

Exam 3 Solutions

Note that each problem has three versions, each with different numbers and answers (separated by |). The numbers for each question and its correct answer are listed in order for all three versions.

1. An RLC circuit is driven by an AC source with emf given by $\mathcal{E} = (12.4 | 29.3 | 32.4) \sin(120\pi t)$ V. The current is measured to be $i = 4.9 \sin(120\pi t - 0.43)$ A | $i = 5.6 \sin(120\pi t - 0.67)$ A | $i = 6.5 \sin(120\pi t - 0.92)$ A. What is the average power dissipated by the resistor in watts?

Answer: 27.6 | 64.3 | 63.8

Solution: The average power supplied to the circuit is $P = \frac{1}{2} I_m \mathcal{E}_m \cos(\phi)$, where ϕ is measured in radians. This average power is dissipated in the resistor because on average the power into the capacitor and inductor is zero.

2. A capacitor consists of parallel plates separated by vacuum. The plates are squares with sides of length 12 | 14 | 16 cm and separated by 2.2 mm. An electric field uniformly distributed over the plates has a strength (in V/m) given by $E(t) = 500t^3 - 5250t + 2000$ | $E(t) = 600t^3 - 6250t + 2000$ | $E(t) = 700t^3 - 7250t + 2000$, where t is measured in seconds. At time $t = 4$ s, what will be the displacement current in amperes between the plates?

Answer: 2.39×10^{-9} | 3.91×10^{-9} | 5.97×10^{-9}

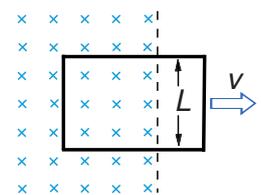
Solution: The displacement current is $i_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 A \frac{dE}{dt}$, where A is the area of the square plates.

3. An atom with magnetic moment $\mu = 9.274 \times 10^{-24} \text{ J T}^{-1}$ is placed in a magnetic field of magnitude 1.2 | 1.3 | 1.4 T. Initially its magnetic moment points in the opposite direction of the magnetic field. Then the moment reverses its direction so that it is pointing parallel to the magnetic field. In the process the energy of the atom changes by $-\Delta E$, and a photon is emitted with frequency $f = \Delta E/h$, where h is Planck's constant given in the table of constants. What is f ?

Answer: 3.36×10^{10} Hz | 3.64×10^{10} Hz | 3.92×10^{10} Hz

Solution: The potential energy of a magnetic moment pointing in the opposite direction as the magnetic field is $U_i = -\vec{\mu} \cdot \vec{B} = +\mu B$. The potential energy of a magnetic moment in the same direction as the magnetic field is $U_f = -\vec{\mu} \cdot \vec{B} = -\mu B$. Thus, the change in energy is $U_f - U_i = -2\mu B = -\Delta E$, and the frequency of the photon is $f = \Delta E/h = 2\mu B/h$.

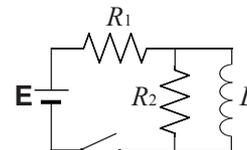
4. The figure shows a rectangular loop of rigid wire of height $L = 30 | 40 | 20$ cm, placed perpendicular to a 2 | 3 | 4 tesla magnetic field. The total resistance of the loop is 3 | 2 | 0.5 Ω . What is the force required to pull the loop out of the field at a constant velocity of $v = 3 | 2 | 0.5$ m/s, as shown?



Answer: 0.36 N | 1.44 N | 0.64 N

Solution: Using Faraday's law of induction the induced emf in the circuit is $\frac{d\Phi_B}{dt} = B \frac{dA}{dt} = BvL$. Thus, the current in the loop is $i = BvL/R$, where R is the resistance of the loop. The force on a current carrying wire is $\vec{F} = i\vec{L} \times \vec{B}$. The forces on the top and bottom segments of the rectangular loop cancel, but the left side of the loop experiences a force $F = iLB = vL^2 B^2/R$.

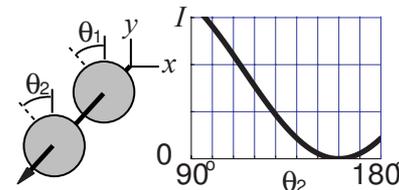
5. In the circuit shown, $\mathcal{E} = 1.5 \mid 3.0 \mid 2.0$ V, $R_1 = 3.0 \mid 2.0 \mid 4.0$ Ω , $R_2 = 2.0 \mid 5.0 \mid 3.0$ Ω , and $L = 1.0$ H. What is the current through R_2 immediately after the switch is closed?



Answer: 0.30 A \mid 0.43 A \mid 0.29 A

Solution: The current can not change instantaneously through an inductor. Thus, right after the switch is closed the current in the inductor is still zero. A current $i = \mathcal{E}/(R_1 + R_2)$ flows through both R_1 and R_2 .

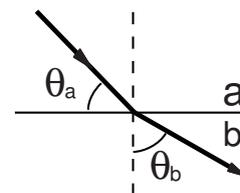
6. Unpolarized light is sent into a system of two polarizing sheets, as shown in the left frame of the figure. The angles θ_1 and θ_2 of the polarizing directions of the sheets are measured counterclockwise from the positive direction of the y axis. Angle θ_1 is fixed but angle θ_2 can be varied. The right frame of the figure shows the intensity of the light emerging from sheet 2 as a function of θ_2 . Give the intensity of the emerging light as a percentage of the light's initial intensity when $\theta_2 = 90^\circ \mid 120^\circ \mid 100^\circ$.



Answer: 44% \mid 21% \mid 38%

Solution: The transmitted light through both filters is zero at $\theta_2 = 160^\circ$. This means that θ_1 is perpendicular to 160° , i.e. $\theta_1 = 70^\circ$ (or equivalently 250°). The general form for the transmitted light through both filters with unpolarized incoming intensity I_o is $I = \frac{1}{2}I_o \cos^2(\theta_2 - \theta_1)$.

7. The diagram shows a ray of light from medium a , whose index of refraction is 1.89, entering medium b . The angle θ_a is $50^\circ \mid 40^\circ \mid 50^\circ$ and θ_b is $60^\circ \mid 60^\circ \mid 65^\circ$. (The two angles are not to scale in the diagram, and note the definition of the angles in the figure.) What is the index of refraction of medium b ?



Answer: 1.40 \mid 1.67 \mid 1.34

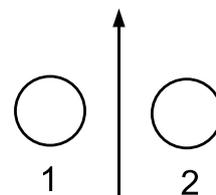
Solution: The angle of the incoming light relative to the perpendicular is $90^\circ - \theta_a$. Snell's law implies that $n_a \sin(90^\circ - \theta_a) = n_b \sin(\theta_b)$ so $n_b = n_a \sin(90^\circ - \theta_a)/\sin(\theta_b)$.

8. Light with intensity $13 \mid 6.5 \mid 13$ W/m² is totally reflected backwards from a surface of area $A = 1.2 \mid 1.2 \mid 2.4$ cm². What is the force on the surface by the light?

Answer: 1×10^{-11} N \mid 5×10^{-12} N \mid 2×10^{-11} N

Solution: The radiation pressure for light which is reflected backwards is $2I/c$. Multiply by the area to get the force $F = A2I/c$.

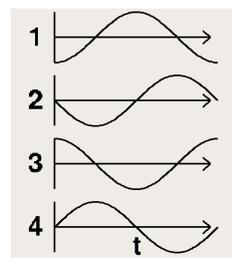
9. The current in the center wire is going upward | downward | downward and increasing | increasing | decreasing in magnitude. What is the direction of the induced current in loop 1 and loop 2, respectively?



Answer: clockwise, counterclockwise | counterclockwise, clockwise | clockwise, counterclockwise

Solution: For the first version of this problem the wire has current flowing up so the magnetic field is coming out of the page on the left and into the page on the right. The current is increasing so the magnetic field is increasing as well. To oppose the change in the magnetic field, we wish to create a magnetic field which is going into the page on the left and out of the page on the right.

10. The current in a series RLC circuit is shown in graph 4 | 3 | 2. Which graph corresponds to the voltage across the capacitor?



Answer: graph 1 | graph 4 | graph 3

Solution: In a series RLC circuit the current leads the voltage by $\pi/2$, or in other words the voltage lags the current by $\pi/2$. In the first case if graph 4 is the current, then graph 1 is the graph which lags by $\pi/2$.