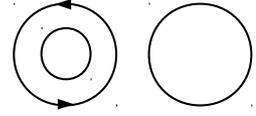


Exam 3 Solution

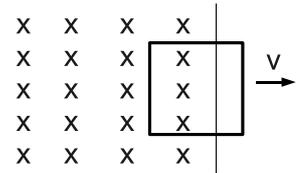
1. The current i increases in the loop labeled in the figure. What is the direction of the induced current in the smaller inner loop and the loop at the right? (List the direction of the induced current for the smaller loop first.)



Answer: clockwise, counterclockwise

Solution: The magnetic field due to the current i is going out of the page inside the loop and into the page outside the loop. Since the current is increasing, the flux going out of the page increases for the small loop, and the flux into the page increases for the right loop. To oppose this (Lenz's law) the induced current for the small loop is clockwise and for the right loop is counterclockwise.

2. A square loop of side 2 cm moves with velocity 3 m/s at $t = 0$ as it exits a region of uniform magnetic field 5 T into the page (see figure). If the loop has resistance 0.4 Ω , what is the magnitude of the magnetic force on it at $t = 0$?



Answer: 7.5×10^{-2} N

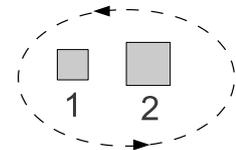
Solution: In this problem the flux through the square loop is decreasing because the area inside the loop where there is a magnetic field is decreasing. The rate of change of the area is $dA/dt = (2\text{cm})(3\text{m/s}) = 0.06\text{m}^2/\text{s}$, and the rate of change in flux is $d\Phi_B/dt = BdA/dt = 0.3\text{V}$. Consequently, the induced current is $0.3\text{V}/0.4\Omega = 0.75\text{A}$. The induced current is clockwise to oppose the change in flux. In particular there is a current going upwards on the left side of the loop. Using $\vec{F} = i\vec{L} \times \vec{B}$, the force on that segment of the loop is $(0.75\text{A})(0.02\text{m})(5\text{T}) = 0.075\text{N}$ to the left. The other segments create forces which cancel, and hence this is also the net force on the loop.

3. A generator consists of a circular coil of wire with 500 turns and radius 2 cm. It rotates in a uniform magnetic field of magnitude 3 T with frequency $f = 60$ Hz. What is the voltage or emf amplitude produced by the generator?

Answer: 711 V

Solution: In this problem the angle between the coil and the magnetic field is changing: $\Phi_B(t) = 500(3\text{T})(\pi(0.02\text{m})^2) \cos(\omega t)$, where $\omega = 2\pi f$ is the angular frequency. Taking the derivative of the flux and using Faraday's law the induced emf is $-d\Phi_B(t)/dt = \omega(1.88\text{Wb}) \sin(\omega t)$. The amplitude is $\omega(1.88\text{Wb}) = 710\text{V}$.

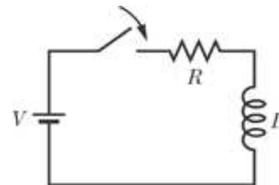
4. The magnetic field is 1 T into the page and 2 T out of the page in regions 1 and 2 respectively (see figure). The magnitude of both fields is increasing at a rate of 0.005 T/s. If region 1 has area 0.4 m^2 and region 2 has area 0.8 m^2 , what is $\oint \vec{E} \cdot d\vec{s}$ for the path of integration shown in the figure?



Answer: $-2 \times 10^{-3}\text{V}$

Solution: The flux is increasing going out of the page at a rate $(0.8\text{m}^2 - 0.4\text{m}^2)(0.005\text{T/s}) = 0.002\text{Wb/s} = 0.002\text{V}$. Thus, $-d\Phi_B/dt$ is going into the page and has magnitude 0.002 V. For the path of integration shown the positive direction for the flux is going out of the page so the integral is negative.

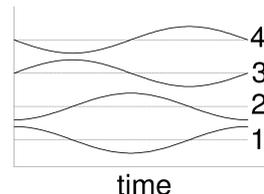
5. At $t = 0$ the switch is closed in the RL circuit at right. If $L = 4mH$, $R = 3\Omega$, and $\mathcal{E} = 6V$, what is the voltage across the inductor at time $t = 2ms$?



Answer: 1.3 V

Solution: The current is equal to $i = (\mathcal{E}/R)(1 - e^{-t/\tau_L})$ so the voltage across the inductor is $L di/dt = (L\mathcal{E}/R)e^{-t/\tau_L}/\tau_L = \mathcal{E}e^{-t/\tau_L}$. Using $\tau_L = L/R$, at $t = 2ms$ the voltage across the inductor is 1.33 V.

6. If graph 2 at right is the current as a function of time in an RLC series circuit, which graph represents voltage across a capacitor?



Answer: graph 4

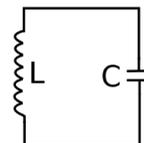
Solution: The voltage across the capacitor is q/C . The charge, q , is the integral of the current or equivalently the current is the derivative of the charge. The derivative of graph 4 is indeed graph 2. Some of you may prefer to use that graph 2 looks like $-\cos(\omega t)$. The integral of $-\cos(\omega t)$ is $-(1/\omega)\sin(\omega t)$, which is what graph 4 looks like.

7. In a series driven RLC circuit the current leads the voltage. Using the convention $V(t) = \mathcal{E}_m \sin(\omega_d t)$ and $I(t) = I_m \sin(\omega_d t - \phi)$, what is the sign of ϕ , and the relation between ω_d and the resonance frequency, $1/\sqrt{LC}$?

Answer: $\phi < 0$, $\omega_d < 1/\sqrt{LC}$

Solution: For the current to lead the voltage $-\phi > 0$ and hence $\phi < 0$. The current leads the voltage for a circuit in which capacitive reactance is larger (ICE of Eli the Ice man): $1/(\omega_d C) > \omega_d L$. Another way to see this is that $\tan \phi = (X_L - X_C)/R < 0$. Either of these equations imply that $\omega_d^2 < 1/(LC)$.

8. In an LC circuit (see figure) the amplitude of the current oscillations is 2A and the amplitude of the voltage oscillations across the capacitor is 3V. If the capacitance is $5\mu F$, what is the inductance?



Answer: 11 μH

Solution: For an LC circuit the charge on the capacitor as a function of time has the form $q = Q \cos(\omega t)$. The current as a function of time is $i = dq/dt = -\omega Q \sin(\omega t)$. The problem gives us the amplitude of the current oscillations, $2A = \omega Q$, the amplitude for the voltage across the capacitor, $3V = Q/C$, and the capacitance, $C = 5\mu F$. These can be combined to find $Q = 15\mu C$, $\omega = 1.33 \times 10^5 Hz$, and $L = 1/(\omega^2 C) = 11\mu H$.

9. A series RLC circuit is driven by a voltage of amplitude $\mathcal{E}_m = 5V$. If $L = 5H$, $C = 3\mu F$, $R = 2k\Omega$, and the angular frequency is $\omega = 1000s^{-1}$, what is the amplitude of the voltage oscillations across the capacitor?

Answer: 0.33 V

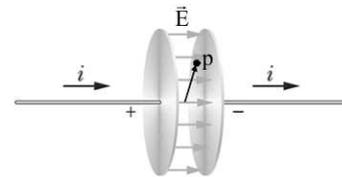
Solution: For this circuit $X_L = \omega L = 5k\Omega$, $X_C = 1/(\omega C) = 0.333k\Omega$, and $X_R = R = 2k\Omega$. The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the current amplitude is $I = \mathcal{E}/Z$. The amplitude oscillations across the capacitor are $IX_C = \mathcal{E}X_C/Z$.

10. In a transformer used to take an AC voltage source of amplitude 120 V down to a voltage amplitude of 5 V, which can be used to run an electronic device, what is the ratio of the number of turns in the primary coil to the number of turns in the secondary coil, N_p/N_s ?

Answer: 24

Solution: Because $V_p/V_s = N_p/N_s$, $N_p/N_s = 24$.

11. A current of 2 A flows into a capacitor with circular plates of radius $R = 7$ mm. What is the magnitude of the magnetic field at point P in between the plates at a radial distance $r = 4$ mm from the center of the plates?



Answer: 3.3×10^{-5} T

Solution: The magnetic field between the plates is determined by the displacement current enclosed in the path of integration: $\oint B \cdot ds = \mu_o i_{d,enc}$. The left hand side of this equation is $2\pi r B$. Because $r < R$, the enclosed displacement current is not equal to 2 A but only the fraction enclosed in a circle of radius r : $i_{d,enc} = 2A(\pi r^2)/(\pi R^2)$. The magnetic field is equal to $B = \mu_o i_{d,enc}/(2\pi r)$.

12. For the same geometry as in the previous problem, what is the magnitude of the magnetic field at a radius $r = 10$ mm from the center of the plates?

Answer: 4.0×10^{-5} T

Solution: The magnetic field is again equal to $B = \mu_o i_{d,enc}/(2\pi r)$ with $r = 10$ mm; however, here $i_{d,enc}$ is equal to the full displacement current, 2 A, because $r > R$.

13. A common refrigerator magnet is an example of a

Answer: ferromagnet

Solution: A refrigerator magnet is an example of a permanent magnet which does not need an external magnetic field to develop a magnetic dipole. These are called ferromagnets.

14. Which of the Maxwell equations explains why a voltage is created when a loop of wire turns in a magnetic field?

Answer: $\oint E \cdot ds = -d\Phi_B/dt$

Solution: This Maxwell equation is Faraday's law, which is the physical basis behind electrical generators.

15. A cube has six faces, which we label as 1,2,...,6. If the flux through faces $i = 1, \dots, 5$ is $(i - 2)$ in Webers, what is the flux through face 6?

Answer: -5 Wb

Solution: Because $\oint B \cdot dA = 0$, the net magnetic flux through any surface is zero. The flux through faces 1,2,...,5 is $-1 + 0 + 1 + 2 + 3 = 5$ Wb so the remaining face must have flux -5 Wb.

16. The electric field in a plane electromagnetic wave is $E_z = E_m \sin(ky + \omega t)$. An electric field of 3.0 kV/m in the (-z) direction is measured at some point and time along the travel path of the wave. What is the magnetic field at the same point and time?

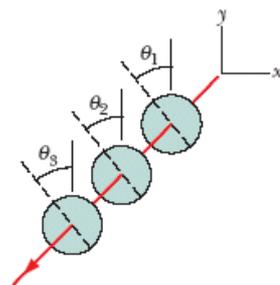
Answer: $1.0 \times 10^{-5} T \hat{i}$

Solution: The ratio of the electric to magnetic field is $E/B = c$ so if $E = 3000 V/m$, $B = 1 \times 10^{-5} T$. The direction the wave is traveling is in the $-\hat{j}$ direction. Since $\vec{E} \times \vec{B}$ is in the same direction and the electric field is in the $-\hat{k}$ direction, the magnetic field must be in the x-direction: $-\hat{k} \times \hat{i} = -\hat{j}$.

17. Unpolarized light of intensity I_o is sent through three polarizers. If the polarization axes of the polarizers is $\theta_1 = 0^\circ$, $\theta_2 = 30^\circ$, and $\theta_3 = -30^\circ$, what is the intensity of the light emerging from all three polarizers?

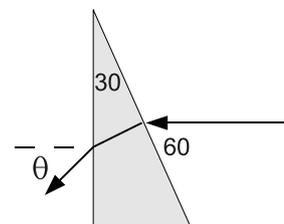
Answer: $0.094 I_o$

Solution: The intensity of the emerging light is $0.5 I_o \cos^2(\theta_1 - \theta_2) \cos^2(\theta_2 - \theta_3)$.

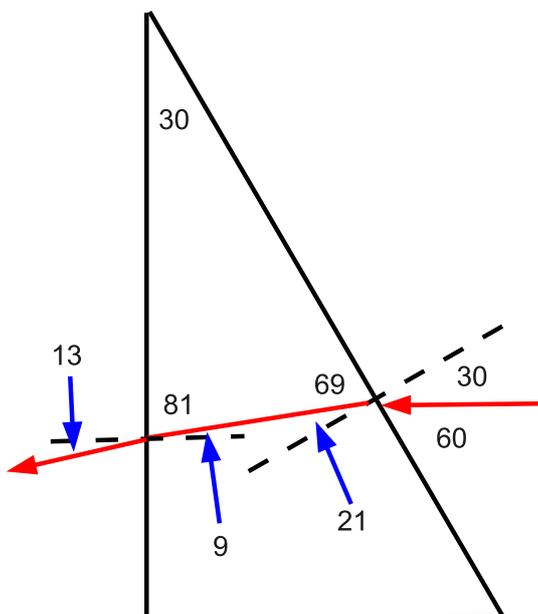


18. Light is incident on a triangular piece of glass with index of refraction 1.4. What is the angle θ in the figure at which the light emerges? Assume the index of refraction of air is 1. (The figure is not drawn to scale so do not try to read the angle directly from it.)

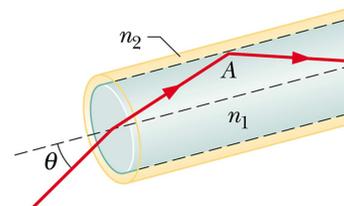
Answer: 13°



Solution: For this problem and ones like it remember that the angles for Snell's law are measured relative to the normal to the interface. At the first interface the angle relative to the normal is 30° for the incoming light. Applying Snell's law to this interface, $1 \sin(30) = 1.4 \sin(A)$, we see that the transmitted angle is $A = 20.92^\circ$. Now doing some trigonometry illustrated in the large figure below the incoming angle for the second interface is 9.08° . Applying Snell's law to the second interface, $1.4 \sin(9.08) = 1 \sin(\theta)$, yields $\theta = 12.76^\circ$.



19. An optical fiber has a plastic core ($n_1 = 1.5$) surrounded by a plastic sheath ($n_2 = 1.4$). A light ray is incident on one end of the fiber at angle θ . The ray is to undergo total internal reflection at point A, where it encounters the core-sheath boundary. What is the maximum value of θ that allows total internal reflection at A?



Answer: 33°

Solution: The critical angle for total internal reflection at A is $\sin^{-1}(1.4/1.5) = 68.96^\circ$. Using trigonometry, the angle at the air/glass interface is thus $90 - 68.96 = 21.04^\circ$. Using Snell's law, $1 \sin(\theta) = 1.5 \sin(21.04)$, the maximum value of θ that allows total internal reflection at A is 32.6° .

20. A space craft has a reflecting sail of area 5000 m^2 . Electromagnetic radiation of intensity 1200 W/m^2 is incident on the sail. What is the net force on the sail?

Answer: 0.04 N

Solution: The force on the sail is the radiation pressure times the area of the sail. For a reflecting surface the radiation pressure is $2I/c$. Thus, the net force is $(5000 \text{ m}^2)2(1200 \text{ W/m}^2)/(3 \times 10^8 \text{ m/s})$.