Exam 1 Solution

1. A solid conducting sphere of 15 cm radius is located at the center of a spherical conducting shell, whose inner radius is 21 cm and outer radius 42 cm. At a distance of 50 cm from the center of these concentric conductors, the electric field is 2.1×10^5 V/m and is pointing outward. The shell has a net charge of $+3.1$ μC. What is the net charge on the sphere at the center in μC?

   Answer: $+2.7$

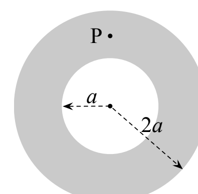
   Solution: A spherical Gaussian surface of radius 50 cm encloses a charge of $4\pi(0.5m)^2 E\epsilon_o = 5.8\mu C$. Since the shell has a net charge of $3.1\mu C$, the sphere has a net charge of $5.8 - 3.1 = 2.7\mu C$.

2. A long, straight wire is uniformly charged with a linear charge density of 5.0 μC/m. The wire runs along the axis of a cylinder of radius 2.0 cm and length 30.0 cm. What is the total electric flux through the cylinder surfaces in N·m^2/C?

   Answer: 1.69×10^5

   Solution: From Gauss's Law, the electric flux is equal to the charge enclosed divided by ϵ_o. The charge enclosed is $(5\mu C/m)(0.3m) = 1.5\mu C$ so the total electric flux is $(1.5\mu C)/\epsilon_o$.

3. The figure shows a uniformly charged, nonconducting spherical shell of inner radius a and outer radius $2a$. If the electric field at the outer radius is E, what is the electric field at point P with radius $r = 1.5a$?



   Answer: $0.6E$

   Solution: Here we apply Gauss's law twice: once for a spherical surface of radius $2a$ and once for a spherical surface of radius $1.5a$. Let the magnitude of the electric field at $2a$ be E, and the magnitude of the electric field at $1.5a$ be E'. The charge density is denoted by ρ.

$$\oint \vec{E} \cdot \hat{n} dA = 4\pi(2a)^2 E = Q_{enc}/\epsilon_o = \rho(4\pi/3)\left((2a)^3 - a^3\right)/\epsilon_o$$

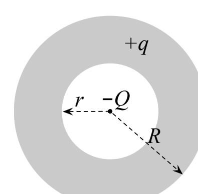
$$\oint \vec{E} \cdot \hat{n} dA = 4\pi(1.5a)^2 E' = Q_{enc}/\epsilon_o = \rho(4\pi/3)\left((1.5a)^3 - a^3\right)/\epsilon_o$$

   Divide the second equation by the first:

$$\frac{(1.5)^2 E'}{2^2 E} = \frac{(1.5)^3 - 1}{2^3 - 1},$$

   which implies that $E' = 0.603E$.

4. A thick conducting spherical shell has inner radius r and outer radius R, as shown in the figure. A point charge of $-Q$ is located at the center of the sphere and a charge of $+q$ is placed on the conducting shell. The charge on the outer surface of the conducting shell is:



   Answer: $q - Q$

   Solution: A Gaussian surface just larger than R will enclose a charge of $q - Q$, while a Gaussian surface just smaller than R will enclose no net charge because the electric field is zero inside a conductor. Consequently, the charge on the outer surface of the shell must be $q - Q$.

5. Suppose we have an insulating spherical ball of uniform charge density ρ and radius R. At what radius or radii from the center of the sphere is the electric field strength reduced by a factor of 4 from the electric field strength at the surface?

Answer: $R/4$ and $2R$

Solution: Applying Gauss's law, the flux through a sphere of radius $r < R$ is

$$\Phi = (\vec{E} \cdot \hat{n}) 4\pi r^2 = \frac{Q_{enc}}{\epsilon_o} = \frac{1}{\epsilon_o}\frac{4\pi}{3} r^3 \rho \Rightarrow \vec{E} \cdot \hat{n} = \frac{1}{3\epsilon_o} r\rho.$$

The flux through a sphere of radius $r > R$ is

$$\Phi = (\vec{E} \cdot \hat{n}) 4\pi r^2 = \frac{Q_{enc}}{\epsilon_o} = \frac{1}{\epsilon_o}\frac{4\pi}{3} R^3 \rho \Rightarrow \vec{E} \cdot \hat{n} = \frac{1}{3\epsilon_o}\frac{R^3}{r^2} \rho.$$

The electric field at the surface, $r = R$, is $\vec{E} \cdot \hat{n} = \rho R/(3\epsilon_o)$. To determine where the electric field is 1/4 of the value at the surface, set $\vec{E} \cdot \hat{n}$ for the $r < R$ and $r > R$ cases equal to $(1/4)\rho R/(3\epsilon_o)$ and solve for r.

6. Two lead nuclei each with charge $+82e$ approach one another head-on from a great distance. Initially each has kinetic energy 50.0 MeV (1 MeV $= 1.6 \times 10^{-13}$ J). What is the closest distance in fm (1 fm $= 10^{-15}$ m) the nuclei will approach one another?

Answer: 97

Solution: Initially since the nuclei are far apart their potential energy is zero, $U_i = 0$. Their initial kinetic energy is $2 * 50 \times 1.6 \times 10^{-13} J$. The factor of two is because both nuclei have kinetic energy. Finally, their kinetic energy is zero, and their potential energy is $k(82e)^2/r$. Use conservation of energy, $K_i + U_i = K_f + U_f$, to solve for $r = k(82e)^2/K_i$.

7. Particles 1, 2 and 3 are on the x-axis at $x = 0$ cm, 5 cm, and 10 cm, respectively. The charges of particles 1 and 2 are each 5 μC. The electrical potential energy of the three-particle system is 0, where potential energy at infinity is defined to be 0. What is the charge of particle 3 in μC?

Answer: -3.3

Solution: The net potential energy of this charge configuration is

$$U = \frac{k(5\mu C)(5\mu C)}{5cm} + \frac{k(5\mu C)Q}{5cm} + \frac{k(5\mu C)Q}{10cm},$$

where Q is the unknown charge. Setting $U = 0$, we get the following equation for Q: $0 = (5\mu C) + 1.5Q$.

8. The rectangle shown in the figure has sides 5 cm and 8 cm. The charges are $q_1 = 6\mu$C and $q_2 = 9\mu$C. How much work is required to move a charge $q_3 = 4.0\mu$C from point A to point B?

Answer: $+0.81$ J

Solution: Compute the initial and final potential energy of the charge configuration. The work is equal to $W = -(U_f - U_i)$.

$$U_i = \frac{k(6\mu C)(4\mu C)}{8cm} + \frac{k(9\mu C)(4\mu C)}{5cm} + \frac{k(6\mu C)(9\mu C)}{\sqrt{89}cm}$$

$$U_f = \frac{k(6\mu C)(4\mu C)}{5cm} + \frac{k(9\mu C)(4\mu C)}{8cm} + \frac{k(6\mu C)(9\mu C)}{\sqrt{89}cm}$$

Note that two of the terms are the same in the initial and final cases.

9. A thin plastic rod 8.2 cm long carrying charge -0.022μC is bent into a circle. What is the potential at the center of the ring, assuming $V = 0$ at infinity? (1 KV = 1000 volts)

Answer: -15.2 KV

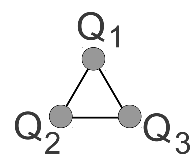
Solution: The potential is $V = \int kdq/r = kq/r$ because r is constant along the circle. The radius can be found from the circumference: $2\pi r = 8.2cm$.

10. The potential (in volts) in a region is given by $V(x, y, z) = 2x^2 - 5y^2 + 4x^3z^2$, where x, y and z are expressed in meters. What is the magnitude of the electric field in V/m at the point (1,1,1)?

Answer: 20.5

Solution: The three components of the electric field are $E_x = -\partial V/\partial x = -4x - 12x^2z^2 = -16\,V/m$, $E_y = -\partial V/\partial y = 10y = 10\,V/m$, and $E_z = -\partial V/\partial y = -8x^3z = -8\,V/m$. The magnitude of the electric field is $\sqrt{16^2 + 10^2 = 8^2} = 20.5$.

11. Three charges are placed at the corners of an equilateral triangle of side 3 cm. If $Q_1 = 2\mu C$, $Q_2 = -1\mu C$, and $Q_3 = 3\mu C$, what is the magnitude of the net force on Q_3?



Answer: 50 N

Solution: Make a table showing the components of each of the forces and then add the components.

	F_x	F_y
F on 3 by 1	$+k(2\mu C)(3\mu C)/(3cm)^2\ \cos(60)$	$-k(2\mu C)(3\mu C)/(3cm)^2\ \sin(60)$
F on 3 by 2	$-k(1\mu C)(3\mu C)/(3cm)^2$	0
F total	0 N	52 N

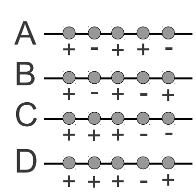
The directions were obtained using opposite charges attract and like charges repel.

12. On the x-axis a $4\mu C$ charge is at $x = 0$ cm, and a $2\mu C$ charge is at $x = 5$ cm. What is the force on a $-1\mu C$ charge placed at $x = 3$ cm?

Answer: $+5N\hat{\imath}$

Solution: The force on the $-1\mu C$ charge by the $4\mu C$ is $k(1\mu C)(4\mu C)/(3cm)^2 = 40N$ in the $-\hat{\imath}$ direction. The force on the $-1\mu C$ charge by the $2\mu C$ is $k(1\mu C)(2\mu C)/(2cm)^2 = 45N$ in the $+\hat{\imath}$ direction. Thus, the net force is $(45 - 40)N\hat{\imath}$.

13. Five charges are equally spaced along the x-axis. Each charge has the same magnitude, e, but some of the charges are $+e$ and some are $-e$. Four different configurations of charge are labeled A, B, C, D in the figure at right. Rank the magnitude of the force on the middle charge for the different configurations with largest first and smallest last.



Answer: C, D, A, B

Solution: For case C all the forces on the middle charge are in the same direction, producing the maximum force. For case B all of the forces on the middle charge cancel, producing zero net force. This leaves only cases A and D. For case D the two plus charges on the ends cancel, leaving only the forces due to the second and fourth charges. For case A the force due to the two end charges do not cancel; however, these forces act to oppose the force due to the second and fourth charges. Thus, case D has a larger force.

14. A dipole is allowed to rotate freely like the molecules in water. Which of the following statements is true?

Answer: The dipole will be attracted to both a positive and a negative charge.

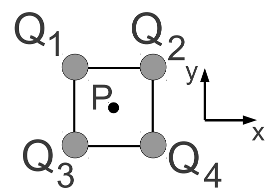
Solution: For the case of a positive charge the negative end of the dipole will rotate towards the positive charge, and the dipole will be attracted to the positive charge. Similarly, for the case of a negative charge the positive end of the dipole will rotate towards the negative charge, and the dipole will be attracted to the negative charge.

15. Two charges with charge $Q < 0$ are separated by 3 cm. There is a 1 N repulsive force between the charges. How many excess electrons are needed to make one of the charges, Q?

Answer: 2×10^{12}

Solution: The magnitude of the force is $1N = kQ^2/(3cm)^2$. This implies that the charge is $Q = -3.16 \times 10^{-7}C$, and the number of excess electrons is $|Q|/e = 2 \times 10^{12}$.

16. Four charges are placed on the corners of a square of side 2 m as shown in the figure. If $Q_1 = 1\mu C$, $Q_2 = 3\mu C$, and $Q_3 = Q_4 = 2\mu C$, what is the magnitude of the electric field at point P at the center of the square?

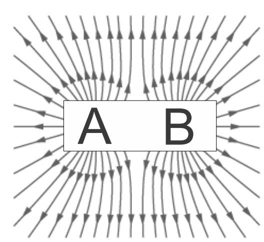


Answer: 6400 N/C

Solution: The electric fields due to Q_1 and Q_4 partially cancel, as do the electric fields due to Q_2 and Q_3. Add the remaining electric fields as vectors:

	E_x	E_y
$\vec{E}_1 + \vec{E}_4$	$-k(1\mu C)(\sqrt{2}m)^2\ \cos(45)$	$+k(1\mu C)(\sqrt{2}m)^2\ \sin(45)$
$\vec{E}_2 + \vec{E}_3$	$-k(1\mu C)(\sqrt{2}m)^2\ \cos(45)$	$-k(1\mu C)(\sqrt{2}m)^2\ \sin(45)$
$\vec{E}$ total	$-2k(1\mu C)(\sqrt{2}m)^2\ \cos(45)$	0

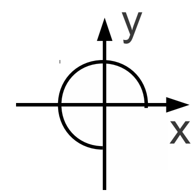
17. The electric field lines of two point charges, A and B, are shown in the figure. Which of the following is true of the point charges?



Answer: Both point charges are positive.

Solution: Electric field lines go away from positive charges and towards negative charges. The electric field lines are going away from both A and B.

18. A line of charge in the form of an arc in a circle of radius 2 m is centered about the origin as shown in the figure. If the charge per unit length on the line is $\lambda = 5nC/m$, what is the electric field at the origin?



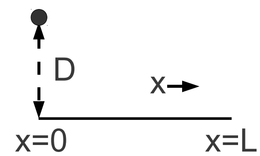
Answer: $+22.5V/m\,\hat{\imath} - 22.5V/m\,\hat{\jmath}$

Solution: Let θ be the usual angle measured with respect to the positive x-axis. The arc extends from $\theta = 0$ to $\theta = 3\pi/2$. By symmetry the x and y-components of the electric field at the origin will be the same magnitude. Thus, we only need to compute E_x.

$$E_x = -\int_0^{3\pi/2} \frac{k\lambda r d\theta}{r^2} \cos\theta = -\frac{k\lambda}{r}\sin(\theta)\Big|_0^{3\pi/2} = +\frac{k\lambda}{r}$$

$E_y = -E_x$ because the electric field goes away from positive charges.

19. A line of charge of length L and charge per unit length, λ, is located on the x-axis as shown in the figure. Which expression below gives the x-component of the electric field, E_x, at a distance D from the left end of the line?



Answer: $-\int_0^L \frac{kx\lambda}{(x^2+D^2)^{3/2}} dx$

Solution: The magnitude of the electric field due to an element of charge $dq = \lambda dx$ at x is kdq/r^2, where $r = \sqrt{x^2 + D^2}$. To get the x-component of the electric field we must multiply by the trigonometric factor $-x/r$, where the minus sign is because the electric field due to dq is in the negative x-direction. Thus, the integrand is $(kdq/r^2)(-x/r)$.

20. A proton is located at $x = 0$ and an electron is located at $x = 2$ m. There is a uniform electric field of $5V/m\,\hat{\imath}$. If at $t = 0$ they are released from rest, at what time would they reach the same x-position? (Do not include the attraction between the proton and the electron.)

Answer: $2.1\mu s$

Solution: The proton accelerates to the right $(+\hat{\imath})$ with acceleration $a_p = eE/m_p$, and the electron accelerates to the left with $a_e = eE/m_e$. They pass each other when their positions are the same: $x_p = 0.5a_pt^2 = x_e = 2m - 0.5a_et^2$, which implies a time of $t = \sqrt{4m/(a_p + a_e)}$.