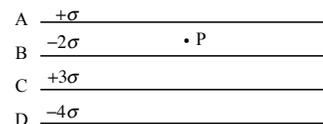


Exam 1 Solution

1. Four parallel infinite sheets of charge spaced by 4 cm between adjacent sheets are shown edgewise in the figure. Their surface charge densities are as indicated with $\sigma = 2.0\mu\text{C}/\text{m}^2$. What is the magnitude of the electric field (in N/C) at point P located midway between two sheets as shown? (Hint: Electric fields are vectors that add as such.)



Answer: 4.5×10^5

Solution: Sheet A produces an electric field of magnitude $\sigma/(2\epsilon_0)$ going away from it, which means the electric field is pointing towards the bottom of the page at point P. Sheet B produces an electric field of magnitude $2\sigma/(2\epsilon_0)$ going towards it, which implies that the electric field is pointing towards the bottom of the page at point P. Similarly, at point P the electric fields due to sheets C and D are $3\sigma/(2\epsilon_0)$ toward the top of the page and $4\sigma/(2\epsilon_0)$ toward the bottom of the page. Adding these four fields together produces a net electric field of $4\sigma/(2\epsilon_0)$ toward the bottom of the page.

2. Suppose we have an insulating spherical ball of uniform charge density ρ and radius R . At what radius or radii from the center of the sphere is the electric field strength reduced by a factor of 16 from the electric field strength at the surface?

Answer: $R/16$ and $4R$

Solution: Applying Gauss's law, the flux through a sphere of radius $r < R$ is

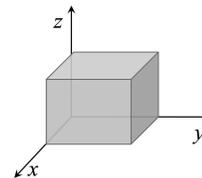
$$\Phi = (\vec{E} \cdot \hat{n})4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0} \frac{4\pi}{3} r^3 \rho \Rightarrow \vec{E} \cdot \hat{n} = \frac{1}{3\epsilon_0} r \rho.$$

The flux through a sphere of radius $R > r$ is

$$\Phi = (\vec{E} \cdot \hat{n})4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0} \frac{4\pi}{3} R^3 \rho \Rightarrow \vec{E} \cdot \hat{n} = \frac{1}{3\epsilon_0} \frac{R^3}{r^2} \rho.$$

The electric field at the surface, $r = R$, is $\vec{E} \cdot \hat{n} = \rho R/(3\epsilon_0)$. To determine where the electric field is 1/16 of the value at the surface, set $\vec{E} \cdot \hat{n}$ for the $r < R$ and $r > R$ cases equal to $(1/16)\rho R/(3\epsilon_0)$ and solve for r .

3. A non-uniform electric field given by $\vec{E} = (5.5\hat{i} - 2.1\hat{j} + (4.6z^2 - 3)\hat{k})$ N/C pierces a cube with sides 3 m, as shown in the figure. The cube has its rear corner at the origin. What is the total charge inside the cube?



Answer: +3.3 nC

Solution: By Gauss's law the net flux is equal to Q_{enc}/ϵ_0 . The flux through each face of the cube is $(\vec{E} \cdot \hat{n})(3\text{ m})^2$, where the normal vector points outwards. For the front face the normal component of the electric field is $(\vec{E} \cdot \hat{n}) = 5.5$ N/C, while for the back face it is $(\vec{E} \cdot \hat{n}) = -5.5$ N/C. These two fluxes will cancel in computing the net flux. Similarly, the flux for the right and left faces will cancel. Only the fluxes through the top and bottom faces do not cancel. The flux through the top face ($z = 3$) is $(4.6 \cdot 3^2 - 3) 3^2$, and the flux through the bottom face is $-(4.6 \cdot 0^2 - 3) 3^2$. Combine these to find the charge enclosed is $\epsilon_0 4.6 \cdot 3^4$.

4. Members of a space mission discover a mysterious alien artifact consisting of a solid conducting sphere of radius 4 cm floating inside a conducting shell with inner and outer radii 10 cm and 12 cm, respectively. Scientists measure the electric field strength at different places, obtaining $E = 4500 \text{ N/C}$ pointing inward at $r = 7 \text{ cm}$ and $E = 7600 \text{ N/C}$ pointing outward at $r = 15 \text{ cm}$. What is the total charge on the outer shell?

Answer: +21.5 nC

Solution: For a Gaussian spherical surface of radius 7 cm, the charge enclosed is that of the sphere, while for a Gaussian spherical surface of radius 14 cm, the charge enclosed includes both the sphere and the shell. Apply Gauss's law,

$$\Phi = (\vec{E} \cdot \hat{n})4\pi r^2 = -4500(4\pi 0.07^2) = \frac{Q_{\text{sphere}}}{\epsilon_0}$$

$$\Phi = (\vec{E} \cdot \hat{n})4\pi r^2 = 7600(4\pi 0.15^2) = \frac{Q_{\text{sphere}} + Q_{\text{shell}}}{\epsilon_0}.$$

Solve the first equation for Q_{sphere} , and then use that to find Q_{shell} in the second equation.

5. A long solid copper cylinder has radius 1 cm and length of 90 cm. The field at the surface of the cylinder is 9000 N/C going outwards. What is the charge on the cylinder?

Answer: 4.5 nC

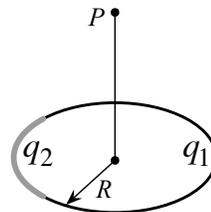
Solution: Use a cylindrical Gaussian surface on the surface of the copper rod. The electric field near the ends is nearly parallel to the surface, and the surface area of the ends is much smaller than that of the side. Thus, the flux from the ends is much smaller than that from the side, which is equal to $\Phi = 2\pi(0.01 \text{ m})(0.9 \text{ m})(9000 \text{ N/C})$. The charge enclosed is $\epsilon_0\Phi$.

6. Two isolated conducting spheres are separated by a large distance. Sphere 1 has a radius of R and an initial charge $3Q$ while sphere 2 has a radius of $3R$ and an initial charge $7Q$. A very thin copper wire is now connected to the spheres. Charge flows between the spheres until they reach the same electrical potential. How much charge will be transferred from sphere 2 to sphere 1? (Note that the charge transferred can be positive, negative or zero.)

Answer: $-Q/2$

Solution: The net amount of charge on both spheres does not change because charge is conserved. Thus, in the end $q_1 + q_2 = 10Q$. Also, in the end they have the same potential so $kq_1/R = kq_2/(3R)$, which implies that $q_2 = 3q_1$. Solve these two equations to find in the end $q_1 = 2.5Q$, and $q_2 = 7.5Q$. The charge on sphere 1 has changed by $-Q/2$, which means that $-Q/2$ flowed to sphere 1 from sphere 2.

7. A plastic rod has been bent into a circle of radius $R = 5.5 \text{ cm}$. It has a charge $q_1 = +4.2 \text{ pC}$ uniformly distributed along three-quarter of its circumference and a charge $q_2 = -4q_1$ uniformly distributed along the rest of the circumference (see figure). Assuming $V = 0$ at infinity, what is the electric potential in volts at point P, which is on the central axis of the circle at distance $D = 6.5 \text{ cm}$ from the center?



Answer: -1.33

Solution: All points on the circumference are $r = \sqrt{R^2 + D^2}$ from point P. Thus, the potential at point P is

$$V = \int \frac{k dq}{r} = \frac{k}{r} \int dq = \frac{k}{r}(q_1 + q_2).$$

8. A spherical conducting drop of water carrying a charge of 25 pC has a potential of 380 V at its surface relative to infinity. If three water droplets with the same charge and radius merge, what is the potential at the surface of the new droplet?

Answer: 790 V

Solution: The potential at the surface of a spherical conducting water droplet is kQ/r . Let Q and r be the charge and radius of one of the three initial identical water droplets. The charge on the merged droplet is $3Q$. The volume of the merged droplet is $3(4\pi/3)r^3 = (4\pi/3)r_m^3$, where r_m is the radius of the merged droplet. From this equation it follows that $r_m = 3^{1/3}r$. Consequently, the potential of the merged droplet is $k(3Q)/(3^{1/3}r) = 3^{2/3}kQ/r = 3^{2/3} 380$ V.

9. A proton located at $x = 1$ m is released along the positive x direction in a electric potential of the form $V(x) = 5 - 3/x$, where x is measured in meters and V is measured in volts. What minimum initial velocity is needed for the proton to be able to reach infinity?

Answer: 2.4×10^4 m/s

Solution: The initial potential energy of the proton is $U_i = |e|V(1) = 3.2 \times 10^{-19} J$. At infinity the potential energy of the proton is $U_f = |e|V(+\infty) = 8.0 \times 10^{-19} J$. To find the minimum initial velocity needed for the proton to be able to reach infinity set $K_f = 0$. Using conservation of energy, $U_i + K_i = U_f + K_f$, the initial kinetic energy is then $4.8 \times 10^{-19} J = (1/2)m_p v_i^2$.

10. Refer to the previous problem. What is the x component of the force acting on the proton at $x = 3$?

Answer: -5.3×10^{-20} N

Solution: The x-component of the force on the proton is $|e|E_x = -|e|dV/dx = -|e|3/x^2$. At $x = 3$ the force is -5.3×10^{-20} N.

11. Two identical conducting spheres A and B carry identical charges. They are separated by a distance much larger than their diameters. The magnitude of the electrostatic force between the spheres is F . A third, identical conducting sphere is first touched to earth ground and then touched to sphere A and moved away. The magnitude of the electrostatic force between spheres A and B is now:

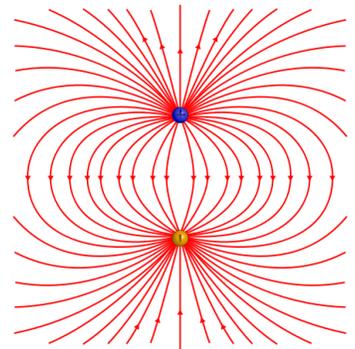
Answer: $\frac{1}{2}F$

Solution: The charge on sphere A is split between the new sphere and A. The charge on sphere thus goes down by a factor of 2, and the force between A and B goes down by a factor of 2.

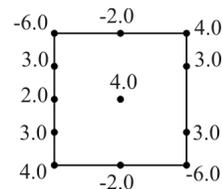
12. An electric dipole centered at the origin of a Cartesian coordinate system is oriented so that the line between the two charges lies along the z axis with the positive charge on the $+z$ side of the origin. The direction of the force experienced by a charge $+q$ that lies on the x axis at a distance far from the origin is:

Answer: $-\hat{k}$

Solution: Electric field lines go from positive to negative charges. The field lines for a dipole are shown in the figure at right, where the z -axis is towards the top of the page. Along the x -axis the field is pointing downwards (negative z -direction).



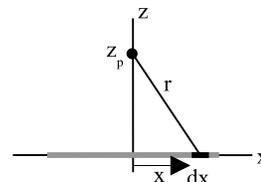
13. In the figure, the central particle of charge 4.0 nC is surrounded by the square array of charged particles (also in units of nC). The square has side length of 1 mm. The magnitude of the net force on the central particle due to the others is (Hint: if you don't use symmetry to greatly reduce the work of solving this problem come back to it last).



Answer: 0.29 N

Solution: The forces due to charges opposite one another on the square cancel because they have the same magnitude, but opposite directions. The only charge without another charge opposite it is the 2 nC charge on the left. Thus, the magnitude of the net force on the central particle is $k(2\text{ nC})(4\text{ nC})/(0.0005\text{ m})^2$.

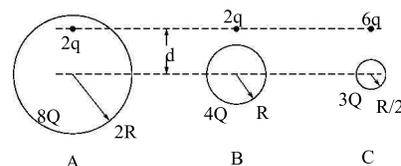
14. In the figure the gray rod has uniform linear charge density λ . The **z component** of the infinitesimal electric field at point z_p due to the infinitesimal element of charge contained in the infinitesimal line element dx can be written $dE_z =$



Answer: $k \frac{z_p \lambda dx}{(x^2 + z_p^2)^{3/2}}$

Solution: The distance from a charge segment $dq = \lambda dx$ to point p is $r = \sqrt{x^2 + z_p^2}$. A charge segment produces an electric field of magnitude kdq/r^2 at point p. To get the z-component of the electric field one must multiply by the trigonometric factor z_p/r . (As a check note that when $x = 0$, $r = z_p$ and $z_p/r = 1$.) Putting the magnitude and the trigonometric factor together, we obtain $dE_z = (kdq/r^2)(z_p/r) = k\lambda dx(z_p/r^3)$. For the choices given on the exam, it also turns out that only the above answer has the correct units of the electric field. Thus, just by checking units, one could have gotten this problem correct.

15. The figure shows 3 insulating hollow spheres that have the indicated charge (aQ) uniformly distributed on their surfaces. In each case there is a point particle of the indicated charge (bq) a distance d from the center of the hollow sphere. Rank order the electrostatic force between the sphere and the point charge for the three cases.



Answer: $C > B > A$

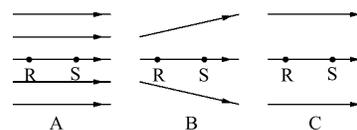
Solution: In case A the force is zero because the charge is inside the hollow sphere. For cases B and C the forces are $k(4Q)(2q)/d^2$ and $k(6q)(3Q)/d^2$, respectively.

16. An electron traveling along the x axis (toward increasing x) has a speed of 4.0×10^6 m/s. At time $t = 0$ it enters a region of space in which there is an electric field of $10.0\text{ N/C}\hat{j}$. At a time $t = 10\mu\text{s}$ later the electron's velocity makes an angle with respect to the $+x$ axis that is:

Answer: -77°

Solution: The force on the electron in the electric field is $-|e|(10\text{ N/C})\hat{j}$, and consequently its acceleration is $a = -|e|(10\text{ N/C})/m_e\hat{j}$. This problem is thus similar to a projectile motion problem from Physics 1. At a time t later the velocity in the x and y directions is $v_x = 4 \times 10^6$ m/s and $v_y = 0 - |a|t$. The angle the velocity vector makes with the x -axis is $\tan^{-1}(v_y/v_x)$.

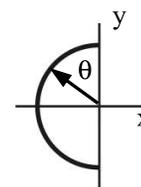
17. The figure shows the electric field lines in three different regions of space. The points labeled R and S are the same distance apart in each region. A proton is let go from rest at point R in each region. Rank order the proton momentum on reaching S for the three cases.



Answer: $A > B > C$

Solution: When the electric field lines are closer together, the electric field is larger. Thus, the electric field in case A is larger than that in case C. In case B the electric field is not going uniformly to the right as in cases A and C. However, between R and S the field is going uniformly to the right with magnitude in between that of cases A and C because the separation of field lines is intermediate between cases A and C. The final momentum of the proton is proportional to the force on the proton, which in turn is proportional to the electric field between R and S. From the above argument the electric fields in the three cases satisfy $E_A > E_B > E_C$.

18. A semi-circular, thin, insulating rod with a total charge of $+80.0 \text{ nC}$ uniformly distributed along its length, has a radius of 10 cm . If we needed to express an infinitesimal element of charge on the rod, dq , in terms of the infinitesimal angular variable $d\theta$ measured in radians, such that $dq = u d\theta$, (as we would if we wanted to determine the electric field at the origin, see figure) then for this case u is:



Answer: 25 nC/radian

Solution: The net charge on the rod is 80 nC , and the angle subtended in radians is π (half of a circle). Consequently, the constant $u = dq/d\theta$ is equal to $80 \text{ nC}/\pi$.

19. Two point particles, with charges of q_1 and q_2 , are placed a distance r apart. The electric field is zero at a point P between the particles on the line segment connecting them. We conclude that:

Answer: q_1 and q_2 must have the same sign but may have different magnitudes.

Solution: In order for the electric fields to cancel the fields due to q_1 and q_2 must be in opposite directions. Since point P is in between the charges, q_1 and q_2 must have the same sign in order for the fields to be in opposite directions. It is not necessary for the charges to be of the same magnitude because point P could be closer to one of the charges than the other.

20. A line of charge extending along the x axis has a continuous charge density given by $\lambda = (5x) \text{ nC/m}$. How much charge is contained on the line from $x = 2$ to $x = 4$ meters?

Answer: 30 nC

Solution: The charge between 2 and 4 meters is

$$q = \int dq = \int_2^4 \lambda dx = \frac{5}{2} x^2 \Big|_2^4 \text{ nC}.$$