

Exam 1 Solutions

1. (*SAT type problem*) A positively charged insulating rod is brought close to an object that is suspended by a string. If the object is attracted toward the rod we can conclude:

- (1) none of these
- (2) the object is positively charged
- (3) the object is an insulator
- (4) the object is a conductor
- (5) the object is negatively charged

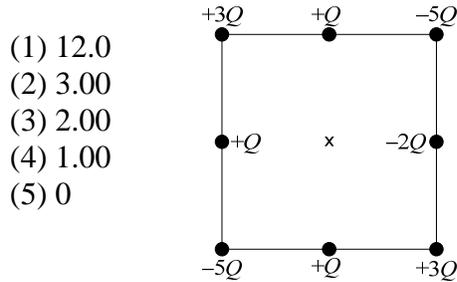
Since either (4) or (5) could explain the phenomenon, none of the answers are conclusive.

2. (*Similar to Webassign and class problem*) Charges $+3Q$ and $-9Q$ are held in place at positions $x = 0$ m and $x = 2$ m, respectively. At what position in x (in m) should a third charge be placed so that it is in equilibrium?

- (1) -2.73
- (2) $+1.33$
- (3) $+3.73$
- (4) -1.33
- (5) -1.86

We see immediately that the equilibrium point should be to the left of the $+3Q$ charge, i.e. $x < 0$. Put a charge q_3 at position x . If L is the distance between the first two charges, then the total force on the third charge at equilibrium is $F_x = -3kQq_3/x^2 + 9kQq_3/(L-x)^2 = 0$. Solving yields $x = -2.73$ m.

3. (*Prob. 22.61*) Charges are arranged on a square of side d as shown in the diagram. The magnitude of the electric field in the center of the square can be written CkQ/d^2 . What is C ?



- (1) 12.0
- (2) 3.00
- (3) 2.00
- (4) 1.00
- (5) 0

All the E fields acting at the center are cancelled except for the charges to the left and right of center. These charges are at distance $d/2$ from the center and act to the left. This gives a total field of $12kQ/d^2$.

4. (*Similar to probs. 24.16, 24.38*) Refer to the previous problem. How much work is required to move a charge $-2Q$ from infinity to the center of the square?

- (1) $+7.31 kQ^2/d$
- (2) $+1.66 kQ^2/d$
- (3) $-12.0 kQ^2/d$
- (4) $+6.00 kQ^2/d$
- (5) 0

The total work done by the external force is the change in potential energy (the work done by the electric force is the negative of the change in potential energy). Initially, $U_i = 0$ and the final value of U is

$$\begin{aligned}
 U_f &= -2kQ(Q - 2Q + Q + Q)/(d/2) - 2kQ(3Q - 5Q + 3Q - 5Q)/(d/\sqrt{2}) \\
 &= (8\sqrt{2} - 4)kQ^2/d = +7.31kQ^2/d
 \end{aligned}$$

Thus the work done by the external force is $W = +7.31kQ^2/d$.

5. (*Prob 24.45*) Two electrons are fixed 1.5 cm apart. Another electron is shot from infinity and stops midway between the two. What is its initial speed in m/s?

- (1) 367
- (2) 233
- (3) 184
- (4) 130
- (5) 81.7

Conservation of energy yields $\frac{1}{2}mv^2 + 0 = 0 + ke^2/(d/2) + ke^2/(d/2) = 4ke^2/d$, with $d = 0.015$ m. Solving for v gives $v = 367$ m/s.

6. 2.0 μC of charge is uniformly distributed on a thin non-conducting disk of radius 10.0 cm. What is the magnitude of the electric field at the center of the disk in V/m?

- (1) 3.6×10^6
- (2) 1.8×10^6
- (3) 5.4×10^6
- (4) 7.2×10^6
- (5) 0

The electric field at the center of the disk can be found from Gauss' law (for z very close to the disk), giving $E = \sigma/2\epsilon_0$ or from the general equation of E vs z that we had in class, which yields the same answer at $z = 0$. This gives $E = 3.6 \times 10^6$ V/m. However, it has been pointed out that exactly at the center $E = 0$, so we are accepting that answer as well.

7. A long, straight wire is uniformly charged with a linear charge density of 4.0 $\mu\text{C}/\text{m}$. The wire runs along the axis of a cylinder of radius 3.0 cm and height 30.0 cm. What is the total electric flux through the cylinder surfaces in $\text{N m}^2/\text{C}$?

- (1) 1.36×10^5
- (2) 3.24×10^5
- (3) 5.74×10^5
- (4) 7.19×10^5
- (5) 0

Gauss' law can be used for this problem, where the total E flux is equal to q/ϵ_0 , where q is the enclosed charge. The charge q can be calculated from the length and the charged line density to yield $q = 4 \times 10^{-6} \times 0.3 = 1.2 \times 10^{-6}$ C. Thus the flux = $q/\epsilon_0 = 1.2 \times 10^{-6} / 8.85 \times 10^{-12} = 1.36 \times 10^5$ Nm^2/C .

8. A solid conducting sphere of 15 cm radius is located at the center of a spherical conducting shell, whose inner radius is 21 cm and outer radius 42 cm. At a distance of 50 cm from the center of these concentric conductors, the electric field is 2.1×10^5 V/m and is pointing outward. The shell has a net charge of $+3.1 \mu\text{C}$. What is the net charge on the sphere at the center in μC ?

- (1) +2.7
- (2) +1.8
- (3) 0.0
- (4) -1.2
- (5) -3.1

We use the fact that the electric field for this total configuration is $E = kQ/r^2$, where Q is the total charge and r is the radius of the point. Solving for Q yields, $5.8 \mu\text{C}$. Since there is $3.1 \mu\text{C}$ on the shell, the charge on the sphere must be $5.8 - 3.1 = 2.7 \mu\text{C}$.

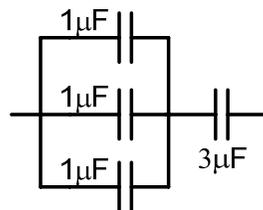
9. A cylindrical capacitor of length L comprises two coaxial cylinders of radii a and b . If all the dimensions are doubled, the capacitance will:

- (1) increase by a factor of 2
- (2) increase by a factor of 4
- (3) decrease by a factor of 2
- (4) decrease by a factor of 4
- (5) be unaffected

The capacitance of a coaxial cylinder is $C = 2\pi\epsilon_0 L / \ln(b/a)$. If the dimensions are all doubled, then the capacitance is doubled.

10. (*Lecture problem, modified*) Four capacitors are connected as shown in the diagram. What is the equivalent capacitance in μF ?

- (1) 1.5
- (2) 3.3
- (3) 4.0
- (4) 6.0
- (5) 0.67



The three capacitors in parallel have a total capacitance of $3 \mu\text{F}$. This capacitance in series with another $3 \mu\text{F}$ capacitor yields $1.5 \mu\text{F}$ as the total capacitance.