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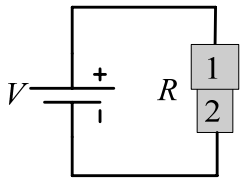
# Exam 2 Solutions

***Note that each problem has three versions, each with different numbers and answers (separated by |. The numbers for each question and its correct answer are listed in order for all three versions.***

1. A battery with emf 13.5 V | 14.1 V | 15.6 V is attached to a resistive rod as shown. The rod consists of two sections of the same material of equal lengths but radii $r_1$ = 4.9 | 5.6 | 6.5 mm and $r_2$ = 4.3 mm. What is the voltage across the thicker section?




**Answer**: 5.87 V | 5.23 V | 4.75 V

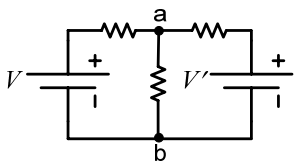
**Solution**:
Because the resistors are in series, the current through each resistor is the same. Letting the resistances be $R_1$ and $R_2$, $i = V / (R_1 + R_2)$, and the voltage across $R_1$ is $V_1 = iR_1 = V [R_1 / (R_1 + R_2)]$. Since both resistors have the same length and are made of the same material, the resistance $R = \rho L / A$ is inversely proportional to the area = $\pi r^2$. Thus, $R_1 / R_2 = r_2^2 / r_1^2$, which can be used to evaluate $V_1 = VR_1 / (R_1 + R_2) = V / (1 + R_2 / R_1)$.

2. Earth's lower atmosphere contains positive and negative ions that are produced by cosmic rays and radioactive elements in the soil. When a downward electric field of magnitude E = 1500 | 1800 | 1900 V/m is applied it acts on singly charged positive ions (number density 620 | 720 | 820 ions/cm$^3$) and singly charged negative ions (number density 330 | 520 | 470 ions/cm$^3$). If the measured current density is $1.50 \times 10^{-12}$ | $1.80 \times 10^{-12}$ | $2.10 \times 10^{-12}$ A/m$^2$, what is the average drift speed, assumed to be the same for both kinds of ions?

**Answer**: 0.99 | 0.91 | 1.02 cm/s

**Solution**:
The positive and negative singly charged ions have drift velocities in opposite directions, so their current densities *add*. The net current density is thus $j = e (n_+ + n_-) v_d$, where $e = 1.6 \times 10^{-19}$ C, $n_+$ is the number density of positive ions, $n_-$ is the number density of negative ions, and $v_d$ is the average common drift speed. Solving yields $v_d = j / e (n_+ + n_-)$

3. Two batteries with emfs $V$ and $V' = 3V \mid 4V \mid 6V$ and three resistors of identical resistance $R$ are connected as shown in the diagram. Find the potential difference $V_a - V_b$.




**Answer**: $4V/3 \mid 5V/3 \mid 7V/3$

**Solution**:
Let $i_1$ flow clockwise in the left loop, and $i_2$ flow counter clockwise in the right loop. The current in the center is $i_1 + i_2$ downward. Kirchoff's loop equations for the left and right loops are

$$0 = V - i_1 R - (i_1 + i_2) R$$

$$0 = V' - i_2 R - (i_1 + i_2) R$$

Adding the two equations yields $0 = (V + V') - 3 (i_1 + i_2) R$. Thus $V_a - V_b = (V + V') / 3$. Alternatively, you can solve for $i_1$ and $i_2$ individually and calculate $V_a - V_b = (i_1 + i_2) R$.

4. A 100 μF capacitor with an initial potential difference of 100 V is discharged through a resistor, when a switch between them is closed at $t = 0$. At $t = 1.5$ s | 0.5 s | 0.8 s, the potential difference across the capacitor is 10 V | 20 V | 25 V. What is the current through the resistor at that time?

**Answer**: 1.5 mA | 6.4 mA | 4.3 mA

**Solution**:
When discharging the voltage across the capacitor decays exponentially as $V = V_{\max} \exp(-t/RC)$, where $V = q/C$ and $V_{\max} = q_{\max}/C$. Taking the natural logarithm of both sides implies that $\ln(V/V_{\max}) = -t/RC$. The current across the resistor is $V/R$, where $V$ is the same voltage as across the capacitor for discharging. Consequently, the current through the resistor is

$$i = V / R = -\ln(V/V_{\max})\, VC / t.$$

5. A proton undergoes a circular motion at a speed of $4 \times 10^6 \mid 7 \times 10^6 \mid 3 \times 10^6$ m/s in a uniform magnetic field of 4 T | 2 T | 0.8 T. What is the period of the motion?

**Answer**: $1.6 \times 10^{-8} \mid 3.3 \times 10^{-8} \mid 8.2 \times 10^{-8}$ s

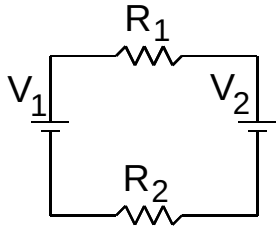
**Solution**:
For circular motion $F = mv^2 / r = qvB$, which implies that $2\pi / T = \omega = v/r = qB / m$. The period $T = 2\pi m / qB$ does not depend on the velocity.

6. A straight wire, mass 15 | 10 | 20 g and 30 | 70 | 50 cm long, is suspended horizontally by a pair of flexible leads in a uniform magnetic field of magnitude 1.2 | 0.5 | 2.5 T. This field is horizontal and makes an angle of 30° | 45° | 60° with the wire. What is the magnitude of the current required to remove the tension in the supporting leads?

**Answer**: 0.82 A | 0.40 A | 0.18 A

**Solution**:
The magnetic force has magnitude $F_B = iLB\sin\theta$. The gravitational force, which is equal in magnitude and opposite in direction, is $F_G = mg$. Setting the magnitude of the two forces equal to each other implies that the current is $i = mg / (LB\sin\theta)$.

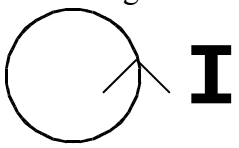
7. In the figure at right $V_1$ = 12 V | 12 V | 6 V, $V_2$ = 6 V | 6 V | 12 V, $R_1$ = 2Ω | 4Ω | 2Ω, and $R_2$ = 1Ω | 2Ω | 1Ω. What is the current in the loop?




**Answer**: 2A clockwise | 1 A clockwise | 2A counterclockwise

**Solution**:
Let $i$ be the current clockwise. Kirchoff's loop rule implies that $0 = V_1 - iR_1 - V_2 - iR_2$. Hence, the clockwise current is $i = (V_1 - V_2) / (R_1 + R_2)$. A negative value means that the current is counterclockwise.

8. A loop of wire carries a current of $i$ = 2 mA in the plane of the page as shown at right. The radius of the loop is 3 cm. A magnetic field of 8 T | 8 T | 4 T is coming out of | going into | coming out of the page. If the direction of current flow is reversed, what is the change in the potential energy, $U_f - U_i$?




**Answer**: $9 \times 10^{-5}$ J | $-9 \times 10^{-5}$ J | $4.5 \times 10^{-5}$ J

**Solution**:
For a single loop the magnitude of the magnetic moment is $\mu = iA = i\pi r^2$. For the loop shown in the figure the magnetic moment is coming out of the page. The potential energy of the loop in a magnetic field is $-\boldsymbol{\mu}\cdot\mathbf{B}$. Thus, if $B$ is out of the page the potential energy is initially negative, and if $B$ is into the page the potential energy is initially positive. Reversing the direction of the current, reverses the direction of the magnetic moment, keeping its magnitude the same. Thus, the change in the potential energy is +/–2 μB = +/–$i\pi r^2 B$, where the + is for $B$ out of the page initially and the – is for $B$ into the page initially.

9. Four wires have current flowing perpendicular to the plane of the page as shown in the figure at right. The currents all have magnitude 2 A, and each wire is a distance $d$ = 0.4 m from the origin. Wires 2 | 1 | 1 and 3 | 2 | 4 have current flowing out of the page, and wires 1 | 3 | 2 and 4 | 4 | 3 have current flowing into the page. What is the magnetic field at the origin?

**Answer**: $(-2\ \hat{\mathbf{i}} - 2\hat{\mathbf{j}})\ \mu\text{T}$ | $(2\ \hat{\mathbf{i}} - 2\ \hat{\mathbf{j}})\ \mu\text{T}$ | $(2\ \hat{\mathbf{i}} + 2\ \hat{\mathbf{j}})\ \mu\text{T}$

**Solution**:
Each wire creates a magnetic field of magnitude $\mu_o(2\text{A}) / (2\pi\ 0.4\text{m}) = 1\ \mu\text{T}$ at the origin. These four magnetic field contributions need to be added as vectors in order to get the net magnetic field. For example, if wire 2 has current flowing out of the page it produces a magnetic field in the $-y$ direction, and if wire 3 has current flowing out of the page it produces a magnetic field in the $-x$ direction. If wire 1 has current flowing into the page, it produces a magnetic field in the $-x$ direction, and if wire 4 has current flowing into the page, it produces a magnetic field in the $-y$ direction. The net magnetic field in this case is $(-2\ \mathbf{i} - 2\ \mathbf{j})\ \mu\text{T}$.

10. An $i$ = 6A current flows uniformly through a straight wire with circular cross-section of radius $R$ = 2 mm. What is the magnitude of the magnetic field inside the wire a distance of $R/2$ | $R/3$ | $R/6$ from the center of the wire?

**Answer**: $3 \times 10^{-4}$ T | $2 \times 10^{-4}$ T | $1 \times 10^{-4}$ T

**Solution**:
Ampere's law states that $\int \mathbf{B}\cdot\mathbf{ds} = 2\pi rB = \mu_o i_{\text{enc}}$. The enclosed current in a loop of radius $r$ about the center of the wire is not the total current for $r < R$. Rather it is a fraction of the total current $i_{\text{enc}} = i\ (\pi r^2)/(\pi R^2) = ir^2 / R^2$. Substituting this expression in the above equation yields a magnetic field of $B = \mu_o ir / (2\pi R^2)$. This can also be written $B = (r / R)\ B_0$, where $B_0 = \mu_o i / (2\pi R)$ is the $B$ field at the wire's surface.