertical component of Earth's magnetic field is $63 \mu\text{T}$ (downward) in some region, which has an area of $3.19 \times 10^5 \text{ km}^2$, and calculate the net magnetic flux (in webers) through the rest of Earth's surface (the entire surface excluding that region). Take the negative direction to be inward.

Solution: The total magnetic flux must be zero, so the compensating flux must be $B \times A = +2.0 \times 10^7$.

3. A generator with an adjustable frequency of oscillation is connected in series to a resistance R , a capacitor of $C = 5.0F$, and an inductor of inductance L . The amplitude of the current produced in the circuit by the generator is maximized when the generator's frequency is 1.4 kHz. What is L (in henries)?

Solution: The amplitude is maximized at resonance, where $f = 1/2\pi\sqrt{LC}$. Solving yields $L = 2.6 \times 10^{-9}$ H.

4. Two straight conducting rails form a right angle where their ends are joined. A conducting bar in contact with the rails starts at the vertex at time $t = 0$ and moves with a constant velocity v along them, as shown in the figure. A magnetic field B points out of the page. If we write the emf as $\mathcal{E} = at^n$, where a and n are constants, what is the value of n ?



Solution: The emf \mathcal{E} is related to the rate of change of magnetic flux by $d\mathcal{E}/dt = -d\phi_B/dt$, where ϕ_B is the magnetic flux. In this case, the B field is constant but the area is changing. If we denote t as the time elapsed from when the bar crossed the vertex, then we see that the area is proportional to t^2 , thus the emf varies like $\mathcal{E} \propto t$. So $n = 1$.

5. Suppose that a parallel-plate capacitor has circular plates with radius $R = 35$ mm and a plate separation of 4.1 mm. Suppose also that a sinusoidal potential difference with a maximum value of 130 V and a frequency of 60 Hz is applied across the plates; that is, $V = 130 \sin(2\pi 60t)$. Find $B_{\max}(R)$, the maximum value of the induced magnetic field that occurs at $r = R$.

Solution: The B field can be calculated from the rate of change of electric flux using $\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_D$, where i_D is the displacement current $i_D = \epsilon_0 d\phi_E/dt$. Here ϕ_E is the electric flux. We are given the voltage V , from which the electric field $E = V/d$ (d is the plate separation) can easily be calculated. The rate of change of the electric flux is $dE/dt = 130(120\pi) \cos(120\pi t)/d$. When $r = R$, the integral is $\int \mathbf{B} \cdot d\mathbf{l} = B(2\pi R)$. Thus the maximum B field is $B_{\max} = \mu_0 \epsilon_0 130(120\pi) \pi R^2/d/2\pi R$ or $B_{\max} = 7800\pi R/dc^2$, where we used $\mu_0 \epsilon_0 = 1/c^2$. Substituting yields $B_{\max} = 2.3 \times 10^{-12}$ T.

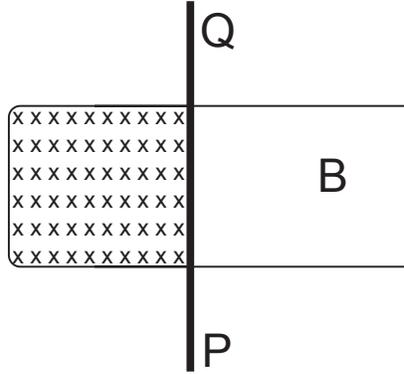
6. An ac generator emf is $\mathcal{E} = \mathcal{E}_m \sin \omega_d t$, with $\mathcal{E}_m = 23.6$ V and $\omega_d = 296$ rad/s. It is connected to a 16.5 H inductor. When the current is a maximum, what is the emf of the generator (in volts)?

Solution: The current lags the emf by 90 degrees (a quarter cycle), so the emf is zero at that time.

7. A beam of partially polarized light can be considered to be a mixture of polarized and unpolarized light. Suppose we send such a beam through a polarizing filter and then rotate the filter through 360 degrees keeping it perpendicular to the beam. If the transmitted intensity varies by a factor of 4.3 during the rotation, what fraction of the intensity of the original beam is associated with the beam's polarized light?

Solution: Let the intensities of the unpolarized and polarized light be 1 and x , respectively. After going through the filter the intensity of the unpolarized light is $1/2$ and the intensity of the polarized light will vary from 0 to x . Thus the ratio of the maximum to minimum intensity of the total beam is $(1/2 + x)/(1/2) = 4.3$. Solving yields $x = 1.65$. The fraction of polarized light is therefore $1.65/2.65 = 0.62$.

8. Consider the arrangement shown in the figure. A conducting rod, PQ, is lying on a U-shaped conducting wire, making good electrical contact with it, and both are supported on a nonconducting table. There is a uniform magnetic field pointing into the table. If the magnetic field strength is increased, which way does the rod move?



Solution: As the magnetic field increases, the current will flow counterclockwise, by Faraday's law. Thus the current in the rod will be moving upwards. The force of this current interacting with the downward magnetic field, using another right-hand rule, moves it to the left. Another way to see this is to apply Lenz' law, which says that the rod moves in the direction needed to offset the increase in the flux, which means it moves to the left to shrink the area to decrease the flux.

9. An isotropic light source is placed 1.8m below the surface of a pool of water ($n = 1.333$). What is the diameter (in meters) of the circle on the surface of the water through which light is transmitted?

Solution: The maximum angle in the water in which light can escape is the critical angle, given by $\sin \theta_c = 1/1.333$, or $\theta_c = 48.8^\circ$. Light can only escape for angles less than the critical angle. Thus the diameter of the circle is $2 \times 1.8 \tan \theta_c = 4.08$ m.

10. A small spherical particle with diameter 0.2mm and density 2 g/cm^3 is suspended over a table by a vertical laser beam. Assume that the particle is totally absorbing. What is the intensity of the laser beam in W/m^2 ?

Solution: The force from the laser beam must be equal to the force of gravity. This gives the condition $IA/c = mg$, where I is the intensity of the laser light, A is the cross-sectional area of the sphere and m is the mass of the sphere. Using the sphere density and radius, we see that $m = 4\pi R^3 \rho / 3$ and $A = \pi R^2$, where ρ is the density and R is the radius. Thus we obtain for the intensity $I = 4\rho Rcg/3 = 7.8 \times 10^8 \text{ W/m}^2$.