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Final Exam Solutions

(First answer is correct)

1. (*Exam 1*) Charges +9Q and –3Q are held in place at positions x = 0 m and x = 2 m, respectively. At what position in x (in m) should a third charge be placed so that it is in equilibrium?

(1) +4.73
(2) –2.73
(3) +1.27
(4) +0.67
(5) –1.86

We see immediately that the equilibrium point should be to the right of the –3Q charge, i.e. x > 2. Put a charge q_3 at position x. If L is the distance between the first two charges, then the total force on the third charge at equilibrium is $F_x = -3kQq_3/(x-L)^2 + 9kQq_3/x^2 = 0$. *Solving yields x = +4.73 m.*

2. (*Exam 1*) A proton and an alpha particle (2 protons and 2 neutrons) are fixed in position 4.5 cm apart. Another proton is shot from infinity and stops midway between the two. What is its initial speed in m/s?

(1) 6.1
(2) 4.9
(3) 370
(4) 13
(5) 82

Conservation of energy yields $\frac{1}{2}m_p v^2 + 0 = 0 + ke^2/(d/2) + 2ke^2/(d/2) = 6ke^2/d$ *, where m_p is the proton mass and d = 0.045 m. Solving for v gives v = 6.1 m/s.*

3. (*Exam 2*) A 23.0 Ω resistor and a capacitor are connected in series and then a 37.0 V potential difference is suddenly applied across them. The potential difference across the capacitor rises to 9.00 V in 1.60 μs. What is the capacitance in μF?

(1) 0.25
(2) 0.11
(3) 0.049
(4) 6.8
(5) 1.9

The voltage across an initially uncharged capacitor is $V(t) = V_0\left(1 - e^{-t/RC}\right)$*, where* $V_0 = 37$ *is the applied voltage. Rearranging yields the equation* $\exp\left(-1.6\times10^{-6}/RC\right) = 1 - V/V_0 = 0.76$*. Taking the log of both sides and solving for C yields* $C = 0.25\ \mu F$*.*

4. (*Exam 2*) In a uniform magnetic field, a proton undergoes a circular motion with a kinetic energy of 4 keV. The radius of the orbit is 5.0 mm. What is the magnetic field in T?

(1) 1.83
(2) 0.0135
(3) 0.23
(4) 2.34
(5) 0.81

The equation describing circular motion is $m_p v^2/R = qvB$ *or* $B = mv/qR$*. The kinetic energy K is* $K = \frac{1}{2}m_p v^2$*, so we can write* $m_p v = \sqrt{2m_p K}$ *and thus* $B = \sqrt{2m_p K}/qR$*. Using* $K = 4\times10^{3}\times 1.6\times10^{-19} = 6.4\times10^{-16}$ *J and the values for the proton charge and mass, we get B = 1.83 T.*

5. (*Exam 3*) A tightly-wound circular wire loop of 5.6 cm radius and 12 turns rotates around its diameter at 25 Hz inside a solenoid of radius 28 cm and 5.2 T B field. The field is perpendicular to the rotation axis of the wire loop. If the wire loop has a resistance of 0.067 Ω, what is the rms current induced in the loop in amps?

(1) 1,020
(2) 1,390
(3) 560
(4) 12,500
(5) 960

The induced emf is given by Faraday's law as $\mathcal{E} = -Nd\Phi_B / dt$ *, where* $\Phi_B = \pi r^2 B \cos \omega t$ *. Taking the derivative, and using* $\omega = 2\pi f$*, we get* $\mathcal{E} = N\pi r^2 2\pi fB \sin 2\pi ft$ *, so the rms emf is* $\mathcal{E}_{\mathrm{rms}} = \sqrt{2} N\pi^2 r^2 fB$ *, which includes a factor of* $1/\sqrt{2}$ *to convert from peak to rms. Using the values given, we get* $\mathcal{E}_{\mathrm{rms}} = 68.3V$ *and* $I_{\mathrm{rms}} = \mathcal{E}_{\mathrm{rms}} / R = 1020\,A$.

6. (*Exam 3*) A laser beam with intensity 4.3×10^6 W/m^2 and wavelength 632.8 nm is aimed vertically upward. What is the maximum radius in nm of a spherical particle (density 4100 kg/m^3) that can be supported by the laser beam against gravity (g = 9.8 m/s^2)? Assume that the particle is totally absorbing.

(1) 270
(2) 490
(3) 1070
(4) 120
(5) 190

The total force on the particle is $F = I\pi r^2 / c$ *and its mass is* $m = 4\pi r^3 \rho / 3$*. Thus to balance the particle against gravity, we must have* $I\pi r^2 / c = 4\pi r^3 \rho g / 3$*. Solving for the maximum value of r yields* $r = 3I / 4\rho gc$*. The values shown yield r = 270 nm.*

7. (*Prob. 35.21*) A double slit experiment is performed with light of wavelength 500 nm. The slits are 1.40 mm apart, and the viewing screen is 4.20 m from the slits. How far apart (in mm) are the bright fringes near the center of the interference pattern?

(1) 1.5
(2) 2.2
(3) 0.93
(4) 2.5
(5) 3.6

For a double slit setup, the bright fringes are specified by $\sin\theta = m(\lambda/d)$, *where d is the slit separation. Since* $\lambda/d = 1/2800$ *is very small, the angles are small and the fringe separation is given by* $\Delta y = L\lambda/d$, *where L is the slit-screen distance. We obtain* $\Delta y = 4.2/2800 = 0.0015$ *m or 1.5 mm.*

8. (*Problem 35.55*) A tanker spill dumps kerosene (n = 1.2) that spreads into a layer 430 nm thick over a large body of water (n = 1.33). If you are scuba diving under the layer, with the Sun directly overhead, for which wavelength (in nm) of visible light is the transmitted intensity strongest?

(1) 688
(2) 516
(3) 572
(4) 470
(5) 344

The transmission is maximized when the reflection is minimized, so destructive interference is required. This gives the condition $2d = \left(m+\frac{1}{2}\right)\lambda/n$, *where d is the layer thickness and n = 1.2 is its index of refraction. This gives the set of wavelengths* $\lambda_m = 2nd/\left(m+\frac{1}{2}\right)$. *Only m = 1,2 give wavelengths in the visible spectrum, 688 and 428 nm, of which only 688 nm is shown.*

9. (*Test bank 35.21*) A light wave with an electric field amplitude of $2E_0$ and a phase constant of zero is to be combined with one of the following waves. Which of these combinations produces the greatest intensity?

(1) wave A has an amplitude of $2E_0$ and a phase angle of zero
(2) wave B has an amplitude of E_0 and a phase angle of π
(3) wave C has an amplitude of $2E_0$ and a phase angle of π
(4) wave D has an amplitude of E_0 and a phase angle of zero
(5) wave E has an amplitude of $5E_0$ and a phase angle of π

The amplitudes of these scenarios are, in order, $4E_0$, E_0, 0, $3E_0$, $3E_0$. *So (1) is the correct answer.*

10. (*Ch. 36, New problem*) The speed of sound in air is 343 m/s under normal conditions. Sound waves of frequency 5700 Hz from a distance source strike a very high fence of total length 20 m with 20 cm wide vertical gaps spaced 1.4 m apart. At what angle is the 3rd order maximum of the diffracted spectrum?

(1) 7.4°
(2) 64.5°
(3) 17.5°
(4) 2.5°
(5) 22.3°

This is a double slit setup with sound waves, in which λ = 343 / 5700 = 0.060 m and the slit separation is d = 1.4. Thus we get $\sin\theta = 3(\lambda / d) = 0.129$*, or* θ *= 7.4°.*

11. (*Test bank 36.50*) A mixture of 450 nm and 900 nm light is incident on a multiple-slit system. Which of the following is true about the bright lines from the two diffraction patterns?

(1) All 900 nm lines coincide with 450 nm lines
(2) Every other 900 nm line coincides with a 450 nm line
(3) None of the 450 nm lines coincide with the 900 nm lines
(4) All of the 450 nm lines coincide with the 900 nm lines
(5) Only the central line (0°) of each spectrum coincides

For a double slit setup, we have $\sin\theta = m(\lambda / d)$*. Note that the m = 1 line for 900 nm overlaps the m = 2 line for 450 nm, the* m_{900} *= 2 line overleaps* m_{450} *= 4, etc. Thus (1) is correct.*

12. (*Problem 36.97*) Two yellow flowers are separated by 68 cm along a line perpendicular to your line of sight to the flowers. Approximately how far (in km) can you distinguish the flowers? Assume the light from the flowers has a single wavelength of 550 nm and that your pupil has a diameter of 6.0 mm.

(1) 6.1
(2) 2.2
(3) 3.4
(4) 5.4
(5) 0.8

The Rayleigh criterion for distinguish close objects is $\Delta\theta = 1.22\lambda / D$*, where D = 0.006 m. Here* $\Delta\theta$ *= 0.68 / L, where L is the distance we are trying to determine. Solving yields L = 6.1 km.*

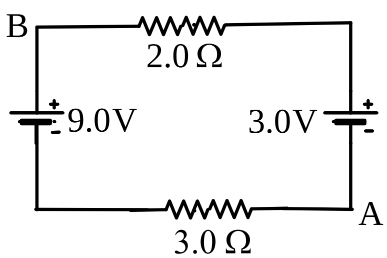
13. (*WebAssign 23.22*) At a perpendicular distance of 9.0 cm from a very long Nylon wire, an electron is released from rest. The wire is straight and is uniformly charged with a density of 6.0 μC/m. What is the magnitude of the electron's initial acceleration in m/s^2?

(1) 2.1×10^{17}
(2) 5.5×10^{15}
(3) 7.3×10^{13}
(4) 4.9×10^{19}
(5) 3.2×10^{21}

From Gauss' law, the electric field for a long line charge is $E = \lambda / 2\pi\varepsilon_0 r$, where λ is the charge per unit length and r is the distance from the wire. The acceleration is a = eE / m, where m and e are the electron mass and charge. Solving yields $a = 2.1 \times 10^{17}$ m/s^2.

14. (*WebAssign 27.6*) In the figure shown, the potential at point A is defined to be 1.0 V. What is the potential at point B in volts?

(1) 6.4
(2) 10.0
(3) 4.0
(4) 7.0
(5) 3.3



The current in the circuit is (9 – 3) / 5 = 1.2 A counterclockwise. Moving from point A to B counterclockwise, $V_B = 1 + 3 + 2 * 1.2 = 6.4$ V.

15. (*WebAssign 32.44*) A small bar magnet has a magnetic dipole moment of 9.3 J/T. What torque (in rm N,m) must be exerted to hold this magnet perpendicular to an external magnetic field of 1.5 T?

(1) 14.0
(2) 3.2
(3) 0.0
(4) 6.2
(5) 1.7

The torque is $\tau = \mu B$, or 14.0 N m.

16. (*WebAssign 33.10*) Assume a TV station acts as a point source broadcasting isotropically at 1.0 MW. What is the intensity (in W/m^2) of the transmitted signal reaching the Moon, which is at 3.85×10^5 km from the Earth?

(1) 5.4×10^{-13}
(2) 3.7×10^{-15}
(3) 6.9×10^{-11}
(4) 8.7×10^{-9}
(5) 2.2×10^{-7}

The intensity is $I = P/A = P/4\pi r^2$, where $r = 3.85 \times 10^8$ m. This gives $I = 5.4 \times 10^{-13}$ W/m^2.

17. (*Ch. 34 lecture problem*) A 176 cm tall person stands still in front of a mirror on a wall. The mirror is 2.0 m from her/him. If the mirror can be positioned anywhere on the wall, what is the minimum height of the mirror (in cm) that allows the person to see herself/himself from top to tow?

(1) 88
(2) 176
(3) 135
(4) 102
(5) 67

The mirror has to be one half the height of the person, as discussed in lecture.

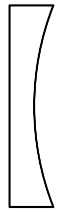
18. (*Testbank 34.22*) The image of an erect candle, formed using a convex mirror is always:

(1) virtual, erect, and smaller than the candle
(2) virtual, erect, and larger than the candle
(3) virtual, inverted, and smaller than the candle
(4) virtual, inverted, and larger than the candle
(5) real, inverted, and larger than the candle

A convex mirror has $f < 0$, so the image is always virtual, and therefore upright and smaller.

19. (*Testbank 34.47*) A hollow lens is made of thin glass, as shown. It can be filled with air, water (n = 1.33), or CS_2 (n = 1.63). The lens will diverge a beam of parallel light if it is filled with:

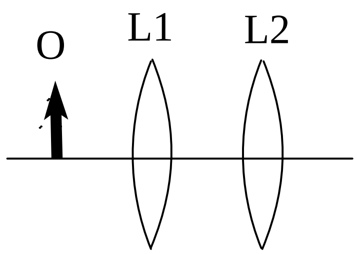
(1) CS_2 and immersed in water
(2) CS_2 and immersed in CS_2
(3) water and immersed in CS_2
(4) air and immersed in air
(5) air and immersed in water



The equation describing the focal length of a lens is $1/f = (n-1)(1/r_1 - 1/r_2)$. *The lens is diverging for n > 1, so (1) is correct.*

20. (*Sample problem 34.4, lecture problem*) In the figure shown, object O is placed in front of two thin symmetrical coaxial lenses L1 and L2, with focal lengths f_1 = 30.0 cm and f_2 = 10.0 cm, respectively. The lens separation is 12.0 cm, and the object is 6.0 cm from lens L1. Where does the system of two lenses produce an image of the object?

(1) 20.5 cm to the right of L2
(2) 6.0 cm to the right of L2
(3) 8.2 cm to the left of L2
(4) 22.4 cm to the left of L2
(5) 10.0 cm to the right of L2



The first lens gives an image location of q = –7.5 cm or 7.5 cm to the left of L1, which is 19.5 cm from L2. The image distance from L2 is then calculated to be +20.5 cm, or 20.5 cm to the right of L2.