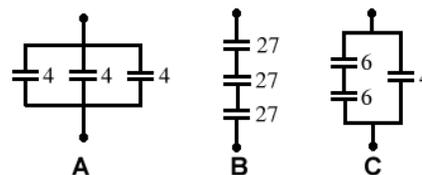


Exam 2 Solution

1. Three capacitor networks labeled A, B & C are shown in the figure with the individual capacitor values labeled (all units μF). Connection to each network is to be made via the round black end terminals. Each network has an equivalent capacitance. The equivalent capacitances of the three networks can be rank ordered, greatest first, in the order:



Answer: A, B, C

Solution: The equivalent capacitances of the three circuits are $C_A = 12\mu\text{F}$, $C_B = 9\mu\text{F}$, and $C_C = 7\mu\text{F}$.

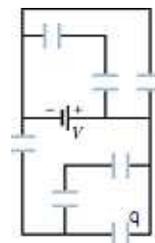
2. The capacitance of a parallel-plate capacitor can be increased by:

Answer: decreasing the plate separation

Solution: The capacitance of a parallel-plate capacitor is $C = \epsilon_o A/d$. Increasing d causes C to decrease.

3. In the circuit shown all capacitors are $6.0\mu\text{F}$ and the power supply is 12V . The charge on the capacitor labeled q is:

Answer: $29\mu\text{C}$



Solution: The effective capacitance of the capacitor with q and the two capacitors in parallel with it is $9\mu\text{F}$. Placing the capacitors in series with a $6\mu\text{F}$ capacitor gives an effective capacitance of $3.6\mu\text{F}$. The charge on this effective capacitor is $(3.6\mu\text{F})(12\text{V}) = 43.2\mu\text{C}$. This is also the charge on the $9\mu\text{F}$ effective capacitor so the voltage across it is $43.2\mu\text{C}/9\mu\text{F} = 4.8\text{V}$. Because capacitors in parallel have the same voltage, the voltage across the q capacitor is 4.8V , and $q = (6\mu\text{F})(4.8\text{V}) = 28.8\mu\text{C}$.

4. A thin, flat, square sheet of metal having an edge length of 2 m has one large flat side coated with a uniform 0.10 mm thick layer of strontium titanate (dielectric constant $\kappa = 310$). A second identical metal plate lies atop the titanate layer symmetrically sandwiching it between the two plates forming a capacitor. Highly conducting metal wires couple electrically to each plate. Using a 12 V power supply, you fully charge the capacitor followed by disconnecting it from the power supply and touching the two wires together to fully discharge the capacitor. Ignoring resistive losses, how many times must you charge and discharge the capacitor to draw one Joule of energy from the supply?

Answer: 127

Solution: The two sheets with a dielectric in between have capacitance $C = \kappa\epsilon_o A/d = 1.1 \times 10^{-4}\text{F}$. The energy stored when the capacitor is placed across a voltage of 12V is $U = \frac{1}{2}CV^2 = 7.9 \times 10^{-3}\text{J}$. The number of times one would have to charge the capacitor to get 1 J of energy is $1\text{J}/7.9 \times 10^{-3}\text{J} = 127$.

5. A wire having a length of 150m, and radius of 0.15mm, has 1V applied across its two ends so that it carries a current with a uniform current density of $2.8 \times 10^5 \text{ A/m}^2$. The resistivity of the wire (in units of $\Omega \cdot \text{m}$) must be:

Answer: 2.4×10^{-8}

Solution: The resistance of a wire is $R = \rho L/A$ so the resistivity is $\rho = RA/L = VA/IL = V/jL$. Using the values given for V , j , and L , the resistivity is $2.4 \times 10^{-8} \Omega \cdot \text{m}$.

6. A 24V DC motor draws 220 mA when raising a 1.0 kg mass. What is the maximum steady state speed that the mass can attain assuming no other losses?

Answer: 0.54 m/s

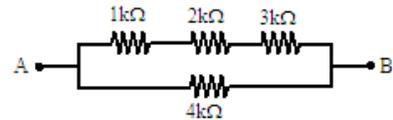
Solution: The power input into the motor is $P = IV$. Thus, the maximum power that can be output to raise the mass is $P = Fv$, where F is the force due to gravity $F = mg$ and v is the velocity of the mass. Solving $P = IV = mgv$ yields a velocity of $v = IV/mg$.

7. Three resistors A, B, and C are connected in parallel with a 12 V power supply. You feel each resistor in turn and B is hottest while C is coolest. The resistance values of the three resistors must be such that:

Answer: $C > A > B$

Solution: All three resistors have the same voltage across them. The power dissipated by a resistor is $P = V^2/R$. The smaller resistance resistor will dissipate more power and will be hotter.

8. The resistor network in the diagram has 12V across its end terminals labeled A and B. The current through, and voltage drop across, the $3k\Omega$ resistor are, respectively:



Answer: 2mA, 6V

Solution: The top three resistors have an effective resistance of $6k\Omega$. The current through them is $12V/6k\Omega = 2mA$. A $2mA$ current through a $3k\Omega$ resistor will have a 6V voltage drop.

9. A 2-terminal circuit has a net (equivalent) resistance between its terminals of 10Ω . To perform its function it requires a minimum of 12V across its 2 terminals. The following non-ideal power supplies are available, where the first number gives its internal (ideal) EMF and the second number gives its internal resistance:

A) 15.6V, 3Ω

B) 14.3V, 2Ω

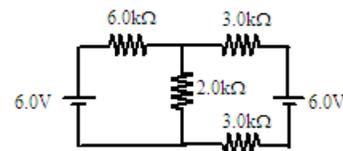
C) 13.2V, 1Ω

Which of these will do the job?

Answer: A & C only

Solution: Let the internal resistance of the battery be R_{int} and the ideal EMF of the battery be \mathcal{E} . The current in the circuit is $\mathcal{E}/(R_{int} + 10\Omega)$. The voltage across the battery and also across the 10Ω resistance is $(10\Omega)\mathcal{E}/(R_{int} + 10\Omega)$. This voltage must be greater than or equal to 12V. For case B the voltage is 11.917V, while for A and C the voltage is exactly 12V.

10. In the multi-loop circuit shown the current through the $2.0\text{k}\Omega$ resistor is (in mA),



Answer: 1.2

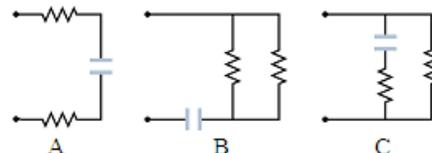
Solution: Applying Kirchhoff's loop rule for i_1 going clockwise in the left loop and i_2 going counterclockwise in the right loop yields

$$0 = 6V - 6k\Omega i_1 - 2k\Omega(i_1 + i_2)$$

$$0 = 6V - 6k\Omega i_2 - 2k\Omega(i_1 + i_2).$$

Add these equations to find that $12V = 10k\Omega(i_1 + i_2)$, where $(i_1 + i_2)$ is the current through the $2k\Omega$ resistor.

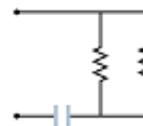
11. In the circuits shown all the capacitors have the same capacitance and all the resistors have the same resistance. At time $t = 0$ the terminals of each circuit are connected to a power supply of the same voltage. Rank order the time taken for the capacitor in each circuit to reach 90% of its full charge, *shortest* time first.



Answer: B, C, A

Solution: The RC time constants for the three cases are $\tau = (2R)C$, $(R/2)C$, and RC , for A, B, C respectively. The resistor in parallel with C does not contribute to the time constant or the charging of the capacitor. The smaller the RC time constant, the more quickly the capacitor charges.

12. In the circuit shown the capacitor is $4.0\mu\text{F}$, and the resistors are each $1.0\text{k}\Omega$. A 12V power supply is connected across the terminals for several minutes. The supply is then disconnected and the terminals touched to each other. How long after the instant the terminals touch does it take for the charge on the capacitor to drop to $3.0\mu\text{C}$.



Answer: 5.5 ms

Solution: The capacitor is initially charged to $Q = (4\mu\text{F})(12\text{V}) = 48\mu\text{C}$. The RC time constant for this problem is $\tau = (0.5\text{k}\Omega)(4\mu\text{F}) = 2\text{ms}$. The capacitor discharges according to $Q(t) = 48\mu\text{C} \exp(-t/\tau)$. Setting $Q(t) = 3\mu\text{C}$, we find that $16 = \exp(t/\tau)$, and $t = \tau \ln(16)$.

13. The orientation of the magnetic field in a region of space is to be determined by monitoring the deflection of a beam of electrons in a cathode ray tube as the tube is rotated in the field. The orientation of \vec{B} (i.e. $\pm\vec{B}$) is along:

Answer: The direction in which the beam travels when it is not deflected

Solution: The force on a charged particle in a magnetic field is $q\vec{v} \times \vec{B}$. When \vec{v} and \vec{B} are either parallel or antiparallel, the force is zero.

14. An electron (mass m_e) and a proton (mass m_p) each have circular trajectories in a uniform magnetic field of magnitude B . The radius of the proton orbit is twice that of the electron. The ratio of the electron to the proton velocities (ie. v_e/v_p) is,

Answer: $\frac{1m_p}{2m_e}$

Solution: For circular motion in a magnetic field $F = qvB = mv^2/r$, which implies that $qB = mv/r$. In these equations all of the quantities are magnitudes, e.g. $v = |v|$ and $q = |q|$. Since $|q|$ and B are the same for the electron and proton, $m_e v_e/r_e = m_p v_p/r_p$. Because $r_p = 2r_e$, the ratio of velocities is $v_e/v_p = (m_p/m_e)(r_e/r_p) = 0.5m_p/m_e$.

15. A long square copper wire lies along the z axis oriented so that a face lies in the x-z plane as shown. It carries current i , in the \hat{k} direction. A Hall voltage measurement shows that there is a voltage between the face in the x-z plane at $y = 0$ and the opposite face (x-z plane at negative y), with the side at $y = 0$ having the higher potential. There must be a magnetic field present oriented in the direction of



Answer: $-\hat{i}$

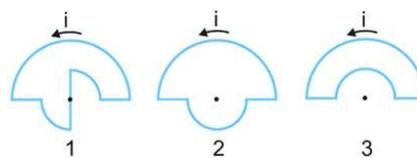
Solution: Because the higher potential is on the right of the wire, the electric field points to the left ($-\hat{j}$). The electric force on an electron is to the right ($+\hat{j}$). This force must be cancelled by the magnetic force on the electron, which must then be to the left ($-\hat{j}$). The magnetic force is $q\vec{v} \times \vec{B}$. Because the current is up, the vector $q\vec{v}$ is also up (\hat{k}). The magnetic field is into the page ($-\hat{i}$) because $\hat{k} \times -\hat{i} = -\hat{j}$.

16. Each of the figures below shows the direction of the dipole moment $\vec{\mu}$ of a current loop in a uniform magnetic field \vec{B} . In which case is the *torque* on the current loop greatest?

Answer:

Solution: The torque, $\vec{\mu} \times \vec{B}$, is greatest when $\vec{\mu}$ and \vec{B} are perpendicular.

17. The figure shows three circuits consisting of straight radial lengths and concentric circular arcs (either half- or quarter-circles of radii r and $2r$). The circuits carry the same current in the indicated direction. Rank the circuits according to the magnitude of the magnetic field produced at the center of curvature (the dot), the greatest first.



Answer: $B_2 > B_1 > B_3$. Because $|B_1| = |B_3|$, we also accepted $B_2 > B_3 > B_1$.

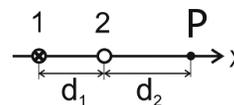
Solution: Let the magnetic field be positive going out of the page. The magnetic field in the three cases is

$$B_1 = \frac{1}{2} \frac{\mu_0 i}{2(2r)} + \frac{1}{4} \frac{\mu_0 i}{2r} - \frac{1}{4} \frac{\mu_0 i}{2r} = \frac{1}{2} \frac{\mu_0 i}{2(2r)} = \frac{1}{4} \frac{\mu_0 i}{2r}$$

$$B_2 = \frac{1}{2} \frac{\mu_0 i}{2(2r)} + \frac{1}{2} \frac{\mu_0 i}{2r} = \frac{3}{4} \frac{\mu_0 i}{2r}$$

$$B_3 = \frac{1}{2} \frac{\mu_0 i}{2(2r)} - \frac{1}{2} \frac{\mu_0 i}{2r} = \frac{-1}{4} \frac{\mu_0 i}{2r}$$

18. Two long straight wires are perpendicular to the page and separated by distance $d_1 = 0.75$ cm, as shown in the figure. Wire 1 carries current 6.50 A into the page. What are the magnitude and direction of the current in wire 2 if the net magnetic field due to the two currents is zero at point P located at a distance $d_2 = 1.50$ cm to the right of wire 2?



Answer: 4.33 A, out of the page

Solution: Wire 2 must have a current flowing out of the page to cancel the field due to wire 1. The condition for the fields due to wire 1 and wire 2 to have the same magnitude is

$$\frac{\mu_0 i_1}{2\pi(d_1 + d_2)} = \frac{\mu_0 i_2}{2\pi d_2},$$

which implies that $i_2 = i_1 d_2 / (d_1 + d_2)$.

19. A long solenoid has 10 turns/cm and carries current i . An electron moves within the solenoid in a circle of radius 2.30 cm perpendicular to the solenoid axis. The speed of the electron is 1.379×10^7 m/s. What is the current i in the solenoid?

Answer: 2.71 A

Solution: For circular motion in a magnetic field $F = qvB = mv^2/r$, which implies that $v/r = qB/m$. This equation allows us to solve for the magnetic field inside the solenoid, $B = 3.41$ mT. The magnetic field inside the solenoid is also equal to $B = \mu_0 n i$, where n is the number of turns per unit length, $n = 10/0.01$ m = 1000 m⁻¹. The only unknown in this equation is i , which is found to be 2.71 A.

20. A uniform current flows through a straight wire with circular cross section of radius R . At what distance from the center of the wire is the value of the magnetic field 1/3 of the magnetic field at the surface of the wire?

Answer: $R/3$ and $3R$

Solution: For $r < R$ Ampere's Law implies

$$\oint \vec{B} \cdot d\vec{s} = 2\pi r B = \mu_0 I_{enc} = \mu_0 i \frac{\pi r^2}{\pi R^2} \Rightarrow B = \frac{\mu_0 i r}{2\pi R^2} = \frac{1}{3} \frac{\mu_0 i}{2\pi R} \Rightarrow r = \frac{R}{3},$$

and for $r > R$ it implies that

$$\oint \vec{B} \cdot d\vec{s} = 2\pi r B = \mu_0 I_{enc} = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r} = \frac{1}{3} \frac{\mu_0 i}{2\pi R} \Rightarrow r = 3R.$$