

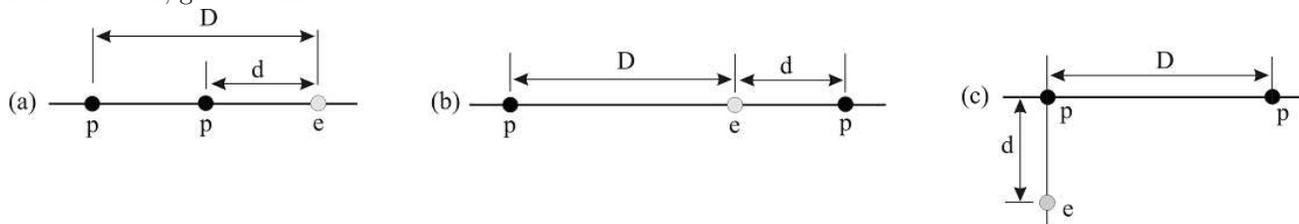
Exam 1 Solution

1. Two identical conducting spheres A and B carry identical charges. They are separated by a distance much larger than their diameters. The magnitude of the electrostatic force between the spheres is F . One of the spheres is connected to earth ground and then disconnected from earth ground. The spheres are now allowed to touch and then separated to their original distance. The magnitude of the electrostatic force between the spheres is now:

Answer: $\frac{1}{4}F$

Solution: Let the charge on each sphere be Q initially. The magnitude of the force between the spheres is $F = kQ^2/r^2$, where r is their separation. When the sphere is grounded, its charge becomes zero. After the spheres touch, they each have charge $Q/2$. Thus, the magnitude of the force between them after they are again placed at separation r is $k(Q/2)^2/r^2 = F/4$.

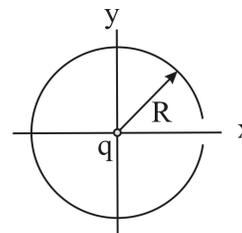
2. For the arrangement of the electron and two protons shown in each case rank the magnitude of the net electrostatic force on the electron, greatest first.



Answer: (a) > (c) > (b)

Solution: The magnitude of the force on the electron in case (a) is greater than the magnitude of the force in (b) because in (a) the forces from the two protons add, while in (b) they subtract: $F_a > F_b$. In case (c) one of the protons is further away at a distance of $\sqrt{d^2 + D^2}$. The forces are not opposed as in case (b) so $F_c > F_b$. On the other hand because the one charge is further away in case (c) the force in case (c) must be smaller than in (a): $F_a > F_c$.

3. As shown in the Figure, a charge of $q = 3C$ lies at the origin surrounded by a spherical shell of charge having a fixed surface charge density of $\sigma = 15\mu C/m^2$. The shell has a radius $R = 10$ km. Where the shell crosses the x axis there is a hole (greatly exaggerated in the Figure) of 0.5 m² area. What vector force does the charge q experience from the shell? (Hint: remember that electrostatic forces obey superposition and consider if there is charge on the shell that is uncompensated.)



Answer: $2 \times 10^{-3} N \hat{i}$

Solution: The hole has no charge. One way to make a neutral region is to take a completely positively charged shell and add a region of negative charge that cancels the positive charge. Thus, the force due to a shell with a hole is the same as that of a positively charged shell with no hole and a negatively charged region where the hole is. The force at the center of a positively charged shell is zero so the net force at the center is just due to the negatively charged region. That region has charge $(-15\mu C/m^2)(0.5m^2) = -7.5\mu C$. The $3C$ charge is attracted to this negative charge so the force is in the \hat{i} direction. The magnitude of the force is $k(3C)(7.5\mu C)/(10^4m)^2$.

4. A charge of -3.0 nC lies fixed at the origin and a charge of $+10 \text{ nC}$ lies along the x axis fixed at $x = 2.0 \text{ m}$. Where along the x axis will an arbitrary charge experience no net electrostatic force from the two fixed charges?

Answer: $x = -2.42 \text{ m}$

Solution: For the force to be zero on an arbitrary charge the electric field must be zero. The electric fields due to the two charges point in the same direction between the charges and opposite directions elsewhere. In order for the fields to cancel one must be closer to the smaller charge. Consequently, the electric field is zero for some $x < 0$. Set the magnitudes of the fields due to the two charges equal to one another:

$$\frac{k(3\text{nC})}{(-x)^2} = \frac{k(10\text{nC})}{(2-x)^2}.$$

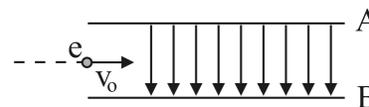
Taking the square root of both sides of this equation remembering that $-x > 0$ and $2-x > 0$, one finds that $\sqrt{3}/(-x) = \sqrt{10}/(2-x)$. This has solution $x = 2\sqrt{3}/(\sqrt{3} - \sqrt{10})$.

5. A coulomb of positive charge on a material body implies an excess or deficit of how many electrons on the body?

Answer: deficit of 6.24×10^{18}

Solution: To make a net positive charge we remove electrons. The number of electrons we must remove is $1C$ divided by the magnitude of the charge of each electron, $1.6 \times 10^{-19}C$.

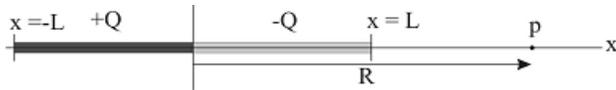
6. Two sheets of opposite charge A & B are separated by 1 cm and produce a uniform electric field between them of 6.0 N/C that points from A to B. An electron enters the region between the sheets from one side, initially equidistant to each sheet ($\frac{1}{2} \text{ cm}$) and initially traveling parallel to the sheets with speed $v_o = 4.0 \times 10^6 \text{ m/s}$ (see figure). How far does the electron travel in the horizontal direction before colliding with one of the sheets and does it collide with A or B?



Answer: 39 cm , A

Solution: The force on an electron is $-e\vec{E}$ so the electron is accelerated toward plate A. The magnitude of the acceleration is $a = eE/m_e$. Since the electron is initially traveling parallel to the plate, it does not have any component of the velocity perpendicular to the plates. The time t it takes to hit plate A satisfies $0.5\text{m} = \frac{1}{2}at^2$. The horizontal distance the electron travels in that time is $v_o t$.

7. A very thin rod of total length $2L$ lies along the x axis with its midpoint at the origin. For points lying to the left of the origin the rod is charged uniformly with total charge $+Q$ (see Figure) while for points to the right of the origin the rod is charged uniformly with total charge $-Q$. The electric field at point p a distance R from the origin ($R > L$) is: (Suggestion: Do most of the other problems before tackling this one. Also, you may wish to use $\int (x - a)^{-2} dx = -(x - a)^{-1}$.)

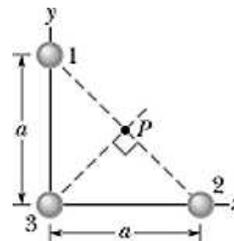


Answer: $-\frac{2kQL}{R^3} \left[\frac{1}{1 - (L/R)^2} \right]$

Solution: Let $\lambda = Q/L$ and perform the integral to evaluate the electric field:

$$\begin{aligned} E_x &= \int_{-L}^0 \frac{k\lambda}{(x - R)^2} dx - \int_L^0 \frac{k\lambda}{(x - R)^2} dx \\ &= k\lambda \left\{ -\frac{1}{0 - R} + \frac{1}{-L - R} + \frac{1}{L - R} - \frac{1}{0 - R} \right\} \\ &= k\lambda \left\{ \frac{2}{R} - \frac{2R}{R^2 - L^2} \right\} \\ &= \frac{2k\lambda}{R} \left\{ 1 - \frac{R^2}{R^2 - L^2} \right\} \\ &= -\frac{2k\lambda L^2}{R} \frac{1}{R^2 - L^2} \\ &= -\frac{2kQL}{R^3} \frac{1}{1 - (L/R)^2}. \end{aligned}$$

8. Three particles are fixed in place and have charges $q_1 = q_2 = -96e$ and $q_3 = -4.0e$. The distance $a = 5.0\mu\text{m}$. The magnitude of the electric field at the point P is:

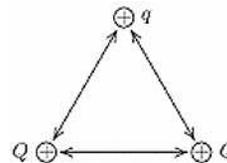


Answer: 460 N/C

Solution: The electric fields due to particles q_1 and q_2 cancel at point P. We are left only with the field due to q_3 , which has magnitude $k|q_3|/(a/\sqrt{2})^2$.

9. Two particles, each with charge Q , and a third particle, with charge q , are placed at the vertices of an equilateral triangle as shown. The total force on the particle with charge q is:

Answer: perpendicular to the bottom side of the triangle



Solution: The charge q is repelled from both charges Q . The horizontal components of the forces on q cancel, and we are left only with the vertical component.

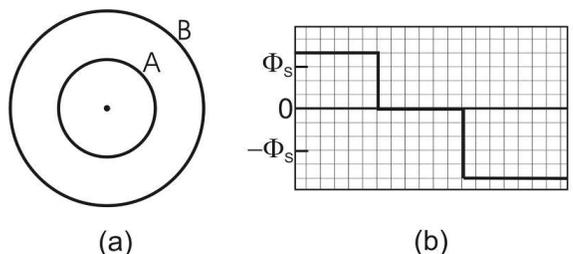
10. A line of charge extending along the x axis has a continuous charge density given by $\lambda = 3x^2 \text{ nC/m}$. How much charge is contained on the line from $x = 3$ to $x = 4$ meters?

Answer: 37 nC

Solution: Integrate the charge density to get the charge:

$$Q = \int \lambda dx = \int_3^4 (3x^2 \text{ nC/m}) dx = x^3 \text{ nC/m} \Big|_3^4 = 37 \text{ nC}.$$

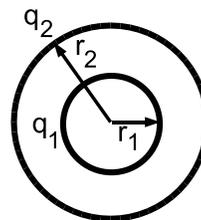
11. A charged particle is suspended at the center of two thin non-conducting concentric shells, as shown in figure (a). Figure (b) gives the net electric field flux Φ through a Gaussian sphere centered on the particle, as a function of the radius of the sphere. The scale of the vertical axis is set by $\Phi_S = 15.0 \times 10^5 \text{ N}\cdot\text{m}^2/\text{C}$. Calculate the net charge of shell B in μC .



Answer: -22.1

Solution: From the figure the electric field in between shells A and B is zero. Hence, the flux through a Gaussian spherical surface in between shells A and B is zero, and there is no charge enclosed. This implies that sum of the charge on shell A and the particle is zero. The net charge on the particle, shell A, and shell B is the same as charge on shell B. From the graph the flux through a Gaussian sphere containing the particle and both shells is $-(5/3)\Phi_S = Q_{enc}/\epsilon_o = Q_B/\epsilon_o$.

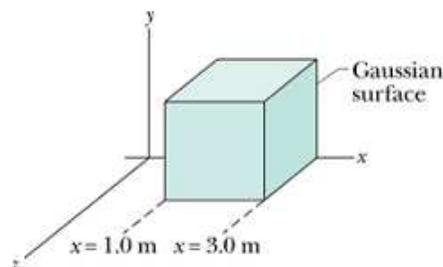
12. Two thin concentric spherical shells made of copper have charge of $q_1 = -46 \text{ nC}$, $q_2 = 69 \text{ nC}$ and radii of $r_1 = 15 \text{ cm}$, $r_2 = 34 \text{ cm}$ as shown in the figure. What is the surface charge density on the outer surface of the second shell with radius r_2 in units of C/m^2 ?



Answer: 1.58×10^{-8}

Solution: Copper is a metal and a good conductor. Consequently, the electric field inside of each shell is zero. The flux through a Gaussian spherical surface inside of the second shell is zero, and so the net charge inside of it is zero. This means that charge on the inside surface of the second shell is $+46 \text{ nC}$. Since the net charge on the second shell is $+69 \text{ nC}$, the charge on the outer surface is $+23 \text{ nC}$. The charge per unit area is then $+23 \text{ nC}/(4\pi r_2^2)$.

13. A non-uniform electric field given by $\vec{E} = 2.51\hat{i} + 0.38(y^3 + 6.22)\hat{j} - 5.47x\hat{k}$ pierces the Gaussian cube as shown on the figure below. What is the net charge (in Coulombs) contained by the cube?



Answer: 1.08×10^{-10}

Solution: By Gauss's Law, the net charge is related to the net flux through the cube by $Q_{enc} = \epsilon_o \Phi$. There are six surfaces to the cube. For the left and right surfaces the flux is $-2.51 A$ and $+2.51 A$, where $A = (2m)^2$ is the area of a side. These fluxes cancel. The flux through the front and rear faces have normal directions of $\pm\hat{k}$ and depend on x : $\vec{E} \cdot \hat{n} = -5.47x$; however, the x dependence is the same for the front and rear surfaces so these fluxes also cancel. The only fluxes which do not cancel are for the top and bottom surfaces.

$$\Phi_{top} = 0.38(2^3 + 6.22)A$$

$$\Phi_{bottom} = -0.38(0^3 + 6.22)A$$

$$\rightarrow \Phi_{net} = 0.38(2^3)A$$

14. An infinitely long plastic cylinder with uniform charge density $\rho = 1.85 \times 10^{-7} \text{C/m}^3$ has a radius $R = 2.74 \text{ m}$. What is the electric field (in N/C) at an axial distance $r = 68.0 \text{ cm}$?

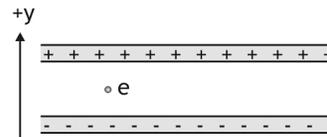
Answer: 7.11×10^3

Solution: Apply Gauss's law to a cylinder of radius r and length L :

$$\oint \vec{E} \cdot \hat{n} dA = \vec{E} \cdot \hat{n} 2\pi r L = \frac{Q_{enc}}{\epsilon_o} = \frac{\rho \pi r^2 L}{\epsilon_o}.$$

This implies that the electric field has magnitude $|\vec{E}| = \frac{\rho r}{2\epsilon_o}$.

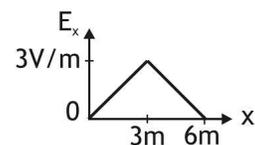
15. An electron is released between two infinite non-conducting sheets that have uniform surface charge densities of $\pm 0.7 \text{ nC/m}^2$ as shown on the figure. The distance between the two sheets is 4.00 mm . What are the magnitude (in units of m/s^2) and the direction of the electron's acceleration?



Answer: 1.39×10^{13} , $+y$

Solution: The electric fields of the two sheets add in the region between the sheets: $E = \sigma/(2\epsilon_o) + \sigma/(2\epsilon_o) = \sigma/\epsilon_o$. The magnitude of the force on the electron is $|F| = e|E|$, and the acceleration is $a = |F|/m_e = e|E|/m_e = e\sigma/(m_e\epsilon_o)$. The direction of the electric field is down because it goes from positive to negative charges, but the force is up ($+y$) because $\vec{F} = q\vec{E}$.

16. The electric field in the x -direction is shown in the figure. If the electric potential at the origin is 4 V, at what value of x for $0 < x < 6$ m is the electric potential zero?



Answer: 2.83

Solution: The voltage at position x is

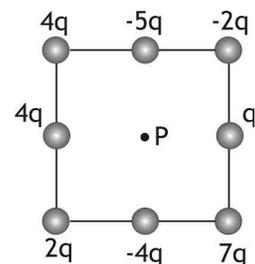
$$V(x) = V(0) - \int_0^x E(x') dx'.$$

The integral over the electric field is the area under the curve in the figure. The area between zero and $3m$ is $0.5(3m)(3V/m) = 4.5V$. Consequently, the voltage at $x = 3m$ is $V(0) - 4.5V = -0.5V$. The voltage is zero somewhere between $x = 0$ and $x = 3$. In the region $0 \leq x \leq 3m$ the electric field has the form $E = (1V/m^2)x$ so the integral is

$$V(x) = 4V - 0.5V/m^2 x^2 = 0.$$

Solving for x one finds that $x = \sqrt{8}m$.

17. Four charges are arranged at the corners of a square with a side $d = 1.5$ m, and four other charges are arranged at the centers of the sides of the same square as shown. If $q = 73$ nC, what is the electric potential at point P at the center of the square?



Answer: 3.3 kV

Solution: The potential due to a single point charge is kQ/r . There are 8 point charges in this figure. Adding up the potential due to each of them yields

$$V = \frac{k(-4q)}{d/2} + \frac{k(11q)}{d/\sqrt{2}},$$

where the first term comes from the charges on the sides and the second term comes from the charges on the corners.

18. Initially three charges are placed on the x -axis as follows: $+1\mu C$ at $x = 0$ cm, $-1\mu C$ at $x = 1$ cm, and $+2\mu C$ at $x = 2$ cm. The positions of the $-1\mu C$ and $+2\mu C$ charges are interchanged. What is the change in the potential energy, $U_f - U_i$?

Answer: 1.35J

Solution: When the $-1\mu C$ charge and the $+2\mu C$ charges are interchanged their distance to the $1\mu C$ charge changes, but not the distance between each other. Consequently, the change in the potential energy is

$$U_f - U_i = \left\{ \frac{k(1\mu C)(-1\mu C)}{2cm} + \frac{k(1\mu C)(+2\mu C)}{1cm} \right\} - \left\{ \frac{k(1\mu C)(-1\mu C)}{1cm} + \frac{k(1\mu C)(+2\mu C)}{2cm} \right\}.$$

19. Two electrons are fixed 3.0 cm apart. A third electron is shot from infinity and comes to a rest midway between the two. What is its initial velocity?

Answer: 260 m/s

Solution: Let the initial velocity of the electron coming from infinity be v . Its final velocity is zero. Also, its initial potential energy is zero because it is infinitely far away. Apply conservation of energy,

$$K_i + U_i = \frac{1}{2}m_e v^2 = K_f + U_f = \frac{ke^2}{0.015m} + \frac{ke^2}{0.015m},$$

to solve for v .

20. The electric potential at points in an x, y plane is given by $V = (2.0V/m^2)x^2 - (5.0V/m^2)y^2$. What is the electric field at the point (1.0 m, 1.0 m)?

Answer: $(-4V/m, 10V/m)$

Solution: The electric field in the x-direction is $-\partial V/\partial x$, and the electric field in the y-direction is $-\partial V/\partial y$. Take the derivatives and evaluate the electric field at $x = y = 1m$.