

Exam 3 Solution

1. An AC power supply drives a 270 mH inductor. The power supply EMF is $\mathcal{E} = \mathcal{E}_m \sin \omega_d t$, where $\mathcal{E}_m = 12V$ and $\omega_d = 120$ rad/s. When the instantaneous current in the circuit is a maximum, the instantaneous emf of the power supply is

Answer: 0 V

Solution: The current through and voltage across an inductor are 90° out of phase. Thus, when the current is a maximum, the voltage is zero.

2. Oxygen:

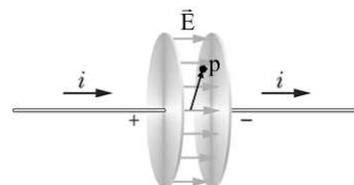
- (A) has an intrinsic magnetic dipole moment,
- (B) is paramagnetic,
- (C) is attracted to the high field point of a non-uniform magnetic field,
- (D) is diamagnetic.

Of these statements *only* the following are true,

Answer: A, B & C

Solution: As shown in the demonstration performed in class, oxygen is attracted to the high field point because it has an intrinsic dipole moment and is paramagnetic.

3. In the figure the instantaneous current charging the capacitor is $i = 1.2$ A. The circular capacitor plates have a radius of 20 cm. The magnitude of the magnetic field at the point labeled p that lies between the plates, 15 cm from the centerline between the plates is



Answer: 9.0×10^{-7} T

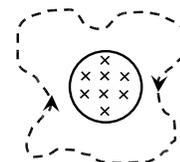
Solution: For this problem use $\oint \vec{B} \cdot d\vec{s} = 2\pi r B = \mu_o i_d$, where i_d is the displacement current through a circle of radius $r = 0.15m$. This is not the total displacement current between the disks, 1.2 A, because the circle of radius $r = 0.15m$ does not enclose all of the electric flux. It encloses a fraction of the electric flux: $(\pi 0.15^2)/(\pi 0.2^2) = (0.75)^2$. Solve for B using $2\pi(0.15m)B = \mu_o(0.75)^2(1.2A)$.

4. The magnetic flux *entering* the circular bottom face of a right circular cylinder is $23.4 \text{ T} \cdot \text{m}^2$ and the magnetic flux *exiting* the sidewall of the cylinder is $63.9 \text{ T} \cdot \text{m}^2$. The magnetic flux on the top face must be

Answer: $40.5 \text{ T} \cdot \text{m}^2$, entering

Solution: The net magnetic flux through the cylinder must be zero. The flux from two of the surfaces is $63.9 - 23.4 = 40.5 \text{ T} \cdot \text{m}^2$ exiting the cylinder. Thus, the remaining surface must have $40.5 \text{ T} \cdot \text{m}^2$ entering the cylinder.

5. The circular region of radius $r = 50$ cm shown in the figure contains a uniform electric field pointing into the page that is increasing according to $E(t) = 2.4 \times 10^5 t$ (in units of V/m), where t is in seconds. For the path indicated by the dotted line what is the magnitude of $\oint \vec{B} \cdot d\vec{s}$, in units of $\text{T} \cdot \text{m}$?



Answer: 2.1×10^{-12}

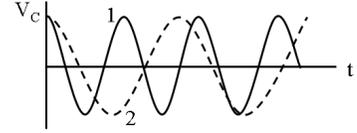
Solution: The integral $\oint \vec{B} \cdot \vec{s}$ is equal to $\mu_o \epsilon_o (d\Phi_E/dt)$, and the rate of change of the flux is $A(dE/dt) = \pi(0.5m)^2(2.4 \times 10^5 \text{ V/m})$.

6. An LC oscillator oscillates at a frequency of 450 Hz. The capacitor has capacitance of $3.00 \mu\text{F}$. The inductor must have inductance of:

Answer: 41.7 mH

Solution: For an LC oscillator the angular frequency is equal to $\omega = 2\pi f = 1/\sqrt{LC}$. Use $f = 450 \text{ Hz}$ and $C = 3.00\mu\text{F}$ to solve for L .

7. Two LC oscillator circuits (1 & 2) have the same capacitance ($C_1 = C_2$) but different inductances ($L_1 \neq L_2$). The plot shows the voltage across the capacitor as a function of time for the respective circuits (dashed curve is for circuit 2). This implies that



Answer: $L_2 > L_1$

Solution: Again for an LC circuit $\omega = 1/\sqrt{LC}$. From the figure oscillator 1 has a greater frequency, $\omega_1 > \omega_2$, which implies that $\sqrt{L_1 C_1} < \sqrt{L_2 C_2}$; however, because $C_1 = C_2$ we must have $L_1 < L_2$.

8. In a sinusoidally driven series RLC circuit the current leads the applied emf. The power dissipated in the resistor can be increased by:

Answer: increasing the inductance (making no other changes)

Solution: If the current leads the applied emf, then $\phi < 0$. This means that $X_L < X_C$. Now the average power dissipated in the resistor is $I^2 R/2 = \mathcal{E}_m^2 R/(2Z^2)$. To increase the power dissipated we need to decrease Z by bringing $X_L = \omega L < X_C = 1/(\omega C)$ closer together. Increasing L does that.

9. For a driven series RLC circuit, as the driving frequency is increased the current amplitude peaks at 3.5A. The inductance is $200 \mu\text{H}$, the capacitance is $12 \mu\text{F}$ and the emf amplitude is 12V. The resistance must be:

Answer: 3.4Ω

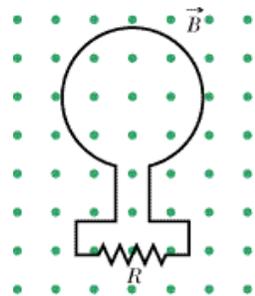
Solution: For a driven RLC circuit, the peak current occurs when $X_L = X_C$. At that point the impedance, Z , is equal to R so the maximum current is $\mathcal{E}_m/R = 12\text{V}/R = 3.5\text{A}$.

10. The primary of an ideal transformer has 100 turns and the secondary has 600 turns. Then:

Answer: the primary current is six times the secondary current

Solution: The ratio of the voltages for the primary and secondary coils is $V_p/V_s = 100/600 = 1/6$. An ideal transformer transfers all the power from the primary coil to the secondary coil. Consequently, $I_p V_p = I_s V_s$, which implies that $I_p/I_s = V_s/V_p = 6$.

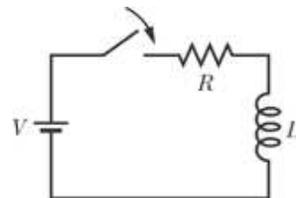
11. In the figure the magnetic flux through the loop increases according to the relation $\Phi_B = 3 + 6t + 3t^2 + t^3$, where Φ_B is in milliwebers and t is in seconds. What is the magnitude of the emf (in millivolts) induced in the loop when $t = 2$ s and the direction of the induced current? (cw = clockwise, ccw = counterclockwise)



Answer: 30, cw

Solution: The magnitude of the emf is equal to $d\Phi_B/dt = 6 + 6t + 3t^2 = 30mV$ at $t = 2$ seconds. From Lenz's law, the induced current opposes the change of flux. Since the flux is increasing out of the page, the induced current produces a flux into the page, which corresponds to a clockwise current.

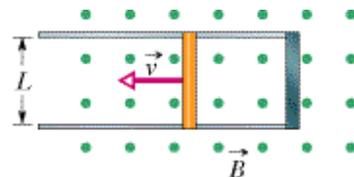
12. In the figure $R = 20 \Omega$, $L = 8H$, and the ideal battery has a voltage of $V = 18$ volts. Right after the switch is closed, what is the rate of change of the current?



Answer: 2.25 A/s

Solution: Right after the switch is close $i = 0$, but di/dt is not zero. All the voltage is dropped across the inductor: $Ldi/dt = V$ or equivalently $di/dt = V/L = 2.25A/s$.

13. In the figure a metal rod is forced to move with constant velocity 80 cm/s along two parallel metal rails connected with a strip of metal at one end. A magnetic field of magnitude $B = 2$ T points out of the page. The rails are separated by $L = 20$ cm, and the rod has resistance 10Ω . What is magnitude and direction of the current in the rod? (cw = clockwise, ccw = counterclockwise)



Answer: 32 mA, cw

Solution: The flux is changing in this circuit because the area of the rectangular loop containing the rod is increasing: $dA/dt = Lv$, where v is the velocity of the rod. The induced emf is $B(dA/dt) = BLv$, and the induced current is BLv/R , where R is the resistance of the rod. Because the flux is increasing out of the page, the induced current produces a current going into the page, which corresponds to a clockwise induced current.

14. A solenoid of length 75 cm and radius 10 cm is wound with 300 turns of wire. If the total energy stored in the solenoid is 400 J, what is the current in the wire?

Answer: 410 A

Solution: The inductance of a solenoid is $L = \mu_0 n^2 Al$, where n is the number of turns per unit length. For the parameters given $L = \mu_0 (300/0.75)^2 (\pi 0.1^2) (0.75) = 4.74mH$. The energy stored in the solenoid is $U = Li^2/2$, which implies that the current is $i = \sqrt{2U/L}$.

15. A battery is hooked up to a series arrangement of a resistor and an inductor. After how many time constants, $\tau_L = L/R$, will the energy stored (not the current) in the inductor be 55% of its final value?

Answer: 1.35

Solution: Because the energy stored in an inductor is proportional to i^2 , for the energy to reach 0.55 of its final value, the current must reach $\sqrt{0.55} = 0.74$ of its final value. For an LR circuit the current is proportional to $(1 - e^{-t/\tau}) = 0.74$. Thus, $e^{-t/\tau} = 0.26$, $e^{t/\tau} = 1/0.26$, and the time is $t = \ln(1/0.26)\tau = 1.35\tau$.

16. A blue-giant star has a luminosity (total power emission) of 2.5×10^{29} W. What is B_{rms} at a distance equal to the earth's distance from the sun (1.5×10^8 km)?

Answer: $61 \mu\text{T}$

Solution: The intensity due to the blue-giant star at a distance $r = 1.5 \times 10^{11}$ m is equal to $I = P/(4\pi r^2)$. Using $E = cB$ and $I = E_{rms}^2/(c\mu_o)$, this intensity is also equal to $I = B_{rms}^2 c/\mu_o$. Set the two intensities equal and solve for B_{rms} .

17. Refer to the previous problem. A black, spherical dust particle of radius $150 \mu\text{m}$ experiences a gravitational force that is 5 times larger than the radiation force. What luminosity is required for the forces to be exactly balanced?

Answer: 1.25×10^{30} W

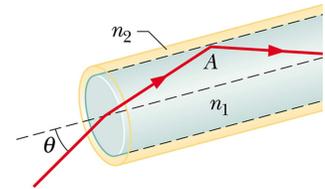
Solution: In order to have the forces balance the radiation force must increase by a factor of 5. Since the radiation force is proportional to the intensity, this means that the luminosity must increase by a factor of five: $5 \times 2.5 \times 10^{29} \text{ W} = 1.25 \times 10^{30} \text{ W}$.

18. A beam of light polarized along the y axis and moving along the $+z$ axis passes through two polarized sheets with axes of polarization oriented 20° and 70° relative to the y axis. The final intensity of the beam is measured to be 71 W/m^2 . What is the initial beam intensity?

Answer: 195 W/m^2

Solution: The angle in Malus's law is the angle between the polarization of the incoming light and the axis of the polarizer. Let the initial intensity be I_o . After passing through the first polarizer, the intensity goes down to $I_o \cos^2(20)$. After passing through the second polarizer the intensity goes to $I_o \cos^2(20) \cos^2(50) = 71 \text{ W/m}^2$. Solve for I_o .

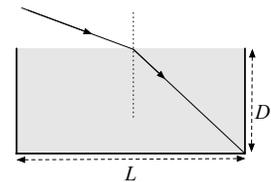
19. The figure shows a simple optical fiber with a plastic core ($n_1 = 1.59$) surrounded by a plastic sheath ($n_2 = 1.54$). A light ray in air is incident from the left with an angle θ relative to the fiber axis. What is the maximum value of θ that will ensure total internal reflection?



Answer: 23.3°

Solution: At point A the critical angle for total internal reflection is $\theta_c = \sin^{-1}(1.54/1.59) = 75.6^\circ$. This angle is measured relative to the normal to the interface. The angle in the plastic core opposite θ is $90 - 75.6 = 14.4^\circ$. Using Snell's law, $\sin \theta = 1.59 \sin(14.4)$, implies that $\theta = 23.3^\circ$.

20. The rectangular water tank in the figure has width $L = 5.00 \text{ m}$. A ray of light striking the midpoint of the tank at an angle 62.0° from the normal refracts and hits the lower corner of the tank. What is the depth D of the tank? $n = 1.333$ for water.



Answer: 2.83 m

Solution: Because the angle of incidence is 62° , the angle of transmission is $\sin^{-1}(\sin(62)/1.33) = 41.6^\circ$. The tangent of 41.6° is equal to $(L/2)/D$, or equivalently $D = (L/2)/\tan(41.6)$.