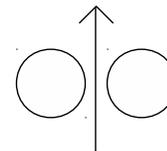


Exam 3 Solution

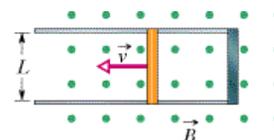
1. The current i in a long wire is going up as shown in the figure, but decreasing in magnitude. What is the direction of the induced current in the left loop and the right loop. (List the direction of the induced current in the left loop first.)



Answer: counterclockwise, clockwise

Solution: Because the current in the wire is going up, the magnetic field is coming out of page on the left side of the wire and into the page on the right side. The magnitude of the flux is decreasing because the current is decreasing. Consequently, via Lenz's law the induced current acts to produce a flux coming out of the page on the left (counterclockwise) and into the page on the right (clockwise).

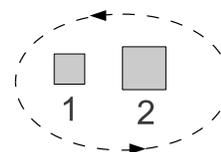
2. A metal rod is forced to move with constant velocity along two parallel metal rails, which are connected with a strip of metal (see figure). A magnetic field of 0.5 T points out of the page. The separation between the rails is $L = 30$ cm, the rod has resistance 40Ω , and velocity 70cm/s . What force must be applied to the rod to keep it moving at the constant velocity?



Answer: $4 \times 10^{-4} N$

Solution: The magnitude of the induced emf is $d\Phi_B/dt = BdA/dt = BvL$, and consequently the induced current is $i = BvL/R$. The magnetic force on the rod is iLB . This is the force which must be applied to keep the rod moving at a constant velocity.

3. The magnetic field is 2 T into the page and 1 T out of the page in regions 1 and 2 respectively (see figure). The magnitude of both fields is decreasing at a rate of -0.005T/s . If region 1 has area 0.3 m^2 and region 2 has area 0.6 m^2 , what is $\oint \vec{E} \cdot d\vec{s}$ for the path of integration shown in the figure?



Answer: $1.5 \times 10^{-3} V$

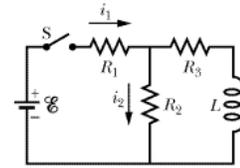
Solution: From the direction of the integral indicated in the figure positive flux is coming out of the page. Thus, the flux is negative in region 1 and positive in region 2; however, the flux is decreasing in magnitude so $d\Phi_B/dt$ is positive in region 1 and negative in region 2. Finally, the integral $\oint \vec{E} \cdot d\vec{s}$ is equal to $-d\Phi_B/dt$ so the contribution from region 1 is negative and the one from region 2 is positive. $-d\Phi_B/dt = 0.005(0.6 - 0.3)$ Volts.

4. A circular loop of wire has radius 20 cm. If the magnetic field inside the loop is uniform and has the time dependence, $B(t) = 1 + 2t + 3t^2$, where t is measured in seconds and B is measured in Tesla. What is the magnitude of the induced emf in the loop at time $t = 3\text{s}$?

Answer: 2.5 V

Solution: The magnitude of the induced emf is $|d\Phi_B/dt| = AdB/dt = \pi(0.2)^2(2 + 6 \cdot 3)$ in Volts.

5. In the figure at right $\mathcal{E} = 120V$, $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 30\Omega$, and $L = 4H$. Immediately after switch S is closed, what is the current through R_2 ?



Answer: 4.0 A

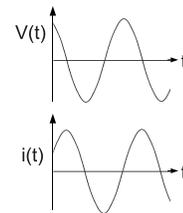
Solution: Because the current changes continuously through an inductor, right after the switch is closed no current flows through the inductor and $i_1 = i_2$. Effectively, we have two resistors in series, $R_1 + R_2 = 30\Omega$, connected across a 120 V battery. The current is $120/30 = 4$ A.

6. An LC circuit has $C = 8\mu F$ and $L = 5mH$. If the maximum voltage on the capacitor is 3 V, what is the maximum current through the inductor?

Answer: 0.12 A

Solution: For this LC circuit the angular frequency is $\omega = 1/\sqrt{LC} = 5000$ Hz. The maximum charge is $(8\mu F)(3V) = 24\mu C$. Taking a derivative of $q(t) = 24\mu C \cos(\omega t)$ gives a maximum current of $(24\mu C)\omega$.

7. For a driven LCR circuit the current and voltage as a function of time are given by the graphs in the figure. For this circuit is $X_L > X_C$ or $X_C > X_L$? Also, does one need to increase or decrease the driving frequency, ω_d , in order to obtain resonance?



Answer: $X_L > X_C$, decrease ω_d

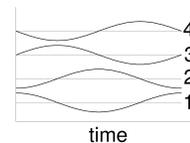
Solution: In the figure the voltage is leading the current. This corresponds to ELI of “ELI the ICE man.” The inductance dominates $X_L > X_C$ or $\omega_d L > 1/\omega_d C$. In order to have resonance, $\omega_d L = 1/\omega_d C$, ω_d must be decreased.

8. A transformer is to take an AC voltage of amplitude 120 V down to an AC voltage of 6 V. If the number of turns in the primary coil is 1000, how many turns are in the secondary coil?

Answer: 50

Solution: For a transformer $V_s/V_p = N_s/N_p$. One is given $V_p = 120V$, $V_s = 6V$, and $N_p = 1000$.

9. In an LC circuit the charge across the capacitor, $q(t)$, is given by graph 2. Which graph represents the derivative of the current, di/dt ?



Answer: graph 1

Solution: The current is the first derivative of the charge, and di/dt is the second derivative of the charge. Graphically, a derivative is the slope. This result can also be obtained by looking at the differential equation for an LC circuit: $d^2q/dt^2 = -\omega^2 q$.

10. Consider an RLC circuit with driving emf amplitude $12V$, resistance $R = 6\Omega$, inductance $L = 1H$, and capacitance $C = 2.0\mu F$. Find the amplitude of the voltage across the resistor at resonance.

Answer: 12 V

Solution: At resonance $X_L = X_C$, and the impedance is $Z = R$ as well as $\phi = 0$. This implies that the net voltage is the voltage across the resistor, and the voltages across the inductor and capacitor, which are 180° out of phase, cancel.

11. Gauss' law for magnetism tells us:

Answer: that magnetic monopoles do not exist

Solution: There are no magnetic "charges" or monopoles.

12. A cylindrical region contains a uniform electric field that is along the cylinder axis and is changing with time. If r is the distance from the cylinder axis the magnitude of the magnetic field within the region is:

Answer: proportional to r

Solution: An electric field which changes in time creates a magnetic field via $\oint B \cdot ds = \mu_o \epsilon_o d\Phi E/dt$. For an integration path of a circle with radius r inside the region where the electric field is changing this becomes $2\pi r B = \mu_o \epsilon_o (\pi r^2)(dE/dt)$ or $B = 0.5\mu_o \epsilon_o r(dE/dt)$.

13. Of the three chief kinds of magnetic materials (diamagnetic, paramagnetic, and ferromagnetic) which are used to make permanent magnets?

Answer: only ferromagnetic

Solution: Diamagnetic and paramagnetic materials only have a magnetic moment when a field is applied, while ferromagnetic materials have a magnetic moment even for zero applied field.

14. What energy in eV is required to flip the spin of an electron in a magnetic field of 12.9 T from antiparallel alignment to parallel alignment? The magnitude of the magnetic moment of an electron is $9.27 \times 10^{-24} J/T$, and an eV is $1.6 \times 10^{-19} J$.

Answer: 1.5×10^{-3}

Solution: Because the energy of a magnetic moment in a magnetic field is $U = -\vec{\mu} \cdot \vec{B}$ the energy required to flip a spin from antiparallel to parallel to the field is $2\mu B$.

15. Which of the following equations, along with a symmetry argument, can be used to calculate the magnetic field between the plates of a charging parallel plate capacitor with circular plates?

Answer: $\oint \vec{B} \cdot d\vec{s} = \mu_o i + \mu_o \epsilon_o d\Phi_E/dt$

Solution: There is a displacement current, $\epsilon_o d\Phi_E/dt$, between the plates which creates the magnetic field.

16. A small rocket of mass 10 kg. emits an intense light beam as a rocket exhaust (because electromagnetic energy U carries momentum U/c). What must be the minimum total power of the beam in gigawatts ($= 10^9 W$) needed to lift the rocket against the force of gravity at the earth's surface? ($g = 9.8 m/s^2$)

Answer: 29

Solution: The gravitational force on the rocket, $(10kg)g$, must equal the force $F = dp/dt$ due to the momentum the light carries. Using $\Delta p = \Delta U/c$, $dp/dt = (dU/dt)/c$, where dU/dt is the power of the beam. (Remember, power is energy per unit time.) Consequently, the power needed is $dU/dt = (10kg)gc$.

17. The speed of light in vacuum depends on which of the following wave properties?

Answer: None of these

Solution: The speed of light in a vacuum is constant at c .

18. If the electric field in a plane electromagnetic wave is given by $E(x, t) = 8500 \sin(33x - \omega t)$, in SI units, the frequency of the wave is:

Answer: 1.6×10^9 Hz

Solution: From the equation we can see that the wave vector is $k = 33 = 2\pi/\lambda$. The frequency is $f = c/\lambda = 33c/(2\pi)$.

19. A 30 W light bulb is placed 1.8m below the surface of a pool of water ($n = 1.333$). What is the *diameter* (in meters) of the circle on the surface of the water through which light is transmitted? (Some of the light is totally internally reflected.)

Answer: 4.1

Solution: The critical angle for total internal reflection at the water-air interface is $\theta = \sin^{-1}(1/1.33) = 48.75^\circ$. Since the light bulb is 1.8m below the surface, this creates a circle of radius $(1.8m) \tan(\theta)$ and diameter $2(1.8m) \tan(\theta)$.

20. An unpolarized beam of light with intensity 72 W/m^2 is incident on a stack of 3 polarizing sheets with each sheet's polarization axis rotated an angle θ relative to that of the previous sheet. If the beam emerging from the stack has intensity 4 W/m^2 , what is its intensity after it has passed through the first 2 sheets?

Answer: 12 W/m^2

Solution: The intensity of the light after it goes through 3 polarizers is $0.5I_o \cos^2(\theta) \cos^2(\theta)$, where θ is the angle between successive polarizers. Setting this equal to 4 W/m^2 with $I_o = 72 \text{ W/m}^2$ yields $\cos^2(\theta) = 1/3$. If there were only two sheets, then the intensity would be $0.5I_o \cos^2(\theta) = 0.5I_o/3$.