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Exam 1 Solutions

1. Two identical conducting spheres A and B carry charges Q and $2Q$ respectively. They are separated by a distance much larger than their diameters. A third identical conducting sphere C is uncharged. Sphere C is first touched to A, then to B, and finally removed. As a result, the electrostatic force between A and B, which was originally F, becomes:

Answer: $5F / 16$

Solution: The initial force between the two charges separated by a distance d is $F = 2kQ^2 / d^2$. After touching, the charges become $Q/2$ and $5Q/4$ and the force is $5kQ^2 / 8d^2 = 5F / 16$.

2. A particle with charge 2 μC is placed at the origin. Another particle, with charge 4 μC, is placed 2 m from the origin on the x-axis, and a third particle, with charge 2 μC, is placed 2 m from the origin on the y-axis. The magnitude of the force on the particle at the origin is:

Answer: 2.0×10^{-2} N

Solution: The force due to the first particle is $kq_1q_2 / d_{12}^2 = 0.0180\,\text{N}$ in the x direction and the force from the second particle is $kq_1q_3 / d_{13}^2 = 0.0090$ in the y direction. The magnitude of the net force is then $\sqrt{0.018^2 + 0.009^2} = 0.020\,\text{N}$.

3. A particle with positive charge Q is on the y-axis a distance $2a$ from the origin, and a particle with positive charge q is on the x-axis a distance d from the origin. The value of d for which the x component of the force on the second particle is the greatest is:

Answer: $\sqrt{2}a$

Solution: The x component of the force is $kQq\cos\theta / \left(4a^2 + d^2\right)$, where $\cos\theta = d / \sqrt{4a^2 + d^2}$. Thus we want to maximize the quantity $kQqd / \left(4a^2 + d^2\right)^{3/2}$. Setting the first derivative to 0 yields $d = \sqrt{2}a$.

4. A 72 nC charge is located at x = 1.50 m on the x-axis and an 8.0 nC charge is located at x = 3.5 m. At what point on the x-axis is the electric field zero?

Answer: 3.0 m

Solution: Since the charges are the same sign, the point where $E_x = 0$ is clearly between them and closer to the 8.0 nC charge. The condition for $E_x = 0$ is $kq_1/\left(x-x_1\right)^2 = kq_2/\left(x_2-x\right)^2$. Cross multiplying and using $\sqrt{q_1/q_2}=3$ we obtain $3\left(x_2-x\right)=x-x_1$ which yields x = 3.0 m.

5. A point charge of 3.0 μC is located on the y-axis at y = 4.0 m, and another point charge of 6.0 μC is located on the x-axis at x = 5.0 m. What is the magnitude of the electric field at the position (5.0 m, 4.0 m)?

Answer: 3.5×10^3 N/C

Solution:

6. An electron gun sends electrons through a region with an electric field of 1.5×10^4 N/C for a distance of 2.5 cm. If the electrons start from rest, how long does it take for the electrons to traverse the gun?

Answer: 4.4 ns

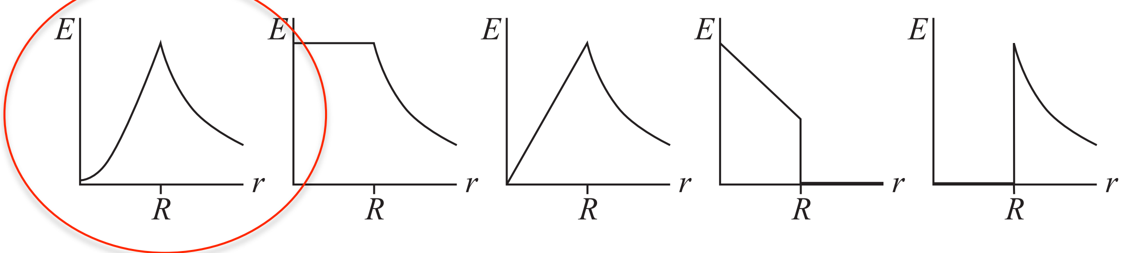
Solution: This is a situation involving uniform acceleration where $d=\frac{1}{2}at^2=\frac{1}{2}\left(Ee/m_e\right)t^2$. Solving yields t = 4.4 ns.

7. A 9.5-cm radius hemisphere contains a total charge of 3.3×10^{-7} C. The flux through the rounded portion of the surface is 4.9×10^4 N·m²/C. The flux through the flat base is:

Answer: -1.2×10^4 N·m²/C

Solution: From Gauss' law, the total flux is Q/ε_0, where $Q = 3.3 \times 10^{-7}$ C is the total charge. The total flux is 3.7×10^4 N·m²/C, which means that the flux through the flat portion must be -1.2×10^4 N·m²/C.

8. A solid insulating sphere of radius R contains a positive charge that is distributed with a volume charge density that does not depend on angle but does increase with distance from the sphere center. Which of the graphs below correctly gives the magnitude E of the electric field as a function of the distance r from the center of the sphere?



Answer: The first one

Solution: Outside the sphere, the E field falls like the square of the distance from the center. By Gauss' law, if the charge distribution were constant, then the E field would rise linearly from the center ($Q_{\text{enc}} \propto r^3$ and $E = kQ_{\text{enc}} / r^2$). Since the volume charge density *increases* from the center, the enclosed charge rises more slowly from the center, which is described only by the first graph.

9. Four charges are placed along a straight line each separated by a distance L from its neighbor. The order of the charges is $+Q$, $-Q$, $+Q$, $-Q$. What is the total potential energy of the system (relative to infinity)?

Answer: $-7kQ^2 / 3L$

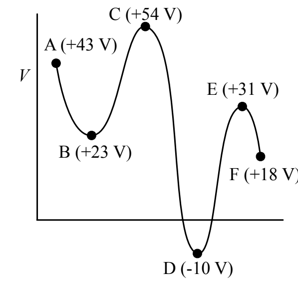
Solution: Let the charges be ordered 1, 2, 3, 4. There are 6 combinations: (12, 23, 24), (13, 24), 14 which give contributions $3\left(-kQ^2 / L\right) + 2\left(+kQ^2 / 2L\right) + \left(-kQ^2 / 3L\right) = -7kQ^2 / 3L$.

10. Three identical particles of charge 1.5 mC and mass 250 g are held in place at the corners of an equilateral triangle of side 15 cm. They are then released simultaneously and fly apart. What are their velocities at the instant they are 60 cm from one another?

Answer: 900 m/s

Solution: Conservation of total energy gives $0 + 3\left(kq^2 / d_0\right) = 3\left(\frac{1}{2}mv^2\right) + 3\left(kq^2 / d_1\right)$, where d_0 and d_1 are the initial and later separations. Solving yields $v = 900$ m/s.

11. Consider a particle with charge −20 μC moving to the left from position E in the figure. How much kinetic energy does it need to reach B?



Answer: 820 μJ

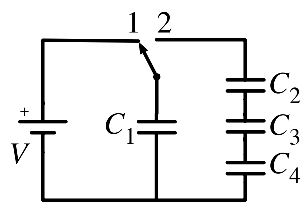
Solution: The charge is negative so the particle leaving E must first clear the potential barrier at D before arriving at B (if the charge were positive, then it would have to climb over the potential energy barrier at C). The kinetic energy required to clear the D potential energy barrier is (31 + 10) × 20 = 820 μJ. (This is the same as the "activation energy" needed by two chemical reactants to clear a potential energy barrier before taking part in an exothermic reaction.) The kinetic energy of the particle when it arrives at point B is $\Delta K = -\Delta U = -q\Delta V = -20\times(23-31) = 160\,\mu\text{J}$.

12. Two isolated conducting spheres are separated by a large distance. Sphere 1 has a radius of R and an initial charge $3Q$ while sphere 2 has a radius of $3R$ and an initial charge $7Q$. A very thin copper wire is now connected to the spheres to allow charge to flow between the spheres. How much charge will be transferred from sphere 2 to sphere 1? (Note that the charge transferred can be positive, negative or zero.)

Answer: $-Q / 2$

Solution: The requirement that the potential be constant on the conductors leads to the condition $kQ_1 / R = kQ_2 / 3R$ or $Q_2 = 3Q_1$. The total charge is $10Q$, which from the constant potential condition must be split as $Q_1 = 2.5Q$ and $Q_2 = 7.5Q$. Since the initial charge on sphere 1 is $3Q$, the charge transferred to it must be $-Q / 2$.

13. All capacitors are identical in the figure. Capacitor C_1 is charged to 48 μC by the emf source when the switch is in position 1. The switch is then moved to position 2 and the charge redistributes among all the capacitors. After this redistribution, the charge on C_1 is:



Answer: 36 μC

Solution: When the switch is moved to position 2, charge on the positive plate moves to the other capacitors until the voltages balance. The equivalent capacitance of the 3 capacitors in series is $C / 3$. Let Q_0 = 48 μC be the initial charge on C_1 and ΔQ be the charge transferred to the three uncharged capacitors. The equal potential condition then yields $(Q_0 - \Delta Q)/C = 3\Delta Q / C$ or $\Delta Q = 12\,\mu\text{C}$. Thus the remaining charge on C_1 is 48 – 12 = 36 μC.

14. Two rectangular parallel plates with dimensions 10 cm × 15 cm are given charges of equal magnitudes 0.80 μC but opposite signs. The electric field within the dielectric material filling the space between the plates is 1.4×10^6 V/m. What is the dielectric constant of the material?

Answer: 4.3

Solution: The electric field near a conducting surface is $E = \sigma / \varepsilon_0$, where $\sigma = Q / A$ is the surface charge density, with Q = 0.80 μC and $A = 0.0015\text{m}^2$. This yields $E = 6.03 \times 10^6$ V/m. The smaller observed electric field implies $\kappa = 6.03 \times 10^6 / 1.4 \times 10^6 = 4.3$.

15. A proton located at x = 1 m is released along the positive x direction in a electric potential of the form $V(x) = 5 - 3/x$, where x is measured in meters and V is measured in volts. What is the x component of the force acting on the proton at x = 3?

Answer: -5.3×10^{-20} N

Solution: The electric field can be calculated from the potential using the basic relation $E_x = -\partial V / \partial x = -3 / x^2 = -1/3$ V/m. The force is $F_x = E_x e = -5.3 \times 10^{-20}$ N.

16. A uniform electric field of 5,000 V/m is directed along the $-y$ direction. The potential at y = 5.0 m on the y-axis is 25,000 V. What is the change in the potential energy of a proton (in J × 10^{-15}) when it is moved from the point (3.0,5.0) to the point (3.0,2.0)?

Answer: –2.4

Solution: The change in potential energy is related to the (constant) electric field by $\Delta U = -e\mathbf{E} \cdot \Delta\mathbf{x}$. Here $\mathbf{E} = -5000\hat{\mathbf{j}}$ and $\Delta\mathbf{x} = -3\hat{\mathbf{j}}$. The change in potential energy is thus $\Delta U = -e \times 5000 \times 3 = -2.4 \times 10^{-15}$ J.

17. A wire of length L has a resistance of 32 Ω. The wire is uniformly stretched to four times its original length, maintaining constant volume. If a length L is now cut from the stretched wire, what is the resistance of the piece that was cut off?

Answer: 128 Ω

Solution: Stretching the wire at constant volume gives 4 times the length and 1/4 the cross sectional area, giving a total resistance of $32 \times 4 \times 4 = 512\Omega$. The resistance of 1/4 of the new wire is $512 \times \frac{1}{4} = 128\Omega$.

18. A heating element is made by maintaining a potential difference of 72.0 V across a Nichrome wire with diameter 2.06 mm. Nichrome has a resistivity of 5.00×10^{-6} Ω·m. If the element dissipates 5470 W, what is its length?

Answer: 63 cm

Solution: The power dissipated by the wire is $P = i^2 R = V^2 / R$, where V = 72 is the applied emf and P = 5470 W is the power. The resistance is given by $R = \rho L / A$, where ρ is the resistivity of the wire, L is its length and $A = \pi r^2$ is its cross sectional area. Solving for the length gives L = 63 cm.

19. Positive charge Q is distributed uniformly throughout an insulating sphere of radius R, centered at the origin. A particle with a positive charge $3Q$ is placed at $x = 2R$ on the x-axis. The magnitude of the electric field at $x = R / 2$ on the x axis is:

Answer: $5Q / 24\pi\varepsilon_0 R^2$

Solution: The net electric field has two components at $R / 2$, an outward directed field from the uniform charge and an inward pointing field from the charge. The field from the charged ball at the surface is $Q / 4\pi\varepsilon_0 R^2$. Since only 1/8 of the full charge is inside $r = R / 2$, its field there is $E_{1x} = Q / 8\pi\varepsilon_0 R^2$. The contribution from the single charge at $R / 2$ is $E_{1x} = -Q / 3\pi\varepsilon_0 R^2$, leaving a net contribution of $-5Q / 24\pi\varepsilon_0 R^2$.

20. A piece of wire is used as a heating element of a household heater. Its energy dissipation rate decreases 3.75% when the temperature of the wire is raised 100° C. What is the temperature coefficient of resistivity of the wire material?

Answer: $3.9 \times 10^{-4}\ {}^\circ\mathrm{C}^{-1}$

Solution: The power dissipated is $P = i^2 R = V^2 / R$ when it is connected to a household electrical outlet. The resistor has a fractional gain in resistance caused by the temperature increase of $\Delta T = 100$, which we call $\alpha\Delta T$. Thus we have $1/\left(1+\alpha\Delta T\right) = 0.9625$, or

$\alpha = 3.9\times10^{-4}\,{}^\circ/\mathrm{C}$.