

Final Exam Solution

1. A small insect starts walking away from a concave mirror along its central axis. Which statement is true?

Answer: The image is initially erect, then flips upside down

Solution: For a concave mirror the object distance, p , and image distance, i , are related via $1/p + 1/i = 1/f$ with the focal length, f , being positive. Near the mirror p is small so $1/p > 1/f$ and the image distance is negative (virtual image). Consequently, the magnification, $m = -i/p$, is positive, and the image is erect. Far from the mirror, $1/p < 1/f$, and the image distance is positive (real image). In this case the magnification, $m = -i/p$, is negative, and the image is inverted.

2. A concave shaving mirror has a radius of curvature of 30 cm. It is positioned so that the (upright) image of a man's face is 3 times the size of the face. How far is the mirror from the face (in cm)?

Answer: 10

Solution: Because the radius of curvature is 30 cm, the focal length is $f = r/2 = 15$ cm. An upright image three times the object size corresponds to a magnification of $m = -i/p = 3$. This implies that $i = -3p$ and $1/p + 1/i = 1/p - 1/(3p) = 2/3p = 1/f$. The object distance is $(2/3)f = 10$ cm.

3. An object is placed 30 cm to the left of a diverging lens of focal length -15 cm. A converging lens of focal length 30 cm is placed 30 cm to the right of the first lens. Where is the final image located relative to lens 2, and what is the overall magnification m ?

Answer: 120 cm to the right; $m = -1.0$

Solution: This is a compound lens problem. Find the image of the first lens, and use that as the object for the second lens. For the first lens we have

$$\frac{1}{30} + \frac{1}{i_1} = \frac{1}{-15}.$$

The image relative to the first lens is -10 cm, which is 10 cm to the left of the first lens and 40 cm to the left of the second lens. Thus, the object distance for the second lens is 40 cm:

$$\frac{1}{40} + \frac{1}{i_2} = \frac{1}{30}.$$

The final image is $i_2 = 120$ cm to the right of the second lens, and the overall magnification is $m = (-i_1/p_1)(-i_2/p_2) = (10/30)(-120/40) = -1$.

4. One of the two slits in a Young's experiment is painted over so that it transmits only one-half the intensity of the other slit. As a result:

Answer: the dark fringes get brighter and the bright ones get darker

Solution: If one of the slits in Young's experiment is covered up, then there will be no interference pattern. Partially covering up a slit reduces the interference: dark fringes get brighter and bright ones get darker.

5. A soap film is illuminated by white light normal to its surface. The index of refraction of the film is 1.50. The intensities of wavelengths 480 nm and 800 nm and no wavelengths in between are to be maximized in the reflected beam. (Take the index of refraction of air to be one.) The thickness of the film is:

Answer: 400 nm

Solution: Because the index of refraction of the film is greater than that of air ($n = 1$) there is an additional phase shift for the reflection inside the soap film. The condition for constructive interference is $2L = (m + 0.5)\lambda/1.5$ or $\lambda = 3L/(m + 0.5)$. The fact that $\lambda = 800$ nm and $\lambda = 480$ nm are consecutive maximum means that if $\lambda = 800$ nm corresponds to m , then $\lambda = 480$ nm corresponds to $m + 1$. Using the m dependence on λ , this implies that $800/480 = (m + 1.5)/(m + 0.5)$, but $800/480 = 5/3 = (2 + 1.5)/(1 + 0.5)$. In other words for $m = 1$ for $\lambda = 800$ nm, and hence $L = (m + 0.5)\lambda/3 = 400$ nm .

6. A battery is used to charge a series combination of two identical capacitors. If the potential difference across the battery terminals is V and total charge Q flows through the battery during the charging process then the charge on the positive plate of each capacitor and the potential difference across each capacitor are, respectively:

Answer: Q and $V/2$

Solution: The charge on capacitors in series is the same (Q). This in turn implies that the voltage drop across each of the identical capacitors is also the same: Q/C . They each drop half the voltage.

7. A current moves from left to right in a horizontal copper wire. Which statement is true?

Answer: The resistivity of the copper wire does not depend on its length.

Solution: Resistivity is an intrinsic property of the material. Like density it does not depend on the geometry or dimensions of the wire.

8. Resistances of 2.0Ω , 4.0Ω , and 6.0Ω and a voltage source with 24 V are all in series. The potential difference across the 2.0Ω resistor is:

Answer: 4 V

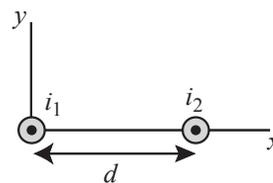
Solution: The net resistance of the three resistors in series is 12Ω . The current is $24V/12\Omega = 2$ A, and the voltage drop across the 2Ω resistor is $(2 A)(2 \Omega) = 4 V$.

9. An electron is launched with velocity v in a uniform magnetic field. The angle θ between the velocity vector \vec{v} and the magnetic field direction is between 0 and 90° . As a result, the electron follows a helix, its velocity vector returning to its initial value in a time interval of:

Answer: $2\pi m/(eB)$

Solution: Let the component of the velocity perpendicular to the magnetic field be v_\perp . The magnitude of the force on the electron is $F = ma = mv_\perp^2/r = ev_\perp B$. Eliminating one of the v_\perp 's from this equation, the angular frequency of rotation perpendicular to the field is $\omega = v_\perp/r = eB/m$. The angular frequency is related to the period via $\omega = 2\pi/T$.

10. In the figure shown, two long straight wires with separation d carry currents i_1 and $i_2 = 2i_1$ out of the page, with i_1 located at $x = 0$. At what point on the x -axis shown is the net magnetic field due to the currents equal to zero?



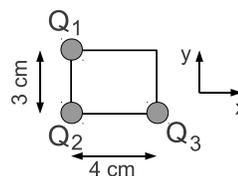
Answer: $d/3$

Solution: On the x -axis between the two wires the magnetic field of wire 1 is in the $+y$ direction, and the magnetic field of wire 2 is in the $-y$ direction. They will cancel when their magnitudes are the same:

$$\frac{\mu_o i_1}{2\pi x} = \frac{\mu_o i_2}{2\pi(d-x)} = \frac{\mu_o 2i_1}{2\pi(d-x)}$$

This implies that $2x = d - x$ or $x = d/3$.

11. Three charges are placed on the corners of a rectangle as shown in the figure. If $Q_1 = -2\mu\text{C}$, $Q_2 = 1\mu\text{C}$, and $Q_3 = 3\mu\text{C}$, what is the x component of the force on Q_3 due to Q_1 and Q_2 ?



Answer: -0.4 N

Solution: The x -component of the force on Q_3 by Q_1 is

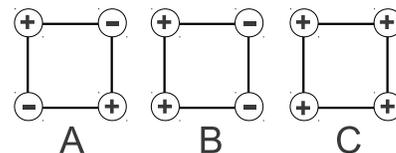
$$F_{31,x} = -k \frac{|Q_3||Q_1|}{(5\text{cm})^2} \frac{4}{5} = -17.280 \text{ N},$$

where the minus sign is because 1 and 3 are attracted and the $4/5$ is from the cosine to compute the x -component. The x -component of the force on Q_3 by Q_2 is

$$F_{32,x} = +k \frac{|Q_3||Q_2|}{(4\text{cm})^2} = 16.875 \text{ N}.$$

The sum of these two is -0.405 N .

12. Four charges are placed on the corners of a square as shown in the figure. All of the charges have the same magnitude, but as indicated in the figure some of the charges are positive and some are negative. In which of the three cases is the electric field at the center of the square zero?



Answer: A,C

Solution: Remember the electric field goes away from positive charges and towards negative charges. In cases A and C the electric fields due to charges on opposite corners cancel one another, and the net electric field is zero. In case B the electric fields due to charges on opposite corners add, and the net electric field is to the right.

13. Two concentric shells of negligible thickness have radius $r_1 = 10 \text{ cm}$ and $r_2 = 20 \text{ cm}$. If the electric field at $r = 15 \text{ cm}$ is directed radially outwards with magnitude 1000 V/m and the electric field at $r = 25 \text{ cm}$ is directed radially inwards with magnitude 500 V/m , what is the *sum* of the charges on the two shells?

Answer: -3.5 nC

Solution: Since we are interested in the net charge on the two shells, use a Gaussian surface that encloses both of them, namely a sphere of radius 25 cm . Then using Gauss's law we obtain $Q_{enc} = 4\pi(0.25\text{m})^2(-500\text{V/m})\epsilon_o = -3.475 \times 10^{-9}\text{C}$. The minus sign is due to the electric field pointing inwards.

14. In a television tube electrons are accelerated through a voltage of 5000 volts. What is the speed of the electrons assuming that they were initially at rest? (Since this is Physics 2, do not use special relativity to calculate the answer even though these speeds are a fraction of the speed of light.)

Answer: 4×10^7 m/s

Solution: Since the electrons are assumed to be accelerated from rest, conservation of energy implies that $(1/2)m_e v^2 = e(5000V)$. Solve for $v = 4.19 \times 10^7$ m/s $\approx 4 \times 10^7$ m/s.

15. A circular loop of wire of radius 2 cm is in the plane of the page. A magnetic field perpendicular to the page is changing with time according to $B(t) = 1 + 3t$, where t is measured in seconds, B is measured in Tesla, and positive field is going out of the page. If the loop has resistance 5Ω , what is the magnitude and direction of the induced current at $t = 1$ second?

Answer: 0.75 mA clockwise

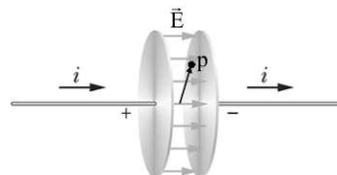
Solution: The flux is increasing out of the page at a rate $d\Phi_B/dt = \pi(2\text{cm})^2(3T/s) = 0.00377V$. To oppose this change in flux the induced current is clockwise. It has magnitude $0.00377V/5\Omega = 0.754mA$.

16. An RLC circuit is driven by a voltage of amplitude 12 V and has $R = 4\Omega$, $L = 2$ H, and $C = 100\mu\text{F}$. At resonance what is the amplitude of the voltage oscillations across the inductor?

Answer: 420 V

Solution: At resonance the driving angular frequency is $\omega_d = 1/\sqrt{LC} = 70.7\text{Hz}$, and the voltages of the inductor and capacitor cancel one another. The net voltage drop is equal to the voltage drop across the resistor. Consequently, the amplitude of the current oscillations is $I = 12V/4\Omega = 3A$. The voltage across the inductor is Ldi/dt and has amplitude $L\omega I = 424V$.

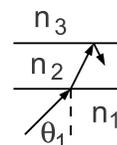
17. A current of 3A flows into circular parallel plate capacitor as shown in the figure. The plates have radius 1.0 cm. What is the magnitude of the magnetic field at point P in between the plates at a radial distance of 0.5 cm from the center of the plates?



Answer: 3.0×10^{-5} T

Solution: The current of 3A leads to a displacement current of 3A between the plates. This displacement current flows uniformly within the plate radius of 1.0 cm. Using $\oint Bds = \mu_o i_{d,enc}$ for a circular path of radius $r = 0.5$ cm, the magnetic field is $B = \mu_o i_{d,enc}/(2\pi r)$. Because the circle has radius 0.5 cm, the displacement current enclosed is $(3A)(\pi r^2)/(\pi(1\text{cm})^2) = 0.75A$.

18. As shown in the figure, light is incident at an angle θ_1 from a material with $n_1 = 1.1$ to a material with $n_2 = 1.3$. What is the smallest value of θ_1 so that there is total internal reflection at the interface between n_2 and $n_3 = 1$?



Answer: 65.4°

Solution: At the first interface using Snell's law we have $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$. At the second interface where there is total internal reflection $n_2 \sin(\theta_2) = n_3 \sin(\theta_3) = n_3$. Putting these two together $n_1 \sin(\theta_1) = n_3$ or $\theta_1 = \sin^{-1}(n_3/n_1)$.

19. Sunlight is incident on a diffraction grating with 400 rulings/mm . What is the highest order, m , that a complete visible spectrum $400 \text{ nm} < \lambda < 700 \text{ nm}$ can be seen?

Answer: 3

Solution: 400 rulings/mm corresponds to a separation between slits of $d = 0.001\text{m}/400 = 2500\text{nm}$. The condition for observing a diffraction line is $d \sin(\theta) = m\lambda$, which implies that $d = 2500\text{nm} > m\lambda$. For the largest wavelength, 700 nm, the largest m for which this inequality is satisfied is $m = 3$.

20. A satellite is 20,000 km above the surface of the Earth. It has a lens of diameter 50 cm. For light of wavelength 500 nm, what is the smallest resolvable distance based on Rayleigh's criterion?

Answer: 25 m

Solution: Rayleigh's criterion states that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. This occurs for $\theta \approx \sin(\theta) = 1.22\lambda/d$. For this problem $\lambda = 500 \text{ nm}$, $d = 50 \text{ cm}$, and $\theta = \Delta x/(20000\text{km})$, where Δx is the smallest resolvable distance. Solve for $\Delta x = 24.4\text{m}$.