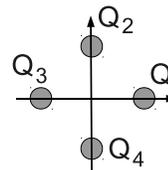


Exam 4 Solution

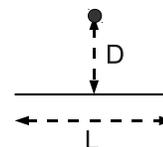
1. Four charges are arranged as shown in the figure. Each charge is 8 cm from the origin. The values of the charges are $Q_1 = +3\mu C$, $Q_2 = +1\mu C$, $Q_3 = +1\mu C$, and $Q_4 = -1\mu C$. What is the magnitude of the force on a fifth charge of $2\mu C$ placed at the origin?



Answer: 8 N

Solution: The x-component of the force is $F_x = k(3\mu C - 1\mu C)(2\mu C)/r^2$, and the y-component of the force is $F_y = k(1\mu C - -1\mu C)(2\mu C)/r^2$, where $r = 8\text{cm}$. The magnitude of the force at the origin is $\sqrt{F_x^2 + F_y^2}$.

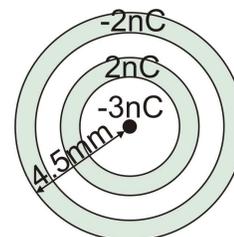
2. A wire segment of length L has constant linear charge density $\lambda > 0$. Which of the following expressions gives the magnitude of the electric field a distance D from the center of the wire (see figure)?



Answer: $\int_{-L/2}^{L/2} \frac{kD\lambda}{(x^2 + D^2)^{3/2}} dx$

Solution: This problem is identical to one from Exam 1 - see that solution.

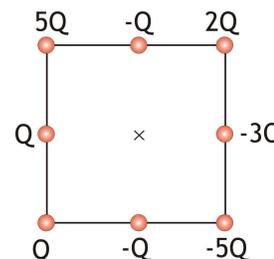
3. A point charge lies at the center of two concentric conducting shells, as shown in the figure. What is the surface charge density of the outer surface of the largest shell?



Answer: $-12\mu C/m^2$

Solution: A Gaussian sphere just larger than the outer shell encloses a net charge of -3 nC , while a sphere inside the outer shell encloses no net charge because the electric field is zero inside a conductor. This means that the net charge on the outer surface of the largest shell is -3 nC . Divide this by the area of a sphere to obtain $\sigma = (-3\text{ nC})/(4\pi r^2)$.

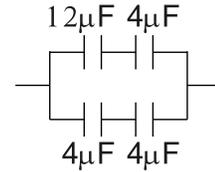
4. Charges are arranged on a square of side 3 cm as shown in the figure. If $Q = 2.3 \times 10^{-7}\text{C}$, how much work is required to move a charge of $-2Q$ from infinity to the center of the square?



Answer: 0.12 J

Solution: The potential, V , at the center of the square is the sum of kQ_i/r_i , where r_i is the distance from charge Q_i to the center. For the corners this distance is $(3\text{cm})/\sqrt{2}$, while for the charges on the sides the distance is $(3\text{cm})/2$. The work to move $-2Q$ from infinity to the center of the square is $(-2Q)V$.

5. If the voltage across the entire capacitor network at right is 5.0 V, what is the charge on the $12\mu F$ capacitor?



Answer: $15\mu C$

Solution: The voltage across the top segment is 5 V, and the effective capacitance of the top segment is $3\mu F$. Thus, the charge on each of the series capacitors of the top segment is $CV = (3\mu F)(5V)$.

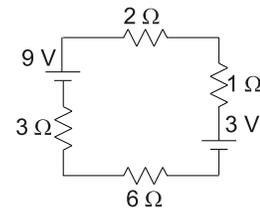
6. A copper wire has a resistivity of $1.69 \times 10^{-8}\Omega \cdot m$. The wire has a circular cross-section with a diameter of 0.255 mm, which corresponds to 30 gauge wire. How long a piece of wire would you need in order for it to have a resistance of 10Ω ?

Answer: 30 m

Solution: The resistance of the wire is $R = 10\Omega = \rho L/A = \rho L/(\pi r^2)$, where $r = 0.255/2$ mm. Solve for L .

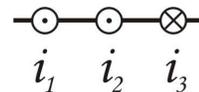
7. What is the magnitude and direction of the current in the circuit at right?

Answer: 0.5 A, clockwise



Solution: Kirchoff's loop rule for a clockwise current i implies $0 = 9V - 3V - 12\Omega i$ or $i = 0.5$ A.

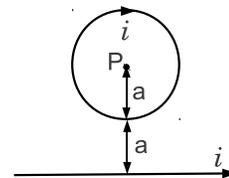
8. Three wires with (nonzero) current flowing perpendicular to the page are spaced equally on a line as shown in the figure. The first wire experiences no force due to the other two wires when:



Answer: $i_3 = 2i_2$

Solution: Wire 1 is attracted to wire 2 with force per unit length $\mu_o i_1 i_2 / (2\pi d)$ and repelled from wire 3 with force per unit length $\mu_o i_1 i_3 / (2\pi 2d)$. In order for these to be equal and opposite we must have $i_2 = i_3/2$, i.e., $i_3 = 2i_2$.

9. A current $i = 2A$ flows in a long straight wire and in a circular loop as indicated in the figure. If the distance $a = 3$ cm, what is the magnitude of the magnetic field at point P at the center of the loop?



Answer: $35\mu T$

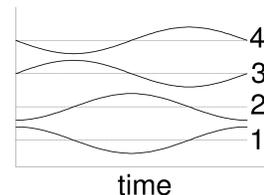
Solution: At the center of the loop the magnetic field due to the loop is $\mu_o i / (2a)$ going into the page, and the field due to the wire is $\mu_o i / (2\pi 2a)$ going out of the page. These two fields partially cancel to yield a net magnetic field of $35\mu T$ at the center of the loop.

10. A circular loop of wire in the plane of the page has radius 3 cm and resistance 0.5Ω . A uniform magnetic field coming out of the page has time dependence $B(t) = (2 - 0.2t)$, where B is measured in Tesla and t is measured in seconds. At $t = 0$ what is the magnitude and direction of the induced current in the loop?

Answer: 1 mA, counterclockwise

Solution: From Faraday's Law the induced voltage is $V = A(dB/dt)$, where $A = \pi r^2$, and the induced current is $V/0.5\Omega$. Since the magnetic field is going out of the page but decreasing, to oppose the change in flux the induced current creates a magnetic field going out of the page, i.e., the current is going counterclockwise.

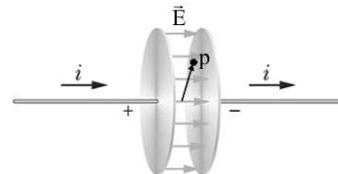
11. If graph 2 at right is the voltage across the capacitor as a function of time in an RLC series circuit, which graph represents current through the resistor?



Answer: graph 3

Solution: The charge on the capacitor is $q = CV$ so graph 2 also represents the charge. The current is $i = dq/dt$, and the derivative of graph 2 is proportional to graph 3.

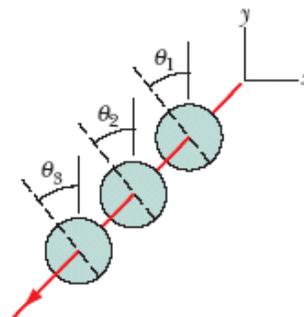
12. A current of 3 A flows into a capacitor with circular plates of radius $R = 5$ mm. What is the magnitude of the magnetic field at point P in between the plates at a radial distance $r = 4$ mm from the center of the plates?



Answer: 9.6×10^{-5} T

Solution: The magnetic field at point P in between the plates is due to the displacement current: $\oint B \cdot ds = 2\pi rB = \mu_0 i_{d,enc}$. The enclosed displacement current is not the full displacement current of 3 A, but a fraction of it: $i_{d,enc} = (3A)(\pi r^2)/(\pi R^2)$.

13. Unpolarized light of intensity I_o is sent through three polarizers. Polarizer 1 has an axis of polarization of $\theta_1 = 0^\circ$, and polarizer 3 has an axis of polarization of $\theta_3 = 30^\circ$. If no light is transmitted, i.e. the final intensity is zero, what are all the possible values of the polarization axis for the second polarizer, θ_2 ?



Answer: 90° and 120°

Solution: In order for no light to get through, polarizer 2 must be perpendicular to either polarizer 1 ($\theta_2 = 90^\circ$) or perpendicular to polarizer 3 ($\theta_2 = 120^\circ$) because the intensity of the light leaving is $0.5I_o \cos^2(\theta_1 - \theta_2) \cos^2(\theta_2 - \theta_3)$.

14. A concave mirror has a radius of curvature of $R = 2$ m. An object is placed 1.25 m in front of the mirror. Is the image real or virtual, upright or inverted, larger or smaller than the object?

Answer: real, inverted, larger

Solution: The focal length of the mirror is $f = R/2 = 1$ m. Because $p = 1.25$ m, the image distance is $i = 5$ m. The image is real because $i > 0$, and larger and inverted because $m = -i/p = -4$.

15. Two converging lenses each with focal length 6 cm are placed on the x-axis at $x = 0$ cm and $x = 12$ cm. An object is placed at $x = -30$ cm. What is the x position of the final image for light going through the two lenses?

Answer: -6 cm

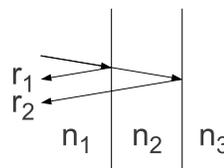
Solution: In this problem $p_1 = 30$ cm so that $i_1 = 7.5$ cm. Because the lenses are 12 cm apart this means that $p_2 = 4.5$ cm and consequently $i_2 = -18$ cm. Since the second lens is located at $x = 12$ cm, the final image is located at 12 cm - 18 cm = 6 cm.

16. Visible light of wavelength 400 nm (blue) passes through two narrow slits of separation $20 \mu\text{m}$. What is the distance between the central maximum and an adjacent maximum on a screen placed 8 meters away?

Answer: 16 cm

Solution: For small angles $\theta \approx \sin \theta \approx \tan \theta$. Because $d \sin \theta = m\lambda$ for constructive interference, the angular separation between adjacent maxima is $\Delta\theta = \lambda/d$. The distance on a screen 8 meters away is $(8m)\Delta\theta$.

17. Light is incident perpendicularly on a thin layer of material 2 that lies between materials 1 and 3. The waves of rays r_1 and r_2 interfere. (The rays are tilted only for clarity.) The indexes of refraction are $n_1 = 1.0$, $n_2 = 1.6$, and $n_3 = 1.8$, and the thickness of layer 2 is 200 nm. What is the wavelength of the incoming light (in media 1) in the visible region, $400\text{nm} \leq \lambda \leq 700\text{nm}$, for which there is completely destructive interference?



Answer: 430 nm

Solution: At both interfaces there is reflection off of a higher index material. Consequently, there are two additional factors of $\lambda_n/2$, and the condition for destructive interference is $2t = (m + 0.5)\lambda_n$. The thicknesses for which there is destructive interference is $\lambda = n\lambda_n = 2nt/(m + 0.5)$. Using $n = 1.6$ and $m = 1$ gives a wavelength of 427 nm, which is in the visible region.

18. Visible light is incident perpendicular to a grating with 400 rulings/mm. What is the longest wavelength that can be seen in the fourth-order ($n = 4$) diffraction pattern?

Answer: 625 nm

Solution: 400 rulings/mm corresponds to a d of $2.5 \mu\text{m} = 2500\text{nm}$. At fourth-order $d \sin \theta = 4\lambda$. Since $\sin \theta$ is at most 1, $4\lambda/d \leq 1$ or $\lambda \leq d/4$.

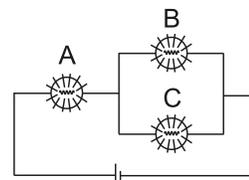
19. Estimate the linear separation of two objects that the human eye can resolve at a distance of 1 meter. Take the diameter of the pupil to be 5 mm, and the wavelength of the light to be 420 nm.

Answer: 0.1 mm

Solution: The Rayleigh criterion for being able to resolve two objects is $\theta_R = 1.22\lambda/d$, where d is the diameter of the aperture (5 mm here). Two objects separated by x at a distance of 1 m from the eye subtend an angle of $x/1\text{m}$. Using $1.22\lambda/d = x/1$ implies that the minimum separation is 0.1 mm.

20. Three identical light bulbs are placed in a circuit as shown at right. Light bulb C is removed, resulting in an open circuit where C was originally. What happens to the brightness of light bulbs A and B?

Answer: Bulb A becomes dimmer, and bulb B becomes brighter.



Solution: This problem is identical to one from Exam 2 - see that solution.