

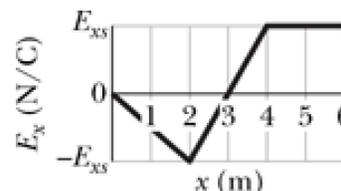
Final Solution

1. An infinitely long cylindrical metal pipe has an electric charge density of $\sigma = 8 \times 10^{-5} \text{ C/m}^2$ spread uniformly along its surface. If the radius of the cylinder is $R = 1 \text{ cm}$, what is the magnitude of the electric field at a radius of $r = 5 \text{ cm}$?

Answer: $1.8 \times 10^6 \text{ N/C}$

Solution: Apply Gauss's Law for a cylinder of radius r and length L : $\oint \vec{E} \cdot \hat{n} dA = 2\pi r L E = q_{enc}/\epsilon_o = (2\pi R L)\sigma/\epsilon_o$, where $2\pi R L$ is the area of the surface of the cylinder which is perpendicular to the electric field. Solving for E we obtain $E = (\sigma R)/(\epsilon_o r)$.

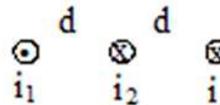
2. The graph shows the x component of the electric field as a function of x (the y and z components of the field are zero). The scale of the vertical axis is set by $E_{xs} = 10 \text{ N/C}$. If the electric potential at $x = 0$ is 20V , what is the electric potential at $x = 3\text{m}$?



Answer: 35V

Solution: For $x = 3 \text{ m}$ $V(x) = 20\text{V} - \int_0^3 E_x dx$, where the integral is the area under the curve between $x = 0$ and $x = 3$. Since the curve in that region is just a triangle the area is one half the base times the height. The area is negative because the curve is negative in this regime. $V(x) = 20\text{V} - (-15\text{V}) = 35\text{V}$.

3. What is the magnitude of the net force per unit length acting on wire 2 in the shown diagram due to the currents in the other wires, if $i_1 = 2 \text{ A}$ out of the page, $i_2 = 4 \text{ A}$ into the page, $i_3 = 1 \text{ A}$ into the page, and $d = 0.05 \text{ m}$?

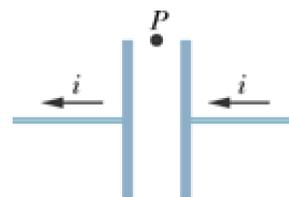


Answer: $4.8 \times 10^{-5} \text{ N/m}$

Solution: Wires 1 and 2 repel each other, while wires 2 and 3 are attracted to each other. Consequently, both forces are to the right.

$$\frac{F}{l} = \frac{\mu_o i_1 i_2}{2\pi d} + \frac{\mu_o i_2 i_3}{2\pi d}$$

4. The figure shows a parallel plate capacitor and the current direction in the connecting wires. The plates are circular with a radius of $R = 1 \text{ cm}$, and the current going into the plates is $i = 4 \text{ A}$. What is the magnetic field (magnitude and direction) between the plates at the shown position P at a distance of $r = 0.8 \text{ cm}$ above the axis defined by the wires?

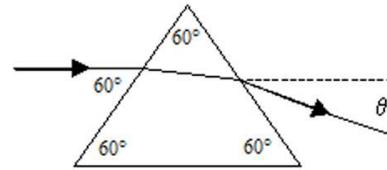


Answer: $64 \mu\text{T}$, into the page

Solution: Between the plates there is only a displacement current so $\oint \vec{B} \cdot d\vec{s} = \mu_o i_d$, where i_d is the displacement current enclosed in the path of integration. The net displacement current following between the plates is 4A . For a path with $r < R$ the fraction of the displacement current enclosed in the path is $\pi r^2/(\pi R^2)$.

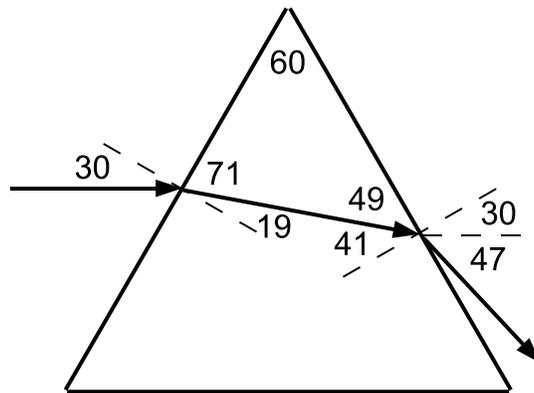
$$2\pi r B = \mu_o (4\text{A}) \frac{r^2}{R^2} \rightarrow B = \frac{\mu_o (4\text{A}) r}{2\pi R^2}$$

5. A light ray traveling in the horizontal direction is incident onto a prism as shown in the figure. At what angle relative to horizontal does the light ray emerge from the second face of the shown prism if the prism has an index of refraction of 1.5 and is surrounded by air? The cross section of the prism is in the shape of an equilateral triangle.



Answer: 47°

Solution: For the equation $n_1 \sin \theta_1 = n_2 \sin \theta_2$, all angles are measured relative to the normal. The angle of incidence for the first interface is 30° . Consequently, the angle of transmission for the first interface is $\theta_1 = 19.47^\circ$, where $1 \sin 30 = 1.5 \sin \theta_1$. Using the sum of the angles in a triangle is 180° and right angles are 90° , the angle of incidence at the second interface is $\theta_2 = 40.53^\circ$ (see figure below). Consequently, the angle of transmission at the second interface is 77° relative to the normal. However, the normal is 30° above the horizontal so the angle of transmission relative to the horizontal is $77 - 30 = 47^\circ$.



6. Light of wavelength 450 nm in air is incident perpendicularly to a soap film ($n = 1.33$) suspended in air. What is the least thickness of the film for which the intensity of the reflection is at a maximum?

Answer: 85 nm

Solution: The condition for constructive interference here is $2t + (\lambda_n/2) = m\lambda_n$. The $(\lambda_n/2)$ factor is due to the extra phase shift when reflecting off of a higher index of refraction. Reflecting off of the top of the film ($n = 1$ to $n = 1.33$) has such an extra phase, while reflecting off of the bottom of the film ($n = 1.33$ to $n = 1$) does not. Solving for t yields $t = (m - \frac{1}{2})\lambda_n/2$. The smallest positive value for t is $\lambda_n/4$.

7. Light of wavelength 600 nm in air is incident perpendicularly to a pair of slits separated by a distance d . If the interference maxima are observed on a screen behind the slits only up to 4th order (but not 5th), what are the possible values for d ?

Answer: $2400 < d < 3000$ nm

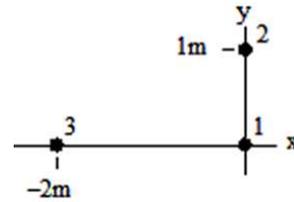
Solution: The condition for constructive interference from a two slit experiment, $d \sin \theta = m\lambda$ can be written as $\sin \theta = m\lambda/d$. Since $\sin \theta$ can be at most one, if the 4th order maxima is seen $4\lambda/d < 1$. On the other hand if the 5th order maxima is not seen, then $5\lambda/d > 1$. Putting these two conditions together: $4\lambda < d < 5\lambda$.

8. A reflection telescope has a mirror diameter of 10 cm. What is the minimum angular separation of a pair of stars that can still be resolved into two objects assuming that the starlight has a wavelength of 500 nm? Neglect atmospheric distortion.

Answer: 6×10^{-6} rad

Solution: The minimum angular separation satisfies $\sin \theta = 1.22\lambda/d$, where here $d = 0.1$ m and $\lambda = 5 \times 10^{-7}$ m.

9. In the figure each of the three particles has a charge of $+10.0\mu\text{C}$. The net vector electrostatic force on particle 1 at the origin due to the other two charges is (magnitude and angle with respect to the $+x$ axis),



Answer: 0.93N , 284°

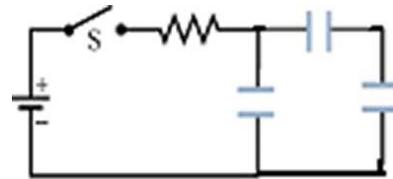
Solution: Using Coulomb's Law the x and y components of the force are $F_x = kq^2/(2m)^2$ and $F_y = -kq^2/(1m)^2$. The magnitude of the force is given by $\sqrt{F_x^2 + F_y^2}$, and the angle with respect to the positive x -axis is given by $\tan^{-1}(F_y/F_x)$. As a check we note that because $F_y < 0$ and $F_x > 0$, the angle should be in the 4th quadrant.

10. A charged particle of mass 1.89×10^{-28} kg and charge $-1.60 \times 10^{-19}\text{C}$ experiences a uniform acceleration of $6.00 \times 10^8\text{m/s}^2$ in the $+\hat{i}$ direction due to a constant electric field. The electric field is,

Answer: $-0.71\hat{i}$ N/C

Solution: Because $\vec{F} = q\vec{E} = m\vec{a}$, the electric field is $\vec{E} = m\vec{a}/q$.

11. For the circuit shown in the figure the power supply is 12V , the capacitors are each $10\mu\text{F}$ and the resistor is $80\text{k}\Omega$. How long after the switch is closed does it take for the total positive charge on the capacitors to reach half its final value?



Answer: 830ms

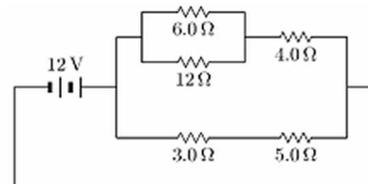
Solution: The effective capacitance of the three capacitors in this circuit is $15\mu\text{F}$. Consequently, the RC time constant is $\tau = RC = 1.2\text{s}$. The charge on the capacitor reaches half its final value when $0.5 = e^{-t/\tau}$ or $2 = e^{t/\tau}$. Taking the natural logarithm of both sides of this equation yields $\ln(2) = t/\tau$.

12. Two conductors are solid cylindrical wires made of the same material and have the same length. Conductor A has radius r . Conductor B has radius $r/3$. The ratio of their resistance, R_A/R_B , is:

Answer: $1/9$

Solution: The resistance of a wire is inversely proportional to its area. Since the ratio of the wire radii is $r_A/r_B = 3$, the ratio of the resistance is $R_A/R_B = (r_A/r_B)^{-2} = 1/9$.

13. For the circuit in the figure the current through the 6Ω resistor is:



Answer: 1A

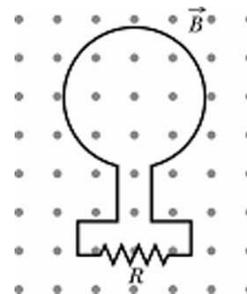
Solution: The effective resistance of the 6Ω , 12Ω , and 4Ω resistors is 8Ω . The current in the top part of the circuit (through the 4Ω resistor) is $12\text{V}/8\Omega = 1.5\text{A}$. The effective resistance of the 6Ω and 12Ω resistors is 4Ω so the voltage across these two parallel resistors is $(1.5\text{A})(4\Omega) = 6\text{V}$. The current in the 6Ω resistor is $6\text{V}/6\Omega = 1\text{A}$.

14. A particle of mass $1.89 \times 10^{-28} \text{kg}$ and charge $-1.60 \times 10^{-19} \text{C}$ moves with velocity $\vec{v} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \text{m/s}$ when it enters a region of uniform magnetic field $\vec{B} = 2T\hat{k}$. The force experienced by the particle at the moment it enters this region is,

Answer: $\vec{F} = 6.4 \times 10^{-19}(-2\hat{i} + \hat{j}) \text{N}$

Solution: The force is $\vec{F} = q\vec{v} \times \vec{B}$. Take the cross product with the given velocity and magnetic field vectors, and multiply by the negative charge.

15. In the figure the magnetic flux through the loop is increasing according to the relation $\Phi_B = 0.060 + 0.070t$, where Φ_B is in Webers and t is in seconds. As indicated in the figure the magnetic field is going out of the page. The resistance is 4.00Ω . What current results in the loop and does it flow toward the left or toward the right through the resistor?



Answer: 17.5 mA, left

Solution: The magnitude of the induced emf is $d\Phi_B/dt = 0.07 \text{V}$. The induced current is $i = V/R = 0.07 \text{V}/4\Omega$. Because the flux coming out of the page is increasing, the induced current acts to oppose the change and is in the clockwise direction. This means that it is going to the left at the resistor.

16. A car radio uses an LC circuit and a variable capacitor to tune to different radio stations. The value of the capacitance to tune to a radio station of 1000kHz is C . What must its value be to tune to a station at 200kHz ?

Answer: 25C (2) 20C (3) 15C (4) 10C (5) 5C

Solution: Because $\omega = 2\pi f = 1/\sqrt{LC}$ for an LC circuit, the frequency is proportional to $1/\sqrt{C}$. The initial capacitance is C , and let the final capacitance be C_f . Then $5 = 1000 \text{kHz}/200 \text{kHz} = (C/C_f)^{-1/2}$, and squaring both sides of this equation yields $25 = C_f/C$.

17. Two identical thin lenses of $+6 \text{ cm}$ focal lengths are separated by 20 cm . An object lies 11 cm from the first lens. The magnification of the 2 lens system is:

Answer: 9

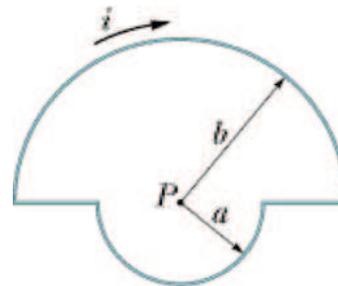
Solution: Using $p_1 = 11 \text{ cm}$ and $f_1 = 6 \text{ cm}$, the image for the first lens is at $i_1 = 13.2 \text{ cm}$. This means that the object for the second lens is at $p_2 = 20 \text{ cm} - i_1 = 6.8 \text{ cm}$. With $p_2 = 6.8 \text{ cm}$ and $f = 6 \text{ cm}$, the final image is at $i_2 = 51 \text{ cm}$. The overall magnification of the two lens system is $(-i_1/p_1)(-i_2/p_2)$.

18. A diffraction grating has $10,000$ evenly spaced rulings (slits) per cm . Assuming normal incidence to the grating, at what angle from normal does the first order line show up on a screen behind the grating for light with a wavelength of 500 nm ?

Answer: 30°

Solution: The separation between the slits is $1 \text{cm}/10000 = 10^{-6} \text{m}$. The first order line shows up at $\sin \theta = m\lambda/d$ with $m = 1$.

19. In the shown figure, a current $i = 50$ mA is set up clockwise in a loop having two radial lengths and two semicircles of radii $a = 5$ cm and $b = 10$ cm with a common center P . What is the magnitude of the magnetic field at point P ?



Answer: 4.7×10^{-7} T

Solution: The two straight segments of the wire do not contribute to the magnetic field at point P . The two semicircular segments both produce fields going into the page at P . Thus, the magnitude of the magnetic field at P is

$$B = \frac{1}{2} \left(\frac{\mu_o i}{2a} + \frac{\mu_o i}{2b} \right).$$

20. An object is placed 6 cm in front of a concave mirror of radius 10 cm. Is the resulting image real or virtual? Is the resulting image inverted or noninverted?

Answer: real, inverted

Solution: A concave mirror with radius 10 cm has a focal length of 5 cm. The object distance is $p = 6$ cm, and consequently the image distance is $i = 30$ cm. Because $i > 0$, the image is real. Because $m = -i/p < 0$, the image is inverted.