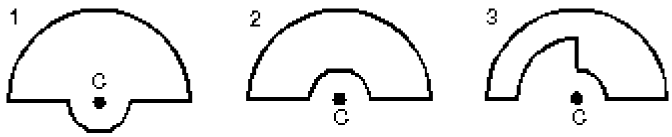
PHY2049 Exam #2 Solutions – Fall 2012

1. The diagrams show three circuits consisting of concentric circular arcs (either half or quarter circles of radii *r*, 2*r*, and 3*r*) and radial segments. The circuits carry the same current. Rank them according to the magnitudes of the magnetic fields they produce at C, least to greatest.



A) 3, 2, 1

Solution: Radial segments don't produce magnetic field at C so we only need to consider arcs. Assume the current is counter clock wise and the magnetic field to be positive pointing out of page. Magnetic field at the center from an arc ϕ of radius R is $\frac{\mu_0 i\emptyset}{4\pi R}$ so we have

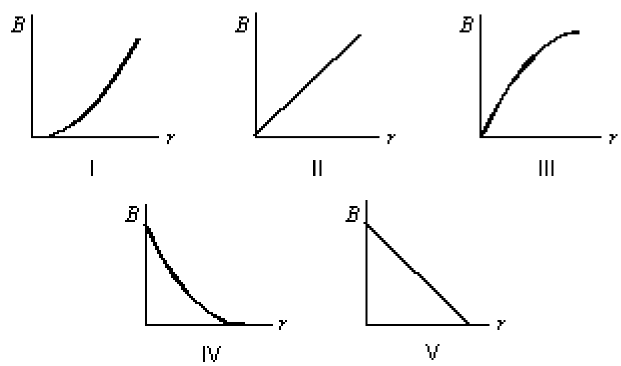
$$(1)\ B = \frac{\mu_0 i\pi}{4\pi(3r)} + \frac{\mu_0 i\pi}{4\pi r} = \frac{1}{3}\frac{\mu_0 i}{r}$$

$$(2)\ B = \frac{\mu_0 i\pi}{4\pi(3r)} - \frac{\mu_0 i\pi}{4\pi r} = -\frac{1}{6}\frac{\mu_0 i}{r}$$

$$(3)\ B = \frac{\mu_0 i\pi}{4\pi(3r)} - \frac{\mu_0 i(\frac{\pi}{2})}{4\pi r} - \frac{\mu_0 i(\frac{\pi}{2})}{4\pi(2r)} = -\frac{5}{48}\frac{\mu_0 i}{r}$$

Therefore the magnitudes of the magnetic fields at C from least to greatest are (3), (2), (1).

2. Which graph correctly gives the magnitude of the magnetic field outside an infinitely long straight current-carrying wire as a function of the distance *r* from the wire?



Solution:

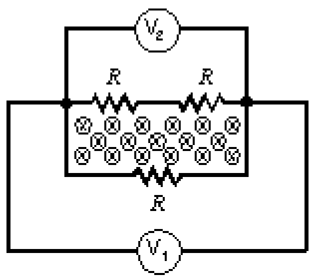
Magnetic field from an infinite long straight current-carrying wire is $\frac{\mu_0 i}{2\pi r}$ at a distance r from wire.

3. Two parallel long wires carry the same current and repel each other with a force *F* per unit length. If both these currents are doubled and the wire separation tripled, the force per unit length

becomes:
A) $4F/3$
Solution:
Magnitude of magnetic force between two current carrying wires is $\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$. If both these currents are doubled and the wire separation tripled, the force per unit length becomes

$$\frac{F'}{l} = \frac{\mu_0 (2i_1)(2i_2)}{2\pi (3d)} = \frac{4}{3}\frac{F}{l}.$$

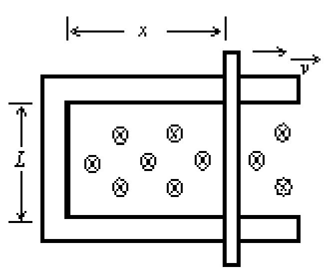
4. A changing magnetic field pierces the interior of a circuit containing three identical resistors. Two voltmeters are connected to the same points, as shown. V_1 reads 1 mV. V_2 reads:



Solution:

Because of the changing magnetic flux, i.e. induced emf, the two voltmeters don't have the same reading. There is no changing magnetic flux in the region between voltmeter 1 and R, neither between voltmeter 2 and R R so voltmeter 1 measures the voltage across R and voltmeter 2 measures the voltage across R R. Let the current through all resistors be i, using ohm's law to determine the voltage across R to be iR and that across R R to be 2iR. Therefore V_2 reads 2mV.

5. A rod with resistance *R* lies across frictionless conducting rails in a constant uniform magnetic field *B*, as shown. Assume the rails have negligible resistance. The magnitude of the force that must be applied by a person to pull the rod to the right at constant speed *v* is:



Solution:

Motional emf is $\mathcal{E} = BLv$. We find the induced current $i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$ count clock wise from Lenz's law. The magnetic force on the rod is $F = iLB = \frac{B^2L^2v}{R}$ toward left using the right hand rule. Therefore a person must pulling the rod with a force to the right with the same magnitude to make the rod moving at constant velocity.

6. An inductance L and a resistance R are connected in series to an ideal battery. A switch in the circuit is closed at time 0, at which time the current is zero. The rate of increase of the energy stored in the inductor is a maximum:
Solution:

The current in the RL circuit is given by $i = i_m\left(1 - e^{-\frac{t}{\tau}}\right)$ with τ=L/R. The energy stored in the inductor is

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}Li_m^2\left(1 - e^{-\frac{t}{\tau}}\right)^2.$$

The rate of increase of the energy is

$$P_L = \frac{dU_L}{dt} = \frac{1}{\tau}Li_m^2\left(1 - e^{-\frac{t}{\tau}}\right)e^{-\frac{t}{\tau}}.$$

To find maximum of P_L, take a derivative of it w.r.t. time and setting to zero.

$$\frac{dP_L}{dt} = -\frac{1}{\tau^2}Li_m^2\left(1 - 2e^{-\frac{t}{\tau}}\right)e^{-\frac{t}{\tau}} = 0.$$

We find

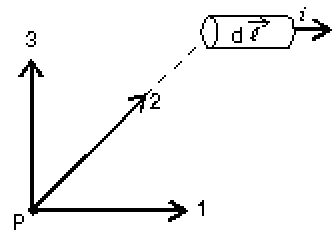
$$\left(1 - 2e^{-\frac{t}{\tau}}\right) = 0; e^{-\frac{t}{\tau}} = \frac{1}{2}; t = \tau\ln(2) = \frac{L}{R}\ln(2).$$

7. The magnetic field at any point is given by $\vec{B} = A\vec{r}\times\hat{k}$, where $\vec{r}$ is the position vector of the point and A is a constant. The net current through a circle of radius R, in the xy plane and centered at the origin is given by:
A) $2\pi AR^2/\mu_0$
Solution:

Consider circle of radius R. The magnetic field at any point on the circle is tangent to the circle with a magnitude AR. Use Ampere's law

$$\mu_0 i_{enc} = \oint \vec{B}\cdot d\vec{s} = \oint ARds = AR\oint ds = A2\pi R^2$$

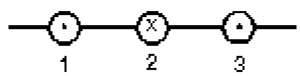
8. In the figure, the current element $i d\vec{\ell}$, the point P, and the three vectors (1, 2, 3) are all in the plane of the page. The direction of $d\vec{B}$, due to this current element, at the point P is:



Solution:

From Biot-Savart law, the magnetic field from a small segment of current carrying wire is proportional to $d\vec{l} \times \hat{r}$ where $\hat{r}$ is a unit vector from current carrying segment to P. Using the right hand rule, the cross product points into the page.

9. The diagram shows three equally spaced wires that are perpendicular to the page. The currents are all equal, two being out of the page and one being into the page. Rank the wires according to the magnitudes of the magnetic forces on them, from least to greatest.



A) 2, 1 and 3 tie

Solution:

The magnetic forces on 2 by 1 and 3 are in opposite directions, same magnitude, so they add up to zero. From the symmetry of the arrangement, the magnitudes of the magnetic forces on 1 and 3 are equal and no vanish.

10 A magnetic dipole is in a uniform magnetic field. The dipole experiences a torque of $20\text{x}10^{-24}$ Nm. The potential energy of the dipole is $15\text{x}10^{-24}$ J. What is the angle between the dipole and the magnetic field (in degrees)?

Solution:

Potential energy of a magnetic dipole in B field and the torque on dipole are

$$U = -\mu B cos\theta;\ \tau = \mu B sin\theta.$$

Divide the two equations to find

$$tan\theta = -\frac{\tau}{U} = -\frac{20 \times 10^{-24}}{15 \times 10^{-24}} = -\frac{4}{3};\ \theta = 126°$$

because the angle between magnetic dipole and magnetic field is between 0° and 180°.

Problem:

An electron and a proton both enter a region of uniform magnetic field with the same kinetic energy. They move perpendicular to the magnetic field. What is the ratio of the radius of the proton orbit to the radius of the electron orbit (r_p/r_e)?

(1) 43 (2) 5.4 (3) 72 (4) 12.5 (5) 9.8

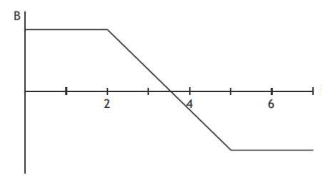
Solution:

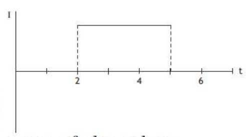
Rotation radius for a charged particle of mass *m*, charge *q*, moving with velocity *v* perpendicular to magnetic field *B* is $r = \frac{mv}{qB} = \frac{1}{qB}\sqrt{2m \cdot KE}$, where *KE* is the kinetic energy of the particle.

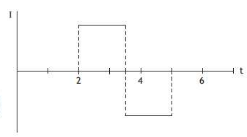
Hence, the ratio of orbit radii for the proton of mass m_p and the electron of mass m_e is $\frac{r_p}{r_e} = \sqrt{\frac{m_p}{m_e}} \approx 43$

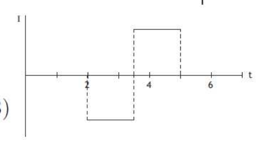
Problem:

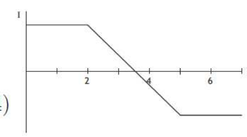
A magnetic field is perpendicularly incident through a circle of wire with resistance R. The magnetic field varies in strength as shown. Which is a possible description of the current induced in the wire?



(1) 

(2) 

(3) 

(4) 

(5) none of the others

Solution: Induced EMF in the loop is $EMF = -\frac{d\Phi_B}{dt} = -\frac{dB}{dt}A$ and must result in current

$$I = \frac{|EMF|}{R} = \left|\frac{dB}{dt}\right|\frac{A}{R}$$, which is depicted in (1).

Problem:

A circular loop of wire with radius 6.0cm and resistance 350mΩ is in the plane of the page. It is sitting in a magnetic field directed perpendicular to the page. The magnetic field strength varies in time and generates a counterclockwise current of 3.2A. Which of the following answers is a possibility for the direction and rate of change of the magnetic field?

(1) into page, $99.0T/s$ (2) out of page, $2.97T/s$ (3) out of page, $99.0T/s$ (4) into page, $2.97T/s$ (5) none of the others

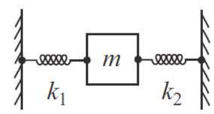
Solution: Induced EMF in the loop is $EMF = -\frac{d\Phi_B}{dt} = -\frac{dB}{dt}A$ and must result in current

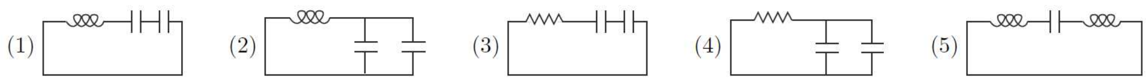
$$I = \frac{|EMF|}{R} = \left|\frac{dB}{dt}\right|\frac{A}{R} = \left|\frac{dB}{dt}\right|\frac{\pi r^2}{R}$$, from where $\left|\frac{dB}{dt}\right| = \frac{IR}{\pi r^2} = 99.0$ T/s.

If the change of the magnetic field is into the page, it leads to the change of the magnetic field flux into the page as well. One aligns the thumb of the right hand with the **opposite of the change of flux** (remember the minus sign above) and finds that the right hand fingers curled around the thumb point counterclockwise. Therefore, the induced EMF and, hence, current are directed counterclockwise, as stated in the problem. Thus, the guess on the direction of the magnetic field change is correct.

Problem:

Consider the mechanical system consisting of two springs and a block, as shown. Which one of the five electrical circuits is the analog of the mechanical system?

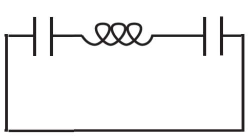




Solution: The mechanical electrical analogies are:

- capacitor ⟵⟶ spring (they store energy in a static form)
- inductor ⟵⟶ massive block (they store energy in a kinetic form when the current flows or the block moves).

These analogies make the following circuit:



which is equivalent to (1)

Problem:

An LC circuit has a capacitance of 30μF and an inductance of 15 mH. At time $t = 0$ the charge on the capacitor is 10μC and the current is 20 mA. The maximum current is:

(1) 25 mA (2) 20 mA (3) 18 mA (4) 35 mA (5) 42 mA

Solution: The LC circuit oscillates with $\omega_0 = 1/\sqrt{LC}$:

$$q(t) = q_{\max} \cdot \sin(\omega_0 t + \varphi)$$
$$I(t) = \frac{dq}{dt} = q_{\max}\omega_0 \cdot \cos(\omega_0 t + \varphi)$$

where φ is an unknown initial phase.

Since we know charge on the capacitor and the current in the circuit at t=0, we can write:

$$q(0) = q_{\max} \cdot \sin(\varphi)$$
$$I(0) = q_{\max}\omega_0 \cdot \cos(\varphi) = I_{\max} \cdot \cos(\varphi)$$

The ratio of the two above equations allows one to find the unknown phase: $\frac{q(0)}{I(0)} = \frac{q_{\max} \cdot \sin(\varphi)}{q_{\max}\omega_0 \cdot \cos(\varphi)} = \frac{\tan(\varphi)}{\omega_0}$

, from where $\tan(\varphi) = \frac{q(0)}{I(0)}\omega_0$

Once we know the phase, we can find the maximum charge: $I_{\max} = \frac{I(0)}{\cos(\varphi)}$

Problem:

An RLC circuit has a resistance of 200Ω and an inductance of 15 mH. Its oscillation frequency is 7000 Hz. At time $t = 0$ the current is 25 mA and there is no charge on the capacitor. After five complete cycles the current is:

(1) 2.1×10^{-4}A (2) 1.8×10^{-6}A (3) 0 (4) 2.3×10^{-3}A (5) 2.5×10^{-2}A

Solution: The RLC circuit oscillates with ω, while oscillations get damped with the characteristic time τ=2L/R:

$$q(t) = q_0 \cdot e^{-t/\tau} \cdot \sin(\omega t)$$

$$I(t) = \frac{dq}{dt} = -q_0 \frac{1}{\tau} \cdot e^{-t/\tau} \sin(\omega t) + q_0 \omega \cdot e^{-t/\tau} \cos(\omega t)$$

Note that there is not an additional phase included in sine/cosine functions because charge has to be zero at t=0. From the second equation, we find $I(0) = q_0 \omega$, which allows us to find q_0.

Therefore, $I(t) = -\frac{I(0)}{\omega} \frac{1}{\tau} \cdot e^{-t/\tau} \sin(\omega t) + I(0) \cdot e^{-t/\tau} \cos(\omega t)$.

At t=5T=5/f (five full periods), sine in the above equation is again zero and cosine = 1.

Therefore, $I(t) = I(0) \cdot e^{-5/(f\tau)}$

Problem:

An RLC series circuit has $L = 100$ mH and $C = 1\mu$F. It is connected to a 1000-Hz source emf is found to lead the current by 75°. The value of R is:

(1) 126 Ω (2) 15.6 Ω (3) 175 Ω (4) 1750 Ω (5) 1810 Ω

Solution: The differences in phases between current and the generator EMF

$\tan\phi = \frac{X_L - X_C}{R}$, where $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.

Therefore, $R = \frac{X_L - X_C}{\tan\phi} = \frac{\omega L - \frac{1}{\omega C}}{\tan\phi} = \frac{(2\pi f)L - \frac{1}{(2\pi f)C}}{\tan\phi}$

Problem:

An ac generator producing 10 V (rms) at 200 rad/s is connected in series with a 50-Ω resistor, a 400-mH inductor, and a 200-μF capacitor. The rms voltage (in volts) across the inductor is:

(1) 10.8 (2) 3.4 (3) 6.7 (4) 10.0 (5) 2.5

Solution: The rms current in an RLC circuit connected in series with an EMF generator is

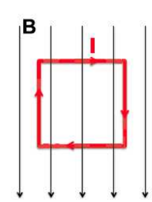
$$I_{RMS} = \frac{EMF_{RMS}}{\sqrt{R^2 + (X_L - X_C)^2}}.$$

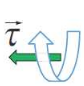
The rms voltage across the inductor is then $V_{RMS} = I_{RMS} X_L = EMF_{RMS} \dfrac{X_L}{\sqrt{R^2 + (X_L - X_C)^2}}$

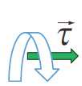
In these equations, $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}$

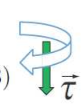
Problem:

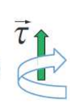
A loop with a current is placed in a magnetic field as shown. What is the direction of torque $\vec{\tau}$ on the loop?



(1) 

(2) 

(3) 

(4) 

(5) the torque is zero

Solution: The direction of the magnetic field force exerted on each of the four sides with currents can be found from $\vec{F} = L\vec{i} \times \vec{B}$ using the right hand rule. The left and right sides of the loop do not experience any force (the current is parallel to the magnetic field). The top side experiences a force into the page, the bottom side—out of the page. This leads to a torque as shown in (1).

The other way to solve it is to use $\vec{\tau} = \vec{\mu} \times \vec{B}$, where μ is the magnetic dipole moment, whose direction is defined by the right hand rule (fingers along the current in the loop, the thumb gives the direction of μ). And then, again, one need apply the right hand rule again to find the direction of the torque.

Problem:

A proton executes helical motion in a uniform B field at a frequency of 820 kHz. It moves at an angle of 30 degrees relative to the direction of the field. What is the magnitude of the B field?

(1) 0.054 T (2) 0.11 T (3) 0.17 T (4) 0.34 T (5) 0.017 T

Solution: Rotation radius for a charged particle of mass *m*, charge *q*, moving with velocity $v_\perp$ perpendicular to magnetic field *B* is $r = \frac{mv_\perp}{qB}$. The period of rotation $T = \frac{2\pi r}{v_\perp} = 2\pi \frac{m}{qB}$ is independent of particles velocity and its orientation with respect to the magnetic field. From here we find $B = 2\pi f \frac{m}{q}$