PHY2049 Exam #1 Solutions – Fall 2012

1. Two identical conducting spheres A and B carry equal charge Q. They are separated by a distance much larger than their diameters. A third identical conducting sphere C carries charge 2Q. Sphere C is first touched to A, then to B, and finally removed. As a result, the electrostatic force between A and B, which was originally F, becomes:

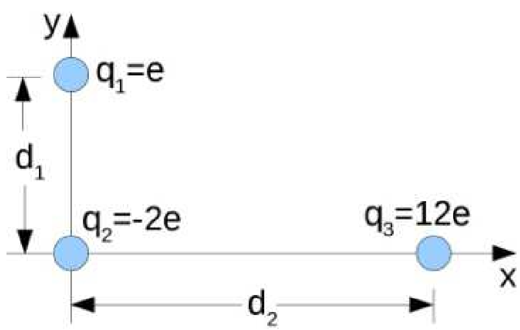
Solution:

After C touches A, $Q_A = Q_C$; $Q_A + Q_C = 3Q \Rightarrow Q_A = Q_C = \frac{3Q}{2}$.

Then C touches B, $Q_B = Q'_C$; $Q_B + Q'_C = \frac{5Q}{2} \Rightarrow Q_B = Q'_C = \frac{5Q}{4}$.

Coulomb's law gives $F' = \frac{kQ_AQ_B}{r^2} = \frac{15}{8}\frac{kQQ}{r^2} = \frac{15F}{8}$.

2. In the figure shown, what is the magnitude of the net electric force (in N) exerted on charge q_2 by charges q_1 and q_3, given that d_1 = 3 nm and d_2 = 6 nm?



Solution:

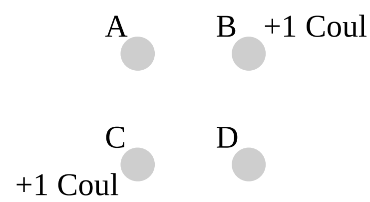
The x and y components of Coulomb force on q_2 are

$$F_x = \frac{24ke^2}{d_2^2} = 1.54 \times 10^{-10} N;\ F_y = \frac{2ke^2}{d_1^2} = 0.51 \times 10^{-10} N$$

The net force is $F = \sqrt{F_x^2 + F_y^2} = 1.6 \times 10^{-10} N$.

3. Four charges are at the corners of a square, with B and C on opposite

corners. Charges A and D, on the other two corners, have equal charge, while both B and C have a charge of +1.0 C. What is the charge on A so that the force on B is zero?



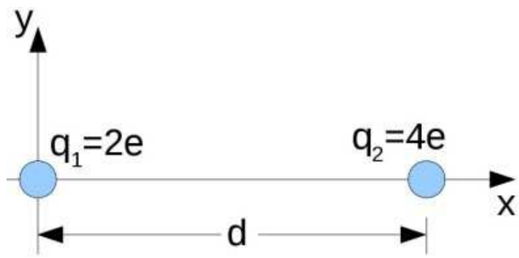
Solution:

Let the side of the square to be a. The Coulomb forces on charge C by A and D add up to a force along the diagonal direction. For the net force on B to be zero, we need

$$2\cos 45° \frac{kQQ_B}{a^2} + \frac{kQ_CQ_B}{\left(\sqrt{2}a\right)^2} = 0.$$

Solving for Q, we have $Q = -\frac{Q_C}{4\cos 45°} = -0.35\ C.$

4. Two charges, q_1 and q_2 lie a distance d = 3 cm apart on the x-axis. Charge q_1 is located at the origin and q_2 is located at x = 3 cm. At what position on the x-axis (in cm) is the magnitude of the electric field equal to zero?



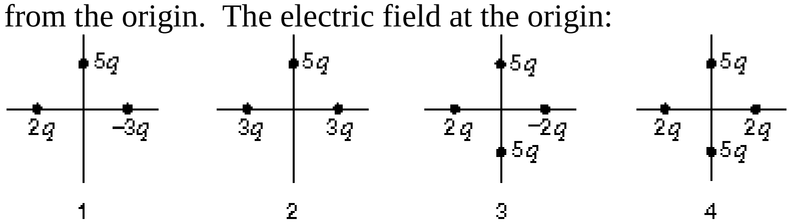
Solution:

The electric field can only vanish between the two positive charges. The fields from two charges must have the same magnitude

$$\frac{k(2e)}{x^2} = \frac{k(4e)}{(d-x)^2}.$$

Solve for x, we find x=1.24 cm.

5. The diagrams below depict four different charge distributions. The charged particles are all the same distance from the origin. The electric field at the origin:



A) is zero for situation 4

6. Three large parallel charged insulating sheets have charge per unit area of $\sigma_1 = 2\ \mu C/m^2$, $\sigma_2 = -4\ \mu C/m^2$, σ_3. What is the charge density of sheet 3, in order for the electric field to be zero in the region between sheets 2 and 3?

Solution:

In the region between sheets 2 and 3, the electric field is given by $E = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0}$. In order for the electric field to be zero, we have $\sigma_3 = \sigma_1 + \sigma_2 = -2\,\frac{\mu C}{m^2}$.

7. A conducting sphere of radius 1 cm is surrounded by a conducting spherical shell of inner radius 3 cm and outer radius 4cm. If the electric field at r=2 cm is going outwards with magnitude 300 V/cm and at r=5 cm is also going outwards with magnitude 300 V/cm. What is the net charge on conducting spherical shell?

Solution:

Make a Gauss surface consists of sphere of radius r_1 =2 cm and sphere of radius r_2 = 5 cm with the conducting spherical shell inside the Gauss surface. Apply Gauss' law, we have

$$\frac{Q_{shell}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = 4\pi r_2^2 E_2 - 4\pi r_1^2 E_1$$

Using the data of the problem, Q_{shell}=7 nC. Alternatively, using shell theorem, we have

$$E_1 = \frac{Q_{sphere}}{4\pi\varepsilon_0\, r_1^2};\ E_2 = \frac{Q_{sphere} + Q_{shell}}{4\pi\varepsilon_0\, r_2^2}$$

Solve them to find charges. Q_{shell}=7 nC.

8. Choose the INCORRECT statement:

A) According to Gauss' law, if a closed surface encloses no charge, then the electric field must vanish everywhere on the surface

9. Charge Q is distributed uniformly throughout an insulating sphere of radius R. The magnitude of the electric field at a point $2R/3$ from the center is:

Solution:

Applying Gauss' law to a spherical surface of radius r (r<R), we have

$$4\pi r^2 E = \frac{\frac{4}{3}\pi r^3 \rho}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \cdot (\frac{r}{R})^3 \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 R^2} \frac{r}{R} \Rightarrow E = \frac{Q}{6\pi\varepsilon_0 R^2}.$$

10. A 3.5-cm radius hemisphere contains a total charge of 6.6×10^{-7} C. The flux through the rounded portion of the surface is 9.8×10^4 N · m^2/C. The flux through the flat base is:

Solution:

Applying Gauss' law to surface of the hemisphere, we have

$$\frac{Q}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = \int_{base} \vec{E} \cdot d\vec{A} + \int_{top} \vec{E} \cdot d\vec{A}$$

$$\int_{base} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} - \int_{top} \vec{E} \cdot d\vec{A} = \frac{6.6 \times 10^{-7}}{8.85 \times 10^{-12}} - 9.8 \times 10^4 = -2.3 \times 10^4 \frac{Nm^2}{C}.$$

11. Two conducting spheres are far apart. The smaller sphere carries a total charge of Q. The larger sphere has a radius three times that of the smaller sphere and is neutral. After the two spheres are connected by a conducting wire, the charges on the smaller and larger spheres, respectively, are:

Solution:

The charge Q will be shared between the two spheres so that the potentials on them (V_1 and V_2) become equal (with difference of potentials becoming zero, there will be no current through the wire connecting the two spheres). The charge acquired by the larger sphere will be q, the charge left on the smaller sphere will be Q-q.

$$V_1 = \frac{Q-q}{r}k + \frac{q}{D}k \approx \frac{Q-q}{r}k$$ (since distance D between the spheres is very large)

$$V_2 = \frac{Q-q}{D}k + \frac{q}{3r}k \approx \frac{q}{3r}k$$ (since distance D between the spheres is very large)

Require $V_1 = V_2$

$$\frac{Q-q}{r}k = \frac{q}{3r}k$$

From where:

$$q = \frac{3}{4}Q$$ (charge on the larger sphere)

$$Q - q = \frac{1}{4}Q$$ (charge on the smaller sphere)

12. A particle with a charge of 5.5×10^{-8}C is fixed at the origin. A particle with a charge of -2.3×10^{-8}C is moved from x = 3.5 cm on the x axis to y = 4.3 cm on the y axis. The change in potential energy of the two-particle system is:

Solution:

Potential energy at the beginning: $U_i = \frac{q_1 q_2}{r_i}k$

Potential energy at the end: $U_f = \frac{q_1 q_2}{r_f}k$

where r_i and r_f are distances between charges at the beginning and at the end.

From where:

$$\Delta U = U_f - U_i = q_1 q_2 k\left(\frac{1}{r_f} - \frac{1}{r_i}\right) = 6.0 \times 10^{-5} \text{ J}$$

13. In separate experiments, four different particles each start from far away with the same speed and impinge directly on a gold nucleus. The masses and charges of the particles are

particle 1: mass m_0, charge q_0
particle 2: mass $2m_0$, charge $2q_0$
particle 3: mass $2m_0$, charge $q_0/2$
particle 4: mass $m_0/2$, charge $2q_0$

Rank the particles according to the distance of closest approach to the gold nucleus, from smallest to largest.

Solution:

Potential energy at the beginning: $U_i = \frac{Qq}{R}k \approx 0$ (particles are far apart)

Kinetic energy at the beginning: $KE_i = \frac{mv^2}{2}$

Potential energy at the end: $U_f = \frac{Qq}{r}k$

Kinetic energy at the beginning: $KE_f = 0$

From energy conservation:

$$r = 2\frac{Qk}{v^2} \cdot \frac{q}{m}$$

Ranking of the closest approach to the gold nucleous comes from q/m:

- particle 3 has the smallest q/m = $1/4 \cdot q_0/m_0$ and, hence, comes the closest
- particle 4 has the largest q/m = $4 \cdot q_0/m_0$ and, hence, comes the last in ranking
- particles 1 and 2 have equal q/m=q_0/m_0 and, hence, come tie in between particles 3 and 4

14. Two conducting spheres have radii of R_1 and R_2. If they are far apart the capacitance is proportional to:

Solution:

The capacitance between two objects is, by definition, C = Q / ΔV, where Q and –Q are charges placed on the two objects and ΔV is the difference of potentials between the two objects produced by the two charges.

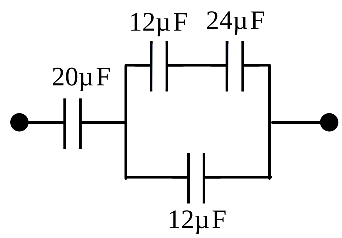
$V_1 = \frac{Q}{R_1}k + \frac{-Q}{D}k \approx \frac{Q}{R_1}k$ (since distance D between the spheres is very large)

$V_2 = \frac{-Q}{R_2}k + \frac{Q}{D}k \approx \frac{-Q}{R_2}k$ (since distance D between the spheres is very large)

$$\Delta V = \frac{Q}{R_1}k - \left(\frac{-Q}{R_2}k\right) = Qk\frac{R_2 + R_1}{R_1 R_2}$$

$$C = \frac{Q}{\Delta V} = \frac{R_1 R_2}{R_2 + R_1} \cdot \frac{1}{k}$$

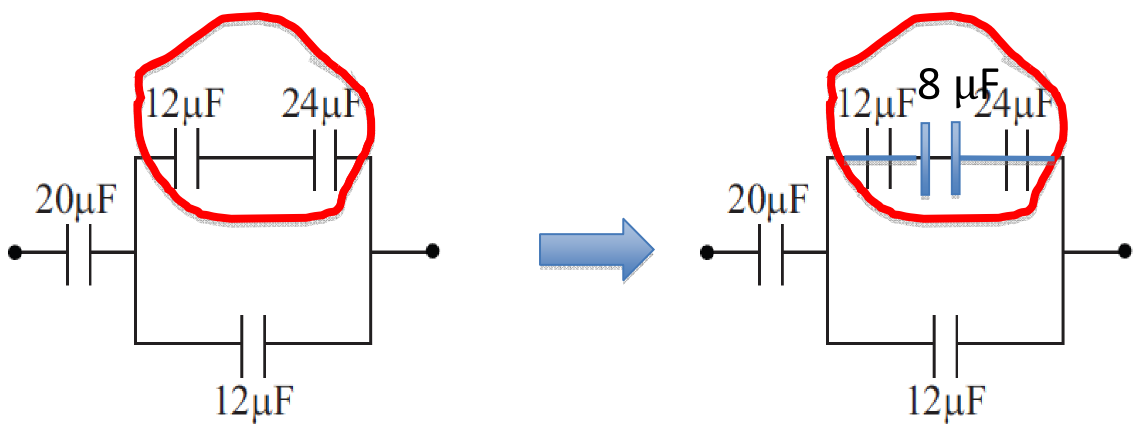
15. What is the equivalent capacitance of the combination shown?



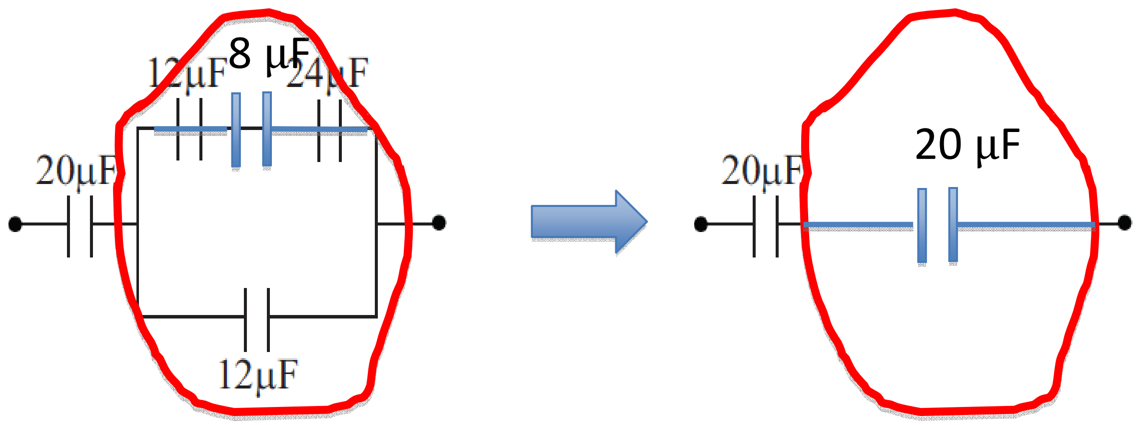
Solution:

The top two capacitors are connected in series:

$1/C = 1/12 + 1/24$. From where $C = 8\ \mu F$



Next step: 8 and 12 µF capacitors are connected in parallel $C = 8 + 12 = 20\ \mu F$



The last step: the two 20 μF capacitors are connected in series. The equivalent capacitance is 10 μF.

16. An air-filled parallel-plate capacitor has a capacitance of 3 pF. The plate separation is then tripled and a wax dielectric is inserted, completely filling the space between the plates. As a result, the capacitance becomes 6 pF. The dielectric constant of the wax is:

Solution:

Initial capacitance $C_i = \dfrac{A\varepsilon_0}{d}$

Final capacitance $C_f = \dfrac{A\varepsilon_0 \kappa}{3d}$

From where: $\kappa = 3\dfrac{C_f}{C_i} = 6$

17. A certain wire has resistance R. Another wire, of the same material, has half the length and half the diameter of the first wire. The resistance of the second wire is:

Solution:

Resistance of the first wire $R = \rho \dfrac{L}{\pi r^2}$

Resistance of the second wire $R_x = \rho \dfrac{(L/2)}{\pi (r/2)^2} = 2\rho \dfrac{L}{\pi r^2} = 2R$

18. A certain resistor dissipates 0.5 W when connected to a 3 V potential difference. When connected to a 1 V potential difference, this resistor will dissipate:

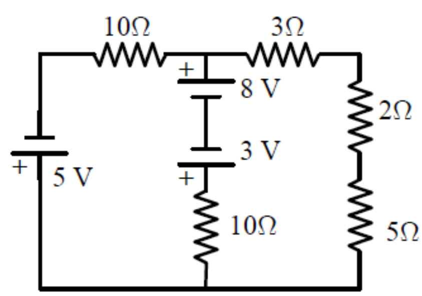
Solution:

Power dessipated when resistor R is connected to $V_1 = 3\,\text{V}: \;\; W_1 = \dfrac{V_1^2}{R}$

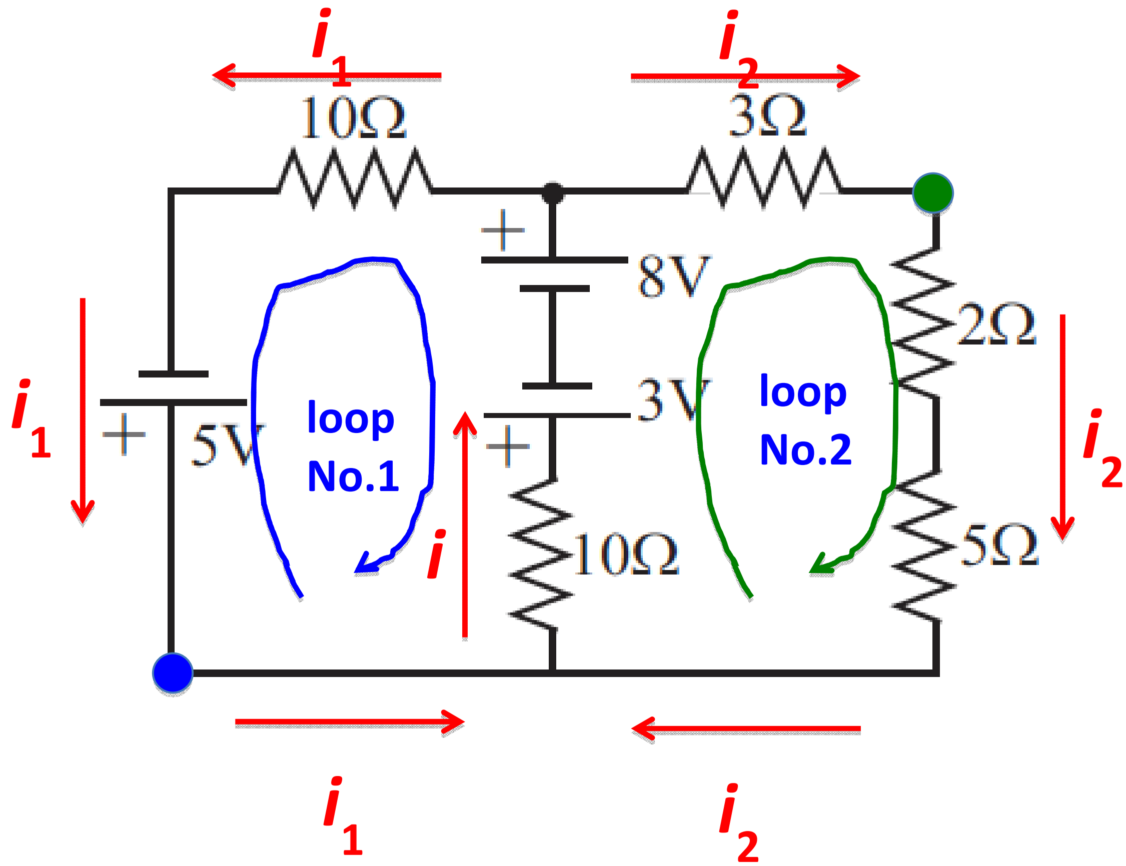
From here : $R = \dfrac{V_1^2}{W_1}$

Power dessipated by the same resistor R connected to $V_2 = 1\,\text{V}: \;\; W_2 = \dfrac{V_2^2}{R} = W_1 \dfrac{V_2^2}{V_1^2} = 0.056\ \text{W}$

19. In the figure shown, what is the current through the 5 Ω resistor?



Solution:



Junction rule for currents:	$i = i_1 + i_2$
Potential rule for the loop No.1:	$-10 \cdot i - 3 + 8 - 10 \cdot i_1 + 5 = 0$
Potential rule for the loop No.2:	$-2 \cdot i_2 - 5 \cdot i_2 - 10 \cdot i - 3 + 8 - 3 \cdot i_2 = 0$

From where one can find that: $i_2 = 0, \quad i = i_1 = 0.5 \text{ A}$

20. A 15 Ω resistor and a 16 μF capacitor are connected in series to a 12 V battery. At t = 0.6 ms after the connection is made, what is the current in the circuit?

Solution:

$$i(t) = \frac{EMF}{R} e^{-t/RC} = 0.066 \text{ A}$$