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Exam 3 Solutions

1. (*Prob. 30.60*) A coil with inductance 1.9 H and a 10 Ω resistor are connected in series to an ideal battery of voltage 95 V. At 0.10 sec after the connection is made, what is current in the circuit (in amps)?

(1) 3.9
(2) 5.6
(3) 8.1
(4) 0
(5) 1.4

The current in a LR circuit is given by $i = \frac{V}{R}\left(1 - e^{-tR/L}\right)$. *Plugging in values for t, R, L and V gives I = 3.9 A.*

2. (*F2005 exam*) A tightly-wound circular wire loop of 6 cm radius and 8 turns rotates around its diameter at 55 Hz inside a solenoid of radius 18 cm and 4.2 T B field. If the wire loop has a resistance of 0.067 Ω, what is the rms current induced in the loop in amps?

(1) 1,390
(2) 330
(3) 173
(4) 12,500
(5) 560

The induced emf is given by Faraday's law as $\mathcal{E} = -Nd\Phi_B / dt$, *where* $\Phi_B = \pi r^2 B \cos\omega t$. *Taking the derivative, and using* $\omega = 2\pi f$, *we get* $\mathcal{E} = N\pi r^2 2\pi fB \sin 2\pi ft$, *so the rms emf is* $\mathcal{E}_{\text{rms}} = \sqrt{2} N\pi^2 r^2 fB$, *which includes a factor of* $1/\sqrt{2}$ *to convert from peak to rms. Using the values given, we get* $\mathcal{E}_{\text{rms}} = 92.8V$ *and* $I_{\text{rms}} = \mathcal{E}_{\text{rms}} / R = 1390\,A$.

3. A series RLC circuit is driven by an AC emf source of a fixed frequency. In order to make this circuit more inductive you need to decrease:

(1) its natural frequency
(2) its resistance
(3) its inductance
(4) its capacitance
(5) the amplitude of the emf

The natural (angular) frequency is given by $\omega = 1/\sqrt{LC}$ *. Decreasing it is equivalent to increasing the inductance (increasing inductive reactance) or increasing the capacitance (decreasing capacitative reactance), both of which increase the relative amount of inductive reactance.*

4. (WebAssign 31.56) A dimmer for a stage light at a theater consists of a variable inductor, whose inductance is adjustable between zero and L_{max}, connected in series with a lightbulb. The electrical supply is 120 V (rms) at 60.0 Hz; the lightbulb is rated at 1.0 kW for this voltage. What L_{max} is required if you want the rate of energy dissipation in the lightbulb to be variable by a factor of 10 from its upper limit of 1.0 kW? Assume that the resistance of the lightbulb does not depend on its temperature.

(1) 0.11 H
(2) 0.45 H
(3) 1.2 H
(4) 65 mH
(5) 21 mH

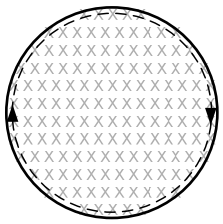
The energy dissipation is given by $P = I^2 R = (V/Z)^2 R = V^2 R/\left(X_L^2 + R^2\right)$*. To get a ratio of 10 to 1,* X_L *must range from 0 to 3R. Using* $X_L = \omega L$*, with* $\omega = 120\pi$*, we see that* $L_{max} = 0.11$ *H.*

5. (*Modified sample problem 31.8*) A series RLC circuit consists of a 0.10 Ω resistor, a 3.0 μH inductor, and a capacitor of an unknown value C. When driven with an AC emf source, the circuit resonates at 100.0 kHz. Find C.

(1) 0.84 μF
(2) 0.16 μF
(3) 2.3 μF
(4) 9.4 μF
(5) No capacitance can cause the circuit to resonate at this frequency.

The natural frequency is given by $f_0 = 1/2\pi\sqrt{LC}$. *To make the circuit resonant, we must have the driving frequency* $f_d = f_0$. *Using* $f_d = 10^5$ *and the given value of L, we calculate C = 0.84 μF.*

6. (*Similar to lecture problem*) In the figure shown, a uniform field (call it field 1) is directed into the page within a circular region. As the strength of this field increases with time, another field (field 2) appears in the clockwise direction as a result. Which of the following can be correct?



(1) Field 1 is an electric field and field 2 a magnetic field.
(2) Field 1 is a magnetic field and field 2 an electric field.
(3) Fields 1 and 2 are both electric fields.
(4) Fields 1 and 2 are both magnetic fields.
(5) No field 1 can produce field 2 in the clockwise direction.

The only possible options are (1) or (2). One can see from the directions that (1) is the answer.

7. (*From lecture*) To which of the following did Maxwell add a new term, completing the theory of electricity and magnetism?

(1) Ampere's law
(2) Gauss' law
(3) Coulomb's law
(4) the Biot-Savart law
(5) Faraday's law

Maxwell added the displacement current term to Ampere's law, as discussed in lecture.

8. (*Similar to WebAssign and lecture*) A laser beam with intensity 10^6 W/m^2 and wavelength 632.8 nm is aimed vertically upward. What is the maximum radius in nm of a spherical particle of graphite (density 2100 kg/m^3) that can be supported by the laser beam against gravity (g = 9.8 m/s^2)? Assume that the particle is totally absorbing.

(1) 120
(2) 490
(3) 1190
(4) 230
(5) 190

The total force on the particle is $F = I\pi r^2 / c$ *and its mass is* $m = 4\pi r^3 \rho / 3$. *Thus to balance the particle against gravity, we must have* $I\pi r^2 / c = 4\pi r^3 \rho g / 3$. *Solving for the maximum value of r yields* $r = 3I / 4\rho g c$. *The values shown yield r = 120 nm.*

9. (*Similar to lecture problem*) A small light bulb is placed 3 cm inside a solid cube of lucite (n=1.50) and turned on. What is the radius (in cm) of the illuminated circle that would be seen by an observer standing above the lucite block?

(1) 2.68
(2) 3.00
(3) 3.35
(4) 4.50
(5) 3.42

Only the light that hits the water surface at less than the critical angle ($\sin\theta_c = 1/n$*) will emerge from the water. The radius of the circle from which light emerges is thus* $r = h\tan\theta_c$*, where h is the depth of the light bulb. A quick calculation yields* θ_c *= 41.8°and r = 2.68 cm.*

10. (*Similar to sample problem 33.3*) A horizontal beam of unpolarized light is incident upon a stack of 4 polarizers with axes of polarization, in order and measured clockwise from the vertical, at 30°, 75°, 120° and 180°. What is the ratio of the intensity of the transmitted beam to that of the incident beam?

(1) 0.031
(2) 0.063
(3) 0.023
(4) 0.047
(5) 0.0063

Let I_0 be the initial intensity. The final intensity I is given by the expression, which depends only on relative angles, $I = I_0 \frac{1}{2}\cos^2 45° \cos^2 45° \cos^2 60° = I_0 / 32$. *The first factor of 1/2 comes from unpolarized light going through the first filter.*