

Exam 3 Solutions

1. The magnetic dipole moment of a substance aligns opposite to the direction of an applied magnetic field. The magnetic property of such a substance can be characterized as:

Answer: diamagnetism

Solution: In diamagnetism the dipole moments point in the opposite direction as the magnetic field, while in paramagnetism the dipole moments align with the magnetic field.

2. A parallel-plate capacitor with circular plates of radius 0.15 m is being charged by a constant current of 3 A. What is the magnitude of the magnetic field in μT between the two plates at a radius of 0.05 m from an axis that passes through the center of both plates?

Answer: 1.3

Solution: Because $\oint \vec{B} \cdot d\vec{s} = \mu_o \epsilon_o (d\Phi_e/dt) = \mu_o i_d$, integrating around a circular path of radius 0.05 m yields $2\pi B(0.05\text{m}) = \mu_o(3\text{A})(0.05^2/0.15^2)$. The factor of $(0.05^2/0.15^2)$ is the fraction of the total displacement current which passes through a circle of radius 0.05 m.

3. The magnitude of the electric field between two parallel circular plates of radius $R = 1$ cm is $E = 1000t$ N/C, where t is in seconds. What is magnitude of the magnetic field that is induced at a radius of $r = 3$ cm? (Hint: $r > R$).

Answer: 1.9×10^{-17} T

Solution: Because $\oint \vec{B} \cdot d\vec{s} = \mu_o \epsilon_o (d\Phi_e/dt)$, integrating around a circular path of radius 3 cm yields $2\pi B(0.03\text{m}) = \mu_o \epsilon_o (\pi 0.01^2 \text{m}^2) 1000 \text{N/C}$. Note that the radius of the path, 3 cm, is different from the radius where the electric field is nonzero, 1 cm.

4. The magnetic dipole moment of an atom is 9.27×10^{-24} J/T. If the dipole is initially aligned parallel to the direction of an applied 3T magnetic field, how much energy does it take to flip the dipole to be aligned opposite to the direction of the field?

Answer: 5.6×10^{-23} J

Solution: The energy of a magnetic dipole in a magnetic field is $U = -\vec{\mu} \cdot \vec{B}$. Changing from aligned with the magnetic field ($U = -\mu B$) to aligned opposite to the magnetic field ($U = \mu B$) is a change in energy of $2\mu B$.

5. A beam of initially unpolarized light is sent into a stack of 3 polarizing sheets, where the polarizing direction of each sheet is rotated $+30^\circ$ with respect to the previous sheet. What percentage of the incident intensity is transmitted through the stack of 3 sheets?

Answer: 28%

Solution: After passing through the first polarizer the intensity goes down by 2. Each subsequent polarizer, reduces the intensity by $\cos^2(30^\circ)$. Thus, the final intensity is reduced by $0.5 \cos^4(30^\circ)$.

6. The light from a professor's laser pointer can be considered as an electromagnetic plane wave. If the laser beam has 5 mW of power and illuminates a circular spot 2.5 mm in radius on a screen, what is the maximum electric field amplitude (E_m) at the screen?

Answer: 440 N/C

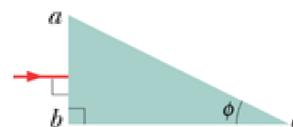
Solution: The intensity of the light is $I = 5mW/(\pi(2.5mm)^2) = 255W/m^2$. It is related to the maximum electric field amplitude via $I = E_m^2/(2c\mu_0)$.

7. Which of the following expressions describes the electric field of an electromagnetic plane wave traveling in the $-z$ direction if the magnetic field is pointing in the $-x$ direction when $x = y = z = t = 0$?

Answer: $-E_m\hat{y}\cos(kz + \omega t)$

Solution: The $(kz + \omega t)$ is for a wave going in the $-z$ direction, while a $(kz - \omega t)$ would be for a wave going in the z direction. The direction of propagation is the same as the direction of $\vec{E} \times \vec{B}$. $-\hat{z} = \hat{E} \times -\hat{x}$ implies that the direction of the electric field at $t = 0$ at the origin is $-\hat{y}$.

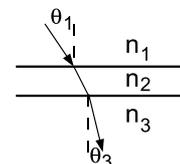
8. A light ray is incident perpendicular to the left face of the shown prism made of glass ($n = 1.6$). What is the largest value of the angle ϕ such that the light ray is totally reflected at the far right face if the prism is surrounded by air?



Answer: 51°

Solution: The light ray goes straight through the first interface (air to glass) because the angle relative to the normal is $\theta = 0$. The angle that the light ray makes with the normal at the glass to air interface is $90^\circ - \phi$. The ray is total reflected for $90^\circ - \phi \geq \theta_c = \sin^{-1}(1/1.6)$, which implies that $\phi \leq 90^\circ - \sin^{-1}(1/1.6)$.

9. In the figure light is incident at an angle $\theta_1 = 30^\circ$ on a boundary between two transparent materials. Some of the light travels through the next layers of transparent materials. If $n_1 = 1.1$, $n_2 = 1.2$, and $n_3 = 1.4$, what is the value of θ_3 ?



Answer: 23°

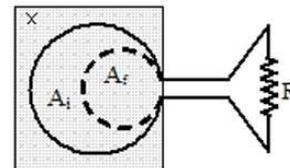
Solution: Because the interfaces are parallel, the transmitted angle at the n_1 to n_2 interface is the same as the incoming angle at the n_2 to n_3 interface (θ_2). Thus, according to Snell's law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2) = n_3 \sin(\theta_3)$. where the angles are measured relative to the normal.

10. A totally reflecting mirror of area $A = 3 \text{ m}^2$ intercepts light from the sun with an intensity of 300 W/m^2 . What radiation pressure is produced on the mirror by the light?

Answer: $2 \times 10^{-6} \text{ N/m}^2$

Solution: The radiation pressure on a totally reflecting surface is $2I/c$, where I here is 300 W/m^2 .

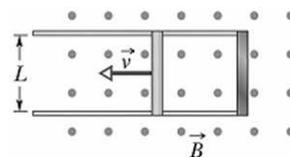
11. An elastic, electrically conducting wire is constrained to form a circular loop with the ends of the wire connected across a 2Ω resistor (see Figure). The region occupied by the circular part of the loop has a uniform magnetic field of 2T (indicated by the grey region) oriented perpendicular to the loop and into the page. The circular part of the loop collapses so that its area collapses uniformly from an initial area of $A_i = 0.10\text{ m}^2$ to a final area of $A_f = 0.07\text{ m}^2$ in one second. Ignoring the small gap in the loop and treating the wires as having no resistance what is the current through the resistor during the collapse and is this current flowing up or down the page through the resistor?



Answer: 30 mA , down

Solution: Using Faraday's law the induced emf is the rate of the change in flux. The flux is changing because the area of the loop is changing. Consequently, $d\Phi_B/dt = B\Delta A/\Delta t = 0.06\text{V}$. The current through the circuit is $i = V/R = 0.03\text{A} = 30\text{mA}$. According to Lenz's law the induced current acts to oppose the change in flux. The flux is going into the page and getting smaller. To oppose this change the induced current makes a flux into the page, which is clockwise, resulting in a downward current at the resistor.

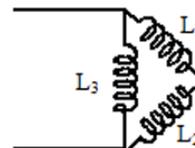
12. The metal rails in the Figure are separated by distance $L = 0.5\text{ m}$ and electrically connected together by a metal strip at one end. The rails and strip have negligible resistance. A resistive metal bar, for which $R = 10\Omega$ that touches the rails (making electrical contact to them) is pulled to the left at a constant speed of 2.0 m/s . A uniform magnetic field of 2.0T directed perpendicular to the area between the rails (out of the page) exists in the region. Neglecting friction, how much power must be expended to maintain this motion of the resistive bar?



Answer: 0.40 W

Solution: Here again the induced emf is due to a change in the area. Consequently, $d\Phi_B/dt = B\Delta A/\Delta t = BvL$, where v is the velocity of the bar. This produces a current of $i = BvL/R$. The power dissipated in the circuit is $P = i^2R = (BvL)^2/R$, which is also the power that needs to be supplied in order to keep the rod moving at a constant velocity.

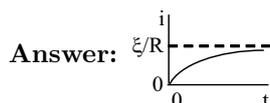
13. In the Figure $L_1 = 20\text{ mH}$, $L_2 = 30\text{ mH}$ and $L_3 = 40\text{ mH}$. The equivalent inductance between the wire ends is:



Answer: 22 mH

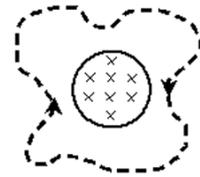
Solution: Inductors add the same way as resistors. L_3 is in parallel with the pair L_1 and L_2 , which are in series. $L_{eff} = (1/L_3 + 1/(L_1 + L_2))^{-1}$.

14. Which plot shows the correct behavior for the current versus time of an RL circuit immediately after it is connected to a battery?



Solution: For an RL circuit or any circuit containing an inductor the current can not change instantaneously. Consequently the current must start at zero. At long times the voltage drop across the inductor is zero because the current is not longer changing. All the voltage is dropped across the resistor so the current saturates at ξ/R , where ξ is the voltage of the battery.

15. The circular region of radius $r = 40$ cm shown in the figure contains a uniform magnetic field pointing into the page that is increasing at a rate of 0.20T/s . For the path and direction indicated evaluate the integral $\oint \vec{E} \cdot d\vec{s}$



Answer: -0.10 V

Solution: The magnitude of the integral $\oint \vec{E} \cdot d\vec{s}$ is $d\Phi_B/dt = \pi(0.4\text{m})^2 0.2\text{T/s} = 0.1\text{V}$. For this path positive flux is going into the page so $\oint \vec{E} \cdot d\vec{s} = -\partial\Phi_B/dt = -0.1\text{V}$.

16. An LC oscillator oscillates at a frequency of 45 Hz. The capacitor has capacitance of $30\mu\text{F}$. The inductor must have inductance:

Answer: 417 mH

Solution: The angular frequency of an LC oscillator is $\omega = 2\pi f = 1/\sqrt{LC}$ so that the inductance is $L = 1/(C(2\pi f)^2)$.

17. An RLC circuit has inductance $L = 22$ mH, capacitance $C = 1.0$ mF and resistance $R = 1.5\Omega$. At what time will the amplitude of the charge oscillations on the capacitor be 50% of their initial amplitude?

Answer: 20 ms

Solution: The amplitude of the charge oscillations is proportional to $e^{-Rt/(2L)}$. The amplitude is 50% of their initial amplitude when $e^{-Rt/(2L)} = 0.5$, which implies that $Rt/(2L) = \ln(2)$.

18. In a sinusoidally driven series RLC circuit the current lags the applied emf. The rate at which energy is dissipated in the resistor can be increased by:

Answer: decreasing the capacitance (making no other changes)

Solution: The rate at which energy is dissipated by the resistor is $i^2 R$. To increase this rate we can increase the current. To increase the current we can decrease the impedance, Z . Because the current lags the emf ($\phi > 0$), $X_L > X_C$, and we increase X_C to decrease Z or equivalently decrease C .

19. For a driven series RLC circuit the resonance curve peaks at 2.5A . The inductance is $220\mu\text{H}$, the capacitance is $10\mu\text{F}$ and the EMF amplitude is 10V . The resistance must be:

Answer: 4Ω

Solution: At the resonance curve peak $Z = R$ and the current amplitude is $2.5\text{A} = (10\text{V})/R$ so R is 4Ω .

20. The primary of an ideal transformer has 100 turns and the secondary has 600 turns. Then:

Answer: the primary current is six times the secondary current

Solution: For a transformer $V_p/V_s = N_p/N_s = 1/6$. For an ideal transformer the power in is equal to the power out: $I_p V_p = I_s V_s$ so that $I_p/I_s = 6$.