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Exam 4 Solutions

1. Which of the following statements is true?

1. In equilibrium all of any excess charge stored on a conductor is on the outer surface.
2. In equilibrium the electric field inside a conductor is zero.
3. In a spherical insulator with radius R on which charge q is uniformly distributed, the electric field at a distance of R/4 from the center of the sphere is ¼ of the field at the surface of the sphere.

a. 1,2,and 3 b. 1 and 2, not 3 c. 1 and 3, not 2 d. 2 and 3, not 1 e. only 2

Solution: 1 and 2 are true, as shown by Gauss' Law. For statement 3, use eq. 23-20 (derived from Gauss' Law) $E=(q/4\pi\varepsilon_0)r/R^3$ Since r=1/4 R, $E_{at\ 1/4\ R}$ is just ¼ of $E_{at\ R}=(q/4\pi\varepsilon_0)(1/R^2)$

2. A long straight wire is 2R long and carries current i upwards in the sketch. What is the magnetic field B at point P which is R away from the center of the wire as shown?

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     |
     | R
↑    |______.P
i    |    R
     | R
     |
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a. $\mu_0 i/2(2)^{1/2}\pi R$ b. $\mu_0 i/2(3)^{1/2}\pi R$ c. $\mu_0 i/4\pi R$ d. $\mu_0 i(3)^{1/2}/4\pi R$ e. $\mu_0 i/2\pi R$

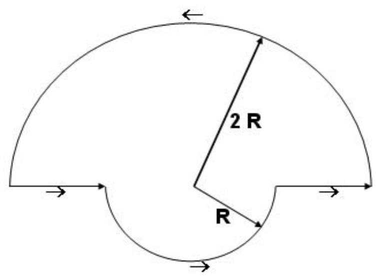
Solution: the field at point P is just double the field produced by either half of the wire, so choose the upper half, solve for B from this segment, and double the result.

$B = 2\int_0^R dB = \mu_0 i/2\pi \int_0^R \sin\theta\, ds/r^2$ where ds is some segment of the wire along its length between 0 and R (see Fig. 29-5, p 767 in the text), θ is the angle between the straight wire and the vector **r** connected ds and point P. The magnitude 'r' is $(s^2 + R^2)^{1/2}$ and $\sin\theta = R/r = R/(s^2 + R^2)^{1/2}$

so $B=\mu_0 i/2\pi \int_0^R R\,ds/(s^2 + R^2)^{3/2}$

This integral gives $s/R^2(s^2 + R^2)^{1/2}$ evaluated between s=0 and R
This gives $B=\mu_0 i/2\pi R \;\; R/(R^2 + R^2)^{1/2} = \mu_0 i/2(2)^{1/2}\pi R$ which is just $1/(2)^{1/2}$ smaller than the result for an infinitely long wire.

3.



A current i goes through the half circle of radius 2R, straight segment of length R, half circle of radius R, and straight segment of length R shown above in a counter-clockwise direction. What is the value of the magnetic field B at the center of the two half-loops?

a. $3\mu_0 i/8R$ b. $\mu_0 i/2R$ c. $\mu_0 i/4R$ d. $3\mu_0 i/16R$ e. $7\mu_0 i/16R$

solution: The contribution of a circular loop of wire of subtended angle Φ is $\mu_0 i\Phi/4\pi R$ where Φ is in radians. The contribution of the two straight segments of wire that go through the center point is zero. Both half circles produce B fields that add, thus $B_{total} = B_{\text{from larger half loop}} + B_{\text{from smaller half loop}} = \mu_0 i\pi/4\pi R + \mu_0 i\pi/4\pi*2R = \mu_0 i(1/4R + 1/8R) = (3/8)\mu_0 i/R$

4.

Two charges, +q and +3 q, are placed along the x-axis a distance 'd' apart (see picture). A third charge which is negative and has magnitude 'Q' is placed on the x-axis at x=0.37 d so that there is no net force in on the negative charge 'Q' from the sum of the Coulomb forces from +q and +3 q. x=0 is at +q.

x=0……..x=0.37d

+q -Q +3 q ----------→ x-direction

x=0 ……………………..x=d

What is the magnitude of Q expressed as a fraction of q so that there is no net force on the +q charge at x=0?
a. 0.41 b. 0.37 c. 0.45 d. ½ e. 1/3

solution: The coulomb force on +q from –Q is $kqQ/(0.37d)^2$ and acts to the right (+x-axis direction.) The Coulomb force on +q from +3q is $kq(3q)/d^2$ and acts to the left (-x-axis direction.) Setting the two equal and solving for Q gives Q=0.41 q.

5. A driven series RLC circuit has the maximum amplitude for the current I for ω_d = 754 radians/sec. If L=10 H, what is C in units of μF?

a. 0.18 b. 6.9 c. 133 d. 0.69 e. 0.069

This is like homework 31-36. At the resonance frequency ω_d where the current is at a maximum, $\omega_d = 1/(LC)^{1/2}$, so we have $754 = 1/(10C)^{1/2}$ or C=0.18 μF

6. Consider a long solenoid with 10,000 turns (windings), where the length is much larger than the radius of the solenoid. If it is desired to wind another, same length solenoid but with double the energy stored compared to the 10,000 turn solenoid, how many total turns should the new solenoid have?

a. 14140 b. 20000 c. 15000 d. 16280 e. 17320

Solution: energy U stored in the B field of the solenoid is ½ Li^2. The inductance L for a long solenoid (L>>radius) is proportional to n^2, where n is the number of turns per unit length. If the length of the new solenoid is the same, then to double the inductance requires $(2)^{1/2}$ more turns, or 14140.

7.

A positively charged particle (for example, a proton) is accelerated from a standing start (velocity(time=0)=0) through a potential difference V. Its final speed is proportional to

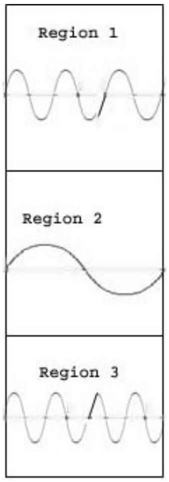
a. $V^{1/2}$ b. V c. V^2 d. 1/V e. $1/V^{1/2}$

solution: the work done on the particle (which is equal to the increase in its kinetic energy) is just the charge of the particle times the potential difference V. Since kinetic energy = ½ m v^2, the velocity is proportional to the square root of V

8. In a double slit interference experiment, monochromatic light (all of one wavelength) passes through the two slits and interferes on the far side, producing alternating areas of brightness (constructive interference) and darkness (destructive interference) on a viewing screen. If λ=500 nm and the angular separation between the central bright spot and the adjacent bright spot is 0.1 radian, what is the separation of the two slits, in units of nm?

a. 5000 b. 500 c. 88 d. 50 e. 2500

Solution: For the maxima, $d\sin\theta = m\lambda$. m=1 for the first adjacent bright spot, θ is give as 0.1 rad, and λ=500 nm, thus d=500 nm/sin(0.1) = 5008 nm (0.1 rad = 5.73^o)



9. Three light rays, all with the same wavelength in vacuum, are shown traveling through 3 regions (all of the same width) with different indices of refraction in the picture at the left. Rank the three regions according to the speeds of the waves, least to greatest.

a. 3,1,2 b. 2,1,3 c. 2,3,1 d. 3,2,1 e. 1,2,3

Solution: We know that the wavelength λ in a medium is equal to the wavelength in free space divided by the index of refraction, n. Also, we know that the speed of light in a medium is the speed in free space divided by n, the index of refraction. Thus, since region three can fit 4 wavelengths in, it has the shortest wavelength (1/4 of the width of the region) and therefore the largest n and the slowest (least) speed. Region one has 3 wavelengths and an intermediate n and speed, while region 2 has the longest wavelength and therefore the smallest n and the greatest speed.

10.

In an RLC circuit, which of the following statements about the voltage and current is true? (Choose only one.)

a. none of the other answers given is correct
b. i_R leads v_R by 90^o
c. i_C lags behind v_C by 90^o
d. i_L leads v_L by 90^o
e. i_R lags behind v_R by 90^o
Solution: As discussed in Chap. 31, i_R and v_R are in phase, and i_C leads v_C by 90^o while i_L lags v_L by 90^o. Thus, a. none of the other answers given is correct.

11. A capacitor with parallel circular plates of radius 12 cm is discharging via a constant current of 12A. At a radius of 4cm from the central axis of the plates, what is the magnitude of the magnetic field in μT?

(1) 6.7 (2) 20 (3) 60 (4) 2.2 (5) 0

Solution:

The magnetic field can be calculated by using Ampere's Law extended with Maxwell's law of induction:

$B = \frac{\mu_0 i_{disp}}{2\pi r}$ *where i_{disp} is the enclosed displacement current at radius r due to the changing electric field. Since the total displacement current is equal to the total current into the capacitor, we just need the fraction inside radius r assuming it is uniformly distributed:*

$$B = \frac{\mu_0 i_{disp}}{2\pi r} = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{\pi R^2}\right)\left(\frac{\pi r^2}{r}\right) = 2\left(10^{-7}\right)(12\text{A})\left(\frac{4\text{cm}}{12\text{cm}}\right)^2 = 6.7\,\mu\text{T}$$

12. A current loop has a magnetic dipole moment of 10^{-3} J/T. If the direction of the dipole moment is initially aligned with the direction of an external magnetic field of magnitude 0.1T, how much work does it take to align the dipole in the direction opposite to the magnetic field?

(1) 2×10^{-4} J (2) 10^{-4} J (3) 10^{-3} J (4) 5×10^{-5} J (5) 0

Solution:

The potential energy of a magnetic dipole in an external magnetic field is given by: $U = -\boldsymbol{\mu}\cdot\mathbf{B}$. So the potential energy difference between aligned and anti-aligned is given by $U = 2\boldsymbol{\mu}\cdot\mathbf{B} = 2\left(10^{-3}\right)(0.1) = 2\times10^{-4}$ J

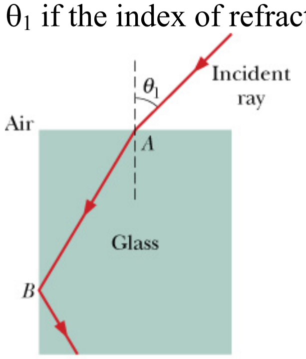
13. The electric field of a plane electromagnetic wave propagating in vacuum is described by $\mathbf{E} = \hat{\mathbf{y}} E_m \sin(kx + \omega t)$. In what direction does the wave propagate?

(1) $-\hat{\mathbf{x}}$ (2) $\hat{\mathbf{x}}$ (3) $\hat{\mathbf{y}}$ (4) $-\hat{\mathbf{y}}$ (5) $\hat{\mathbf{z}}$

Solution:

The wave must travel in the $-\hat{\mathbf{x}}$ direction. To remain at the crest of the electric field maximum, for example, one must follow in the –x direction as t increases.

14. A light ray in air enters a glass slab at point A at incident angle θ_1 and then undergoes total internal reflection at point B, as shown in the figure. What is the maximum value of θ_1 if the index of refraction of the glass is 1.4?



(1) 78º (2) 45º (3) 30º (4) 90º (5) 12º

Solution:

The critical angle in the glass at point B is given by $\theta_{crit} = \sin^{-1}\left(\frac{1}{n}\right) = 45.6^\circ$

The angle θ_2 *for the refraction at point B is just* $\theta_2 = 90^\circ - \theta_{crit}$

So the maximum value of θ_1 *is* $\theta_1 = \sin^{-1}\left[n\sin\left(90 - \theta_{crit}\right)\right] = 78^\circ$

15. A lens is ground from glass with an index of refraction n=1.5 according to the shape shown. The surfaces are spherical, and the magnitude of the radius of curvature for both surfaces is 50 cm. What is the focal length of the lens?

(1) -50 cm (2) 50 cm (3) -25 cm (4) 100 cm (5) 0 cm

Solution:

For a light ray entering a concave surface, the radius is taken to be negative. For a light ray entering a convex surface, it is positive. So using the lens-maker's formula:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{2}\left(\frac{1}{-50} - \frac{1}{50}\right)$$

$$\Rightarrow f = -50\text{cm}$$

It is a diverging lens

16. An object is placed 6 cm in front of a converging lens with a focal length of 5 cm. What is the magnitude of the magnification of the transverse dimensions of the image?

(1) 5.0 (2) 1.2 (3) 0.2 (4) 0.5 (5) 1.8

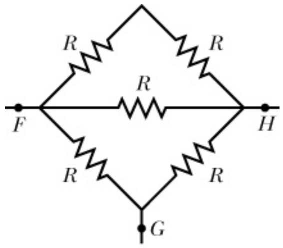
Solution:

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{5} - \frac{1}{6} = \frac{1}{30} \Rightarrow i = 30\text{cm}$$

$$m = -\frac{i}{p} = -\frac{30}{6} = -5$$

So the lateral dimensions are magnified 5 times (and the image is inverted).

17. What is the total resistance, in units of Ω, between points F and G in the section of the circuit shown? All four resistors have R=1Ω.



(1) 5/8 (2) 5/3 (3) ½ (4) 5 (5) 2/5

Solution:

The top two resistors are in series with total resistance R' = 2Ω. This is in parallel with a third resistor, where the combined resistance is:

$$\frac{1}{R''} = \frac{1}{2} + \frac{1}{1} \Rightarrow R'' = \frac{2}{3}\Omega$$

This is in turn in series with another resistor, so R'''=5/3Ω.
Finally this is in parallel with a fifth resistor:

$$\frac{1}{R''''} = \frac{3}{5} + \frac{1}{1} \Rightarrow R'''' = \frac{5}{8}\Omega$$

18. What is the magnitude of the acceleration of an electron located 1 cm above the center of a non-conducting square sheet of charge with a side length of 50 cm? The total charge distributed uniformly across the sheet is 2 x 10^{-8}C, and the mass of the electron is m_e = 9.11 x 10^{-31}kg.

(1) 8 x 10^{14} m/s^2
(2) 3 x 10^{17} m/s^2
(3) 7 x 10^{-16} m/s^2
(4) 2 x 10^{14} m/s^2
(5) 6 x 10^{9} m/s^2

Solution:

The electric field of a non-conductive sheet of charge that is large compared to the distance a charge is above it is given by:

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2A\varepsilon_0} = \left(4\pi \times 9 \times 10^9\right)\frac{2\times10^{-8}}{2(0.5)^2} = 4520$$

The acceleration is given by

$$a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{\left(1.6\times10^{-19}\right)(4520)}{9.11\times10^{-31}} = 8\times10^{14}\ \text{m/s}^2$$

19. A 4μF capacitor is charged to a potential difference of 1V. If the capacitor is connected across a 0.002 H inductor to form an LC circuit, what will be the maximum current in the inductor?

(1) 0.045 A (2) 0.002 A (3) 9×10^{-5} A (4) 2×10^{-6} A (5) 0

Solution:

The charge on the capacitor initially is $q = CV$, and the total energy is

$U_C = \frac{q^2}{2C} = \frac{1}{2}CV^2 = 2\times10^{-6}$ J. *This must equal the maximum energy in the inductor as the LC circuit oscillates:*

$$U_L = \frac{1}{2}Li^2 = U_C = 2\times10^{-6} \text{ J}$$

$$i_{\max} = \sqrt{\frac{4\times10^{-6}}{L}} = 0.045\text{A}$$

20. A 12 V battery is connected through a switch at time t=0 to a 1 F capacitor. There is a series resistor of 5 Ω in the circuit. At what time, in units of seconds (to two significant figures), is the voltage across the capacitor 6 V?

a. 3.5 b. 1.5 c. 2.0 d. 2.5 e. 3.0

Solution: From chap. 27, eq. 27-35, we know that the voltage across the capacitor during charging is $V = V_{battery}(1-\exp(-t/RC))$. Thus, we want to know t for when 1-exp(-t/RC) = 0.5.

1-exp(-t/RC) = 0.5

exp(-t/RC) = 0.5 → take ln of both sides to get -t/RC = ln0.5 = -0.693, so t=RC(0.693) = 3.47 s