Final Exam Solution

1. A charge $+3.00q$ lies fixed at the origin and a second charge of $-2.00q$ lies fixed in the x-y plane at $x = -3.00$, $y = -4.00$. The x component of the force experienced by the $-2.00q$ charge due to the charge at the origin is?

   Answer: $0.144kq^2$

   Solution: The distance between the charges is $5 = \sqrt{3^2 + 4^2}$. The magnitude of the force between the two charges is $k(2q)(3q)/5^2$. Since the $-2q$ charge is attracted to the $+3q$ charge, the x-component of the force on the $-2q$ charge is positive. To get the x-component we need to multiply by the cosine of the angle between the force vector and the x-axis. The cosine is the adjacent over the hypotenuse and equal to 3/5. Thus, the x-component of the force is $(3/5)k(2q)(3q)/5^2$.

2. A circular insulating ring of radius r lies in the x-y plane centered on the origin. The parts of the ring in the negative y half plane are uncharged. The parts of the ring in the positive y half plane are uniformly charged with a total charge Q. The electric field at the origin is:

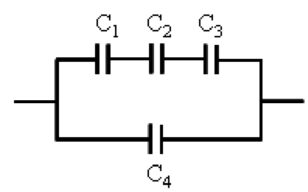
   Answer: $\frac{-2kQ}{\pi r^2}\hat{j}$

   Solution: First, by symmetry the net electric field is in the negative y-direction if we take Q to be positive. The charge per unit length of the half ring is $\lambda = Q/(\pi r)$. Take a small element of charge, $dq = \lambda r d\theta = (Q\pi)d\theta$. This charge produces a field of magnitude kdq/r^2 at the origin. Letting the angle the field makes with the y-axis be θ, the y-component of the electric field at the origin is

$$E_y = -\int_{-\pi/2}^{\pi/2} \frac{k}{r^2}\frac{Q}{\pi}\cos(\theta)d\theta = -\frac{kQ}{\pi r^2}(\sin(\pi/2) - \sin(-\pi/2)).$$

   If Q is negative, then this formula still works, and the evaluated E_y is in positive y-direction because then $-Q > 0$.

3. In the figure the capacitances of the series capacitors are equal. The voltage drop across C_4 is 12 V and the charge on it is 6.0 nC. The charge on the entire network in 9.0 nC. The capacitance of C_2 is?



   Answer: 0.75 nF

   Solution: The voltage across C_1, C_2, and C_3 in series is 12 V. Because these capacitors are in series and equal, their effective capacitance is $C_2/3$. Since the net charge on the network is 9 nC and the charge on C_4 is 6 nC, the charge on the series resistors is 3 nC. Consequently, $C_2/3 = 3nC/12V$, which implies that $C_2 = 9/12$ nF.

4. The resistance measured between the ends of a metal wire having a circular cross-section is R. The wire diameter is halved by going through a series of rollers that preserves the volume of the wire (i.e. the wire gets correspondingly longer as its diameter shrinks with no loss of material). The resistance between the ends of the wire is now:

   Answer: 16R

   Solution: The volume of the wire is $\pi r^2 L$, where r is the radius of the wire and L is the length of the wire. If the radius is decreased by a factor of 2, then the length must be increased by a factor of 4 in order to keep the volume constant. The resistance of the wire $R = \rho L/(\pi r^2)$ becomes $\rho(4L)/(\pi(r/2)^2) = 16R$.

5. A series resistor and capacitor of 5.0Ω resistance and 14 μF capacitance, respectively are hooked up to a 12 V battery. How long does it take the capacitor to charge to 3/4 of its ultimate charge?

   Answer: 97 μs

   Solution: The RC time constant for this circuit is $\tau = RC = 70\mu s$. The capacitor reaches 3/4 of its final charge when $0.75 = (1 - e^{-t/\tau})$ or $0.25 = e^{-t/\tau}$. The solution to this is $t = \ln(4)\tau$.

6. A proton of mass $1.67 \times 10^{-27} kg$ has a velocity (in m/s) of $v = 2.0 \times 10^5 \hat{\imath} + 3.0 \times 10^5 \hat{\jmath}$ when it enters a region of uniform magnetic field $B = 0.10\hat{\jmath}T$. The radius of the spiral path it takes on is:

   Answer: 2.1 cm

   Solution: The proton moves at a constant velocity in the y-direction and in circular motion in the x-z plane. The radius of the circle satisfies $F = mv^2/r = qvB$ with v being the magnitude of the velocity in the x-z plane, $2 \times 10^5 m/s$. Solve for the radius: $r = mv/(qB)$.

7. The variable capacitor of an LC circuit that uses an inductor of 4.0 mH is used to tune a radio to a station broadcasting at a frequency of 1.6 MHz. When it is tuned to the station the capacitance of the circuit is:

   Answer: 2.5 pF

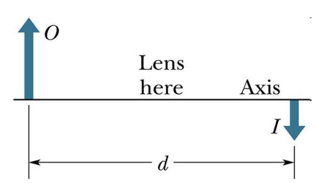
   Solution: An LC circuit oscillates at frequency $f = \omega/(2\pi) = 1/(2\pi\sqrt{LC})$. We are given $L = 4 \times 10^{-3} H$ and $f = 1.6 \times 10^6 Hz$. Solve for C.

8. The $\int \vec{B} \cdot \hat{n} dA$ over 5 of the 6 sides of a cube evaluates to $2.4 \times 10^{-2} T\,m^2$. For the 6th side the $\int \vec{B} \cdot \hat{n} dA$ evaluates to (in $T\,m^2$):

   Answer: -2.4×10^{-2}

   Solution: The net magnetic flux through any closed surface is zero. Consequently, the flux through the 6th side must be the negative of the net flux through sides 1–5.

9. In the Figure (which is not to scale) the lens creating the real, inverted image is not shown. The image to object distance $d = 30$ cm and the image is 1/3 the size of the object. Use these to find the object distance and then use that information to determine that the focal length of the lens is:



   Answer: 5.6 cm

   Solution: From the figure we see that $d = p + i = 30cm$ and also that $m = -i/p = -1/3$. These two equations can be solved for the object and image distances: $p = 22.5$ cm and $i = 7.5$ cm. Use these to find the focal length via $1/f = 1/p + 1/i$.

10. Two identical lenses of 5 cm focal lengths are 35.2 cm apart. Light shines through a film image that is 6 cm from the first lens. How far from the second lens should the screen be placed to form a real focused image, what is the magnification of that image and is the image inverted or upright?

    Answer: 130 cm, 125, upright

    Solution: The object distance for the first lens is $p_1 = 6$ cm. For a lens with focal length 5 cm, the image distance is $i_1 = 30$ cm. The object distance for the second lens is $p_2 = 5.2$ cm, and the image distance for the second lens is 130 cm. This is the final image. From the magnification, $(-i_1/p_1)(-i_2/p_2) = 125$, we can see that the image is upright.

11. At night many people see rings surrounding bright outdoor lamps in otherwise dark surroundings. The rings are the first of the side maxima in diffraction patterns produced by structures in the observer's eye. The central maxima of such patterns overlap the lamp. If the lamp were switched from red to blue light, what would a particular ring become?

Answer: smaller

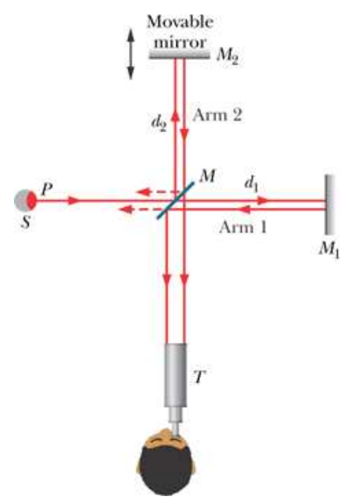
Solution: For the case of diffraction from a circular aperture, we have $\sin\theta = 1.22\lambda/d$. If λ is decreased (going from red to blue light), then the angle also decreases.

12. A sheet of glass ($n = 1.50$) is to be coated with a thin film of oil ($n = 1.25$) so that light with wavelength of 510 nm (in air) is minimally reflected due to destructive interference of the reflections. What is the least thickness of the film?

Answer: 102 nm

Solution: For both the the air/oil and the oil/glass interfaces one is going from a lower index of refraction to a higher one. The phase shift is the same in both cases (π). The condition for destructive interference is thus $(n + 0.5) = 2d/(\lambda/n_2)$, where d is the thickness of the oil film and $n_2 = 1.25$. The minimum value of d for destructive interference is $d = 0.25\lambda/n_2$.

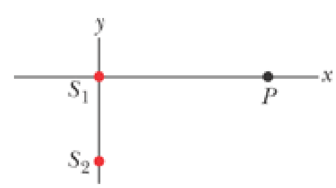
13. A Michelson interferometer is shown at right. There is interference of the light coming from the source at P because of the two possible paths for the light to reach the observer. Initially, d_1 and d_2 are chosen so that there is constructive interference for light of wavelength 500 nm. What is the smallest change in d_2 so that there is completely destructive interference?



Answer: 125 nm

Solution: The difference in path length for the two paths is $\Delta L = 2d_2 - 2d_1$. To go from constructive to destructive interference means that ΔL must change by $\lambda/2$, which corresponds to a change in d_2 of $\lambda/4$.

14. Two isotropic point sources of light (S_1 and S_2) are separated by distance 2.70 μm along the y axis and emit in phase at wavelength 600 nm, and at the same amplitude. A light detector is located at point P at coordinate x_P on the x axis. What is the *greatest* value of x_P at which the detected light is minimum due to destructive interference?



Answer: 12.0 μm

Solution: In the following measure all distances in μm. The difference in path length is $\Delta L = \sqrt{2.7^2 + x^2} - x^2$. For destructive interference the path length difference is $\Delta L = (n + \frac{1}{2})\lambda$. For large x we can expand the square root:

$$\sqrt{2.7^2 + x^2} = x\sqrt{1 + \frac{2.7^2}{x^2}} \approx x\left(1 + \frac{2.7^2}{2x^2}\right) \approx x + \frac{2.7^2}{2x}.$$

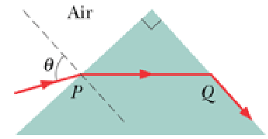
The condition for destructive interference becomes $2.7^2/(2x) = (n + 1/2)\lambda$. The largest value of x for which this is true is $2.7^2/(2x) = \lambda/2$ or $x = 2.7^2/0.6 = 12\mu m$.

15. Monochromatic green light of wavelength 550 nm illuminates two parallel narrow slits 7.70 μm apart. What is the angular separation, $\Delta\theta$ (in degrees), between the third-order and the second-order bright fringes?

Answer: 4.16

Solution: For constructive interference for two slits $m\lambda = d\sin\theta$. For small angles $\sin\theta \approx \theta$ in radians. Thus, in radians the separation between fringes is approximately λ/d and in degrees it is $(180/\pi)(\lambda/d)$. (The exact answer is about 1 percent different from this.)

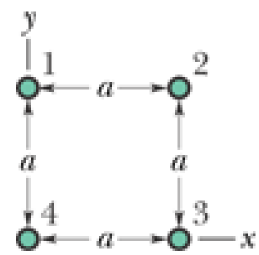
16. Light enters a 90° triangular prism at point P with incident angle θ, and then some of it refracts at point Q with an angle of refraction of 90° as shown. If the index of refraction of the prism is 1.33, what is the incident angle (in degrees)? (Despite appearances, the triangle formed between the 90° corner and the light path through the prism is not an isosceles triangle.)



Answer: 61.3

Solution: At point Q the angle of incidence is $\sin^{-1}(1/1.33) = 48.75°$. This implies that the transmitted angle at point P is $90 - 48.75 = 41.25°$. The angle of incidence at P is thus $\sin^{-1}(1.33\sin(41.25))$,

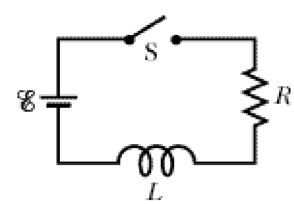
17. Four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 20$ cm. The currents are out of the page for wires 1 and 4 and into the page for wires 2 and 3. Each wire carries current 5.0 A. What is the net magnetic field vector at the square's center?



Answer: 20 $\mu T\hat{j}$

Solution: Each wire produces a magnetic field of magnitude $\mu_o I/(2\pi r)$ at the center of the square, where $I = 5A$ is the current and $r = a/\sqrt{2}$ is the distance to the center of the loop. Wires 1 and 3 produce a field in the direction $(\hat{\imath} + \hat{\jmath})/\sqrt{2}$, while wires 2 and 4 produce a field in the direction $(-\hat{\imath} + \hat{\jmath})/\sqrt{2}$. Adding the fields for all the wires as vectors yields a net magnetic field of $4\mu_o I/(2\pi a)\hat{\jmath}$.

18. The switch S is closed at $t = 0$, initiating the build-up of current in the 15.0 Ω resistor and the 10.0 mH inductor. At what time is the current one third of its long time final value?



Answer: 0.27 ms

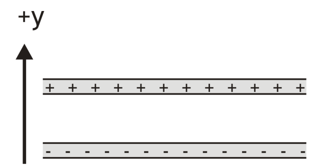
Solution: The time constant for this circuit is $\tau = L/R$. Since the current is increasing from zero, it reaches 1/3 of its final value when $1/3 = (1 - e^{-t/\tau})$. This has solution $t = \tau \ln(1.5)$.

19. The equipotential surfaces associated with a charged point particle are:

Answer: concentric spheres centered at the particle

Solution: The potential for a point charge is $V = kq/r$. Thus, points at a constant distance from the particle have the same potential.

20. Two large, parallel, non-conducting sheets, each with a fixed uniform charge on one side, have surface charge densities of $\sigma_+ = 7.4\mu\text{C/m}^2$ for the positively charged sheet, and $\sigma_- = -5.3\mu\text{C/m}^2$ for the negatively charged sheet. What are the magnitude (in kN/C) and the direction of the electric field in the region above both the sheets?



Answer: 120, $+y$

Solution: The electric field goes away from positive charges and towards negative charges. Since the electric field due to one plate is $\sigma/(2\epsilon_o)$, the electric field above both plates is $(\sigma_+/(2\epsilon_o) - |\sigma_-|/(2\epsilon_o))\hat{\jmath}$.