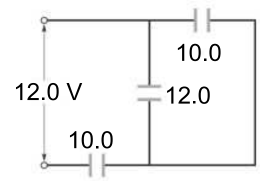
Final Exam Solution

1. A proton traveling along the x axis (toward increasing x) has a speed of 1.0×10^5 m/s. At time $t = 0$ it enters a region of space in which there is an electric field of 100.0 $N/C\ \hat{j}$. At a time $t = 14\mu$s later the proton's velocity makes an angle with respect to the $+x$ axis that is:

   Answer: $+53°$

   Solution: The force on the proton is $|e|(100\ N/C)$ in the y-direction so the acceleration is $a = |e|(100\ N/C)/m_p$ in the positive y-direction. The x-component of the velocity is constant at $v_x = 1.0 \times 10^5$ m/s, and the y-component of the velocity is $v_y = at$. The angle the velocity makes with the x-axis is is $\tan^{-1}(v_y/v_x)$.

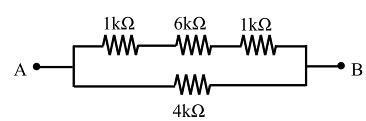
2. In the Figure 12.0 V is applied to the network of capacitors where the capacitance values are in μF. The charge on the positive plate of the upper 10μF capacitor (in μC) is:



   Answer: 37.5

   Solution: The effective capacitance of the 10 μF and 12 μF capacitors in parallel is 22 μF, and the effective capacitance of the entire network is $6.875\mu F$ with charge $(6.875\mu F)(12V) = 82.5\mu C$. Because capacitors in series have the same charge, this is also the charge on the effective 22 μF capacitor resulting from the 10 μF and 12 μF capacitors in parallel. The voltage across this effective capacitor is $Q/C = 82.5\mu C/22\mu F = 3.75V$. Since capacitors in parallel have the same voltage, this is also the voltage across the upper 10 μF capacitor. The charge on that capacitor is $CV = (10\mu F)(3.75V) = 37.5\mu C$.

3. The resistor network in the diagram has 8V across its end terminals labeled A and B. The current through, and voltage drop across, the 6kΩ resistor are, respectively:



   Answer: 1mA, 6V

   Solution: The effective resistance of the top segment is $8k\Omega$ so the current through it is $8V/8k\Omega = 1mA$. This is also the current through the $6k\Omega$ resistor, which has a voltage drop of $IR = (1mA)(6k\Omega) = 6mV$.

4. At time $t = 0$ an ideal 12.00 V power supply is connected to an uncharged series RC circuit in which the resistor is 120Ω and the capacitor is 47.0 pF. The charge accumulated on the positive plate of the capacitor at a time $t = 10RC$ is closest to,

   Answer: 564 pC

   Solution: The charge on the capacitor as a function of time is $(12V)(47 \times 10^{-12}F)(1 - e^{-t/(RC)})$. At $t = 10RC$ the exponential factor is very small, $e^{-t/(RC)} \ll 1$, i.e. the capacitor is very nearly charged to its final value of $(12V)(47 \times 10^{-12}F) = 5.64 \times 10^{-12}C$.

5. An object is reflected in a concave spherical mirror. The object distance p is less than the focal length f ($p < f$). The image can be (the correct answer contains only true statements from the following list of possibilities):

A) real, B) virtual, C) upright, D) inverted, E) larger, F) smaller

Answer: B, C, E

Solution: The equation

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

implies that the image distance is $i = pf/(p - f)$. Because $p < f$, the image distance is negative and the image is virtual (B). The magnification $m = -i/p = f/(f - p)$ is positive and larger than one, meaning that the image is upright (C) and larger (E).

6. A 2.0 cm tall object sits 30 cm from a concave spherical mirror along its central axis. The mirror has a *radius of curvature* of 25 cm. The magnification of the object is

Answer: -0.71

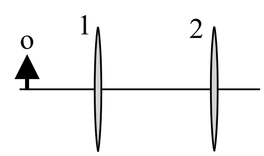
Solution: The focal length of the mirror is $f = r/2 = 12.5$ cm. Using $p = 30$ cm, this implies that the image distance is $i = 21.43$ cm. The magnification is $m = -i/p = -0.71$.

7. Parallel rays sent along the central axis of diverging lens are extrapolated to converge 15 cm from the lens. An object placed 20 cm from the lens can only appear:

Answer: On the same side as the object, 8.6 cm from the lens

Solution: This diverging lens has a focal length of $f = -15$ cm. The object distance is $p = 20$ cm. Consequently, the image distance is $i = -8.6$ cm, which means 8.6 cm on the side of the incoming light and the object.

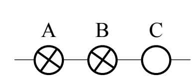
8. The two lenses in the Figure (not to scale) are identical, converging lenses of 15 cm focal lengths. A 2.0 cm tall object is 30 cm away from lens 1. Lens 2 is 50 cm from lens 1. How tall is the image of the object appearing to the right of lens 2?



Answer: 6.0 cm

Solution: Since $f_1 = 15$ cm and $p_1 = 30$ cm, the image distance for the first lens is $i_1 = 30$ cm, which is 20 cm from the second lens. Using $p_2 = 20$ cm, $f_2 = 15$ cm, we find that the image distance for the second lens is $i_2 = 60$ cm. The magnification is $m = i_1 i_2/(p_1 p_2) = 3$ so the final image is $3 \times 2 = 6$ cm tall.

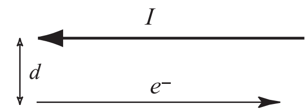
9. Three wires are equally spaced along a line (perpendicular to the page) as shown in the Figure. Wires A and B each carry current i into the page. When wire C carries no current, wire B experiences a force F. With no other currents changed, in order for wire B to experience a force $4F$ in the same direction as the original F, the current in wire C must be set equal to



Answer: $3i$, out

Solution: Wires A and B attract with force per unit length $F/l = \mu_o i^2/(2\pi r)$, where r is the separation between the wires. In order for wire B to experience a force of $4F$, wire C must repel wire B with a force of $3F$. The is means that wire C must have a current opposite to that of wire B and have three times the current.

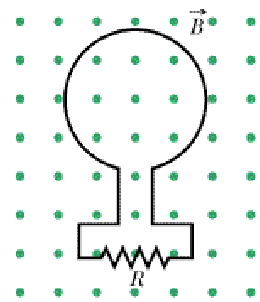
10. An electron moves at a speed of $v = 3 \times 10^7$ m/s parallel to and a distance $d = 20$ cm from a conducting wire carrying a current of $I = 40$ A. If the directions of the electron and current are as shown in the figure, what is the magnitude (in N) and direction of the force experienced by the electron?



Answer: 1.9×10^{-16} toward the wire

Solution: The magnetic field at the electron is $B = \mu_o i/(2\pi d)$ and coming out of the page. The force on the electron is $q\vec{v} \times \vec{B}$ has magnitude $|e|vB$ and is toward the wire.

11. In the figure the magnetic flux through the loop changes according to the relation $\Phi_B = 60 - 36t - 3t^2 + t^3$, where Φ_B is in milliwebers and t is in seconds. What is the magnitude of the emf (in millivolts) induced in the loop when $t = 1$ s and the direction of the induced current? (cw = clockwise, ccw = counterclockwise)



Answer: 39, ccw

Solution: The rate of change of the magnetic flux is equal to $d\Phi_B/dt = -36 - 6t + 3t^2 = -39mV$ at $t = 1$ seconds. From Lenz's law, the induced current opposes the change of flux. Since the flux is decreasing out of the page, the induced current produces a flux out of the page, which corresponds to a counterclockwise current.

12. In a sinusoidally driven series RLC circuit the current lags the applied emf. The power dissipated in the resistor can be increased by:

Answer: decreasing the inductance (making no other changes)

Solution: If the current lags the applied emf, then $\phi > 0$. This means that $X_L > X_C$. Now the average power dissipated in the resistor is $I^2R/2 = \mathcal{E}_m^2 R/(2Z^2)$. To increase the power dissipated we need to decrease Z by bringing $X_L = \omega L < X_C = 1/(\omega C)$ closer together. Decreasing L does that.

13. The magnetic flux *exiting* the circular bottom face of a right circular cylinder is $23.4\ \mathrm{T \cdot m^2}$ and the magnetic flux *entering* the sidewall of the cylinder is $63.9\ \mathrm{T \cdot m^2}$. The magnetic flux on the top face must be

Answer: $40.5\ \mathrm{T \cdot m^2}$, exiting

Solution: The net magnetic flux through the cylinder must be zero. The flux from two of the surfaces is $-63.9 + 23.4 = -40.5\,\mathrm{T \cdot m^2}$ exiting the cylinder. Thus, the remaining surface must have $40.5\,\mathrm{T \cdot m^2}$ exiting the cylinder.

14. A beam of light polarized along the y axis and moving along the $+z$ axis passes through two polarized sheets with axes of polarization oriented 30° and 70° relative to the y axis. The final intensity of the beam is measured to be 61 W/m^2. What is the initial beam intensity?

Answer: 139 W/m^2

Solution: Let the initial intensity be I_o. The intensity after the light passes through the two polarizers is $I_o \cos^2(30°) \cos^2(40°) = 61$ W/m^2. Solve for I_o.

15. Two isotropic point sources emit identical radio waves in phase at wavelength λ. The sources are separated by a distance d on the $x-$axis, and a receiver moves around on a circle of large radius centered on the midpoint between them. It detects 14 points of zero intensity, including two on the $x-$axis, one to the left of the sources and one to the right. What is d?

    Answer: 3.5λ

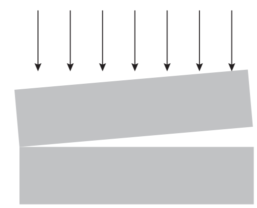
    Solution: There are two zeros of intensity on the x-axis. The twelve remaining zeros of intensity are split evenly among the four quadrants of the circle. There are 3 in the upper right quadrant corresponding to $\Delta L = \lambda/2$, $3\lambda/2$, and $5\lambda/2$. This means that the zeros on the x-axis correspond to a $\Delta L = 7\lambda/2$. We know that the path length difference along the x-axis is $\Delta L = d$ so that $d = 3.5\lambda$.

16. Costume jewelry beads are made of glass with index of refraction 1.55. To make them more reflective, they are coated with a layer of material with index of refraction 1.90. What is the minimum coating thickness needed to ensure that light of wavelength 540 nm and perpendicular incidence will be reflected with maximum constructive interference?

    Answer: 71 nm

    Solution: The condition for constructive interference is $\Delta L = 2d = m\lambda + \lambda/2$, where the $\lambda/2$ comes from the additional phase shift that occurs when reflecting off of a higher index and λ is the wavelength in the film: $\lambda = 540nm/1.9$. The smallest film thickness to satisfy the constructive interference condition is $d = \lambda/4 = (540nm/1.9)/4$.

17. Two rectangular glass plates with $n = 1.65$ (see Figure) of length 4.0 cm are in contact along the left edge and are separated by 0.1 mm at the right edge. The air between the plates acts as a thin film. If light of 540 nm is incident normally from above, what is the distance between fringes as seen from above?

    Answer: 0.108 mm

    Solution: As one moves along the upper plate, let the horizontal distance between fringes be Δx and the vertical distance between fringes be Δy. Δx and Δy are related by the slope: $\Delta y/\Delta x = 0.1mm/4.0cm = 2.5 \times 10^{-3}$. The difference in path lengths for between adjacent fringes is $2\Delta y = \lambda$. Thus, the horizontal distance between fringes is $\Delta x = 0.5\lambda/2.5 \times 10^{-3}$.

18. A double slit arrangement produces interference fringes for 465 nm laser light that are 2.2 mm apart on a screen located 5.0 m from the slits. What is the separation of the slits?

    Answer: 1.06 mm

    Solution: In radians for small angles $\sin\theta \approx \theta$. Because fringes occur for $d\sin\theta = m\lambda$, the angular separation between fringes is $\Delta\theta = \lambda/d$. For this problem, $\Delta\theta = 2.2\ mm/5m = 4.4 \times 10^{-4} = \lambda/d$.

19. A 1.0μm-wide slit is illuminated by light of wavelength 530 nm. Consider a point on a viewing screen that is 9° from the center of the central maximum. What is the phase difference between the waves arriving from the top of the slit to the waves coming from the bottom?

    Answer: 106°

    Solution: The difference in path length between waves arriving from the top and the bottom of the slit is $d\sin(\theta)$. This leads to a phase difference of $2\pi d\sin(\theta)/\lambda$ in radians and of $360\, d\sin(\theta)/\lambda$ in degrees.

20. A satellite telescope orbiting the earth can resolve objects that are 1.1 m apart (Raleigh criterion) from a height of 400 km using 550 nm light. What is the approximate minimum diameter of the telescope?

Answer: 0.24 m

Solution: The Raleigh criterion is $\sin\theta = 1.22\lambda/d$. For small angles as in the previous problem $\sin\theta \approx \theta$. For this problem $\theta = 1.1/4 \times 10^5$, and the wavelength is 550 nm. Thus, the approximate minimum diameter of the telescope is $1.22\lambda/\theta$.