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Exam 1 Solutions

Note that each problem has three versions, each with different numbers and answers (separated by |. The <u>numbers</u> for each question and its <u>correct answer</u> are listed <u>in order</u> for all three versions.

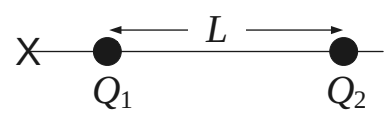
1. A muon (same charge as an electron but 206 times its mass) circles a beryllium nucleus (4 protons, 5 neutrons) at a distance of 0.120 | 0.216 | 0.562 picometers. What is its velocity?

Answer: 6390 km/s | 4770 km/s | 2950 km/s

Solution: The magnitude of the force on the muon is $F = k(4e)(e) / r^2 = ma = mv^2 / r$). Solving for the velocity yields $v = (4ke^2 / mr)^{1/2}$.

2. As shown in the figure (not drawn to scale), two charges Q_1 and Q_2 are held in place a distance L = 2 cm apart. When another charge is placed at point X, located 1.65 cm | 2.32 cm | 0.79 cm to the *left* of Q_1, it is found to be in equilibrium. If Q_1 = 5 μC, what is Q_2?

Answer: –24.5 μC | –17.3 μC | –62.4 μC



Solution: Note that Q_2 must be negative to balance the force due to Q_1 at point X. Let Q_3 be the charge at X. The x-component of the force due to Q_1 is $F_1 = -kQ_1Q_3 / (-x)^2$ and the x-component of the force due to Q_2 is $F_2 = -kQ_2Q_3 / (L - x)^2$. Equating these two yields $Q_2 = -Q_1 (L - x)^2 / x^2$.

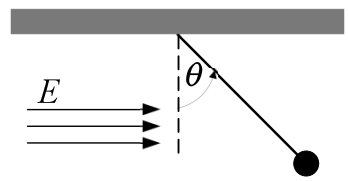
3. Two identical conducting spheres A and B carry charges $2Q$ and $3Q$ | $-3Q$ | $-2Q$, respectively. They are separated by a constant distance much larger than their diameters. A third identical conducting sphere C is uncharged. Sphere C is first touched to A, then to B and finally removed. As a result, the magnitude of the electrostatic force between A and B, initially F, becomes

Answer: $F/3$ | $F/6$ | $F/8$

Solution: Let the initial charges on sphere A and B be Q_{A0} and Q_{B0}. When sphere C is touched to A, $Q_A = Q_C = Q_{A0}/2$. When sphere C is then touched to B, $Q_B = (Q_{B0} + Q_{A0}/2) / 2$. The initial force F was proportional to $Q_{A0}\, Q_{B0}$. The final force F' is proportional to $Q_A\, Q_B = (Q_{A0}/2)(Q_{B0} + Q_{A0}/2)$. Thus the ratio F' / F is $1 + Q_{A0} / (2Q_{B0}) / 4$.

In practice, it might be easier to solve the problem directly, taking the average of the charges when they are touched, rather than deriving this general formula.

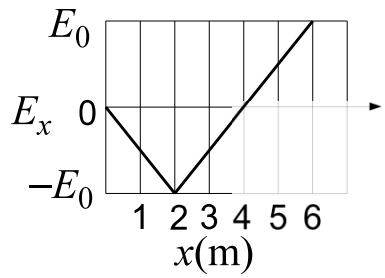
4. In the figure, a small charged ball of mass 4.0 g hangs from a support and makes an angle 35° |48° | 68° with the vertical under the influence of gravity and a horizontal electric field of magni- magnitude E = 2000 V/m. What is the charge on the ball?



Answer: 13.7 μC | 21.8 μC | 48.5 μC

Solution: The force in the x-direction is $F_x = 0 = qE - T \sin\theta$, and the force in the y-direction is $F_y = 0 = T\cos\theta - mg$. Solving yields $q = \tan\theta\ mg / E$.

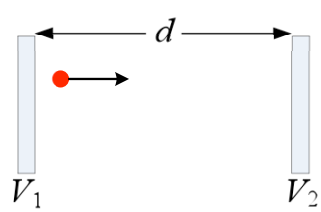
5. A graph of the x component of the electric field as a function of x in a region of space is shown in the figure. The scale of the vertical axis is E_0 = 20 | 10 | 10 V/m. The y and z components of the electric field are zero in this region. If the electric potential at the origin is 10 | 30 | 10 V, what is the electric potential (in V) at x = 6 m?



Answer: +30 V | +40 V | +20 V

Solution: $V(6\text{m}) - V(0\text{m}) = -\int E\,dx$ = –area under curve. The area under the curve is $-0.5\,(2\text{m})(E_0)$, which implies that $V(6\text{m}) = V(0\text{m}) + (E_0)(1\text{m})$.

6. A proton is moving rightward between two parallel charged plates separated by distance d = 1 cm as shown in the figure. The plate potentials are V_1 = 13 V and V_2 = 10 V. If the initial speed of the proton at the left plate is 32 km/s (10 km/s, 18 km/s), what is the speed of the proton just as it reaches plate 2?

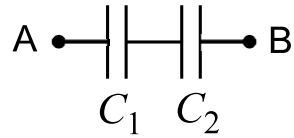


Answer: 40 km/s | 26 km/s | 30 km/s

Solution: Conservation of energy implies that the sum of the kinetic and potential energy is constant for this problem. $E = (1/2)mv_i^2 + eV_1 = (1/2)mv_f^2 + eV_2$. The only unknown is v_f.

7. Capacitors C_1 = 1 F | 2 F | 4 F and C_2 = 2 F | 3 F | 6 F are connected as shown in the figure. If the voltage difference between A and B is 3 V, how much energy is stored in the capacitors?

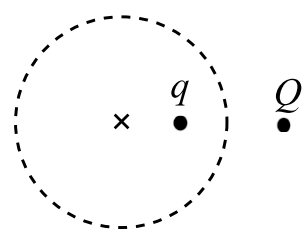
Answer: 3 J | 6 J | 9 J



Solution: These two capacitors are in series. Their effective capacitance is $C_{\text{eff}} = C_1C_2 / (C_1 + C_2)$. The energy stored in the capacitors is $(1/2)C_{\text{eff}}\, V^2$.

8. As shown in the figure, point charges q = 5 C | 4 C | 7 C and Q = –2 C | –1 C | –4 C are separated by a distance of 2 | 4 | 6 m. Consider a spherical Gaussian surface of radius 2 m | 4 m | 6 m, whose center is 1 m | 2 m | 3 m to the left of charge q, as shown by the circle. What is the amount of electric flux (in Nm^2/C) through that surface?

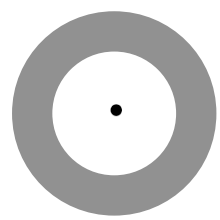
Answer: 5.65×10^{11} | 4.52×10^{11} | 7.91×10^{11}



Solution: The electric flux Φ_E is equal to the charge enclosed q divided by ε_o: $\Phi_E = q/\varepsilon_o$.

9. A point charge is held at the center of a conducting spherical shell, whose inner and outer radii are 3 m | 2 m | 6 m and 5 m | 3 m | 8 m (see figure). The electric fields at the inner and outer surfaces of the shell are 8×10^3 | 3×10^4 | 1.5×10^3 X V/m and 1×10^4 | 7×10^4 | 2×10^3 X V/m, respectively. What is the net charge on the shell?

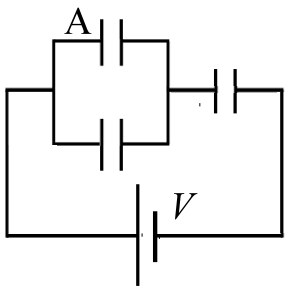
Answer: 19.8 μC | 56.7 μC | 8.2 μC



Solution: From Gauss' Law, the charge on the shell is proportional to the electric flux leaving the shell minus the electric flux entering the shell. Label the inner radius as r_1 and the outer radius as r_2. $Q = \varepsilon_o\,(4\pi(r_2)^2 E_2 - 4\pi(r_1)^2 E_1)$.

10. In the figure at right, all the capacitors have the value of 1 μF | 2 μF | 4 μF, and the voltage of the battery is 1.5 V | 3 V | 6 V. What is the charge on the capacitor marked A?

Answer: 0.5 μC | 2.0 μC | 8.0 μC



Solution: The effective capacitance of the entire circuit is $C_{eff} = 2C/3$, where C is the capacitance of one of the capacitors. This means that the charge on C_{eff} is $2CV/3$. This is the charge on the capacitor on the right. The charge on A is half of this, $CV / 3$, because the charge is split between A and the capacitor below it.