1. $R = \rho \dfrac{L}{A} = 2.0 \times 10^{-6}\Omega\text{m} \times \dfrac{1 \text{ m}}{\pi(1.0 \times 10^{-3} \text{ m})^2} = 0.64 \ \Omega,$

$\mathbf{P} = \mathbf{I}^2 R = (1\text{A})^2 \times 0.64 \ \Omega = 0.64 \text{ W}$

2. $j = nev = \sigma E = \sigma \dfrac{V}{d}$

The drift speed *v* of the electrons is proportional to the strength of the electric field, which is inversely proportional to the length *d* of the conductor along which the potential difference is applied. Thus b(d=L) > c(d=2L) > a(d=3L).

3. $E = \dfrac{V}{d}, \qquad A\left(E = \dfrac{V}{L}\right) = B\left(E = \dfrac{2V}{2L} = \dfrac{V}{L}\right) > C\left(E = \dfrac{2V}{3L}\right)$

4. The currents in the two section are the same. Thus, $\mathbf{I} = \dfrac{\mathbf{14 \ C}}{\mathbf{30 \ sec}} = \mathbf{0.467 \ A}$

5. $V_3 = iR_3 = 12 \text{ A} \times 3 \ \Omega = 36 \text{ V}, \qquad V_1 = V_{AB} - V_3 = 78 \text{ V} - 36 \text{ V} = 42 \text{ V},$

$i_1 = \dfrac{V_4}{R_1} = \dfrac{42 \text{ V}}{1 \ \Omega} = 42 \text{ A}, \qquad i_2 = 42 \text{ A} - 12\text{A} = 30 \text{ A}$ (V_4 is a typo. Meant to be V_1.)

$V_2 = i_2 R_2 = 30 \text{ A} \times 2\Omega = 60 \text{ V}, \qquad V_{2+box} = 36 \text{ V}$ because $V_{2+box} = V_3$.

Thus the “box” is a 24 V battery.

6. $R_{eq} = \dfrac{3R}{4} + \dfrac{R}{2} = \dfrac{5R}{4}, \qquad \varepsilon = 2i \times \dfrac{5R}{4} = \dfrac{5}{2} iR, \qquad i = \dfrac{2\varepsilon}{5R}$ (Note: *i* is defined to be the current through either of the two bottom resistors. So 2*i* is the total current.)

$I_A = 2i \times \dfrac{3}{4} - i = \dfrac{i}{2} = \dfrac{2\varepsilon}{5R} \times \dfrac{1}{2} = \dfrac{\varepsilon}{5R}$ ($2i \times \frac{3}{4}$ flows through *R* at the top right, whereas *i* flows through *R* below it. The difference must be I_A that flows through the ammeter.)

7. Rate of energy transfer from an ideal battery to a resistor or a combination of resistors is given by :

$$P = \mathcal{E}\, i = \frac{\mathcal{E}^2}{R_{net}} \tag{1}$$

where R_{net} is the net resistance attached across the battery. For maximum power transfer, we should minimize R_{net}. In the problem, this is achieved by making R_2 zero. With $R_1 = 16\Omega$.

$$P_{max} = \frac{\mathcal{E}^2}{R_1} = 9\,\mathrm{W}. \tag{2}$$

8. Time constant of the RC circuit is $\tau = RC = 15\,\mathrm{s}$. Charge at any time on the capacitor while charging is given by

$$q(t) = q_0(1 - e^{-t/\tau}) \tag{3}$$

where $q_0 = C\mathcal{E} = 90\,\mu\mathrm{C}$ is the steady state charge. Rate at which charge on the capacitor increases is

$$\frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau}. \tag{4}$$

At $t = 2\,\mathrm{s}$ Eq (5) yields $5.3 \times 10^{-6}\mathrm{A}$.

9. Force on a charged particle moving in an external magnetic and electric field is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = e[4\hat{k} + 2000\hat{\jmath} \times (-0.003\hat{\imath})] = 1.6 \times 10^{-18}\hat{k}\,\mathrm{N} \tag{5}$$

10. The trajectory of the charged particle is a helix. Let us decompose the velocity vector parallel and perpendicular to the magnetic field :

$$v_{\parallel} = v\cos(70^\circ) \qquad v_{\perp} = v\sin(70^\circ) \tag{6}$$

where $v = \sqrt{\frac{2K}{m}}$, K is the kinetic energy (use conversion 1 eV $= 1.6 \times 10^{-19}$J, which can be recalled by noting that $e = 1.6 \times 10^{-19}$C and $U = qV$). $v_{\parallel}$ is responsible for the pitch length and $v_{\perp}$ is responsible for the circular motion with the magnetic field as the central axis. Time period of one cycle is

$$T = \frac{2\pi m}{qB} \tag{7}$$

during this period $v_{\|}$ causes the particle to move along the magnetic field, which is the pitch. Therefore pitch length p is

$$p = v_{\|}\, T = 3.2 \times 10^{-3}\mathrm{m}. \tag{8}$$

11. With no current in wire C, the magnetic field at the location of wire B due to wire A points vertically down, therefore the force F on wire B is to the left. For wire B to experience a force $3F$ in the same direction as F after turning on the current in wire C, the magnetic field due to wire C should also point vertically down at the location of B. Therefore wire C must carry current **out of the page**. The force on B due to A is

$$F = \frac{\mu_0 I^2}{2\pi d}. \tag{9}$$

To experience a force $3F$, the current in wire C should be $2I$, because then the additional force on B due to wire C is $\frac{2\mu_0 I^2}{2\pi d}$, which leads to a net force of $3F$ to the left.

12. Magnetic torque on a current carrying loop due to external magnetic field is

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{10}$$

where $\vec{\mu}$ is the dipole moment vector of the loop, which is perpendicular to the plane of the loop. Therefore for maximum torque, the loop should be parallel to the magnetic field, since $\tau = \mu B \sin(\theta)$ is maximum for $\theta = 90°$.

13. At the center of the pipe the net magnetic field is only due to the wire (magnetic field due to the pipe is zero using Ampere's law), whose magnitude is $\frac{\mu_0 i_w}{2\pi(3R)}$. At P the magnetic field is due to the pipe as well as the wire

$$B_P = B_{pipe} \text{ (to the left)} + B_{wire} \text{ (same direction as at the center of the pipe)} \tag{11}$$

Magnetic field due to the pipe at P can be found using Ampere's law, pick an Ampere's loop which is a circle of radius $2R$ and centered at the pipe's center.

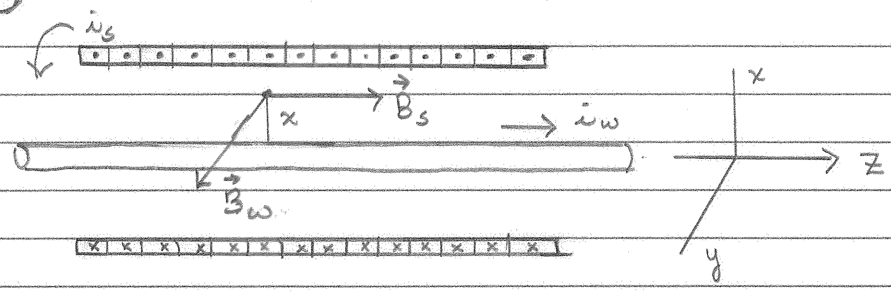
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{enc} \Rightarrow B_{pipe} = \frac{\mu_0 I}{2\pi(2R)} \tag{12}$$

where I is the total current through the pipe. If the magnetic field at the center of the pipe has to be in the opposite direction of the magnetic field at point P then the magnetic field due to the wire must be to the right, which means the wire must carry current out of the page. The magnitude is found by equating the magnitudes of the magnetic fields

$$\frac{\mu_0 i_w}{2\pi(3R)} = \frac{\mu_0 I}{2\pi(2R)} - \frac{\mu_0 i_w}{2\pi R} \Rightarrow i_w = \frac{3}{8} I = 3.75 \text{ mA} \tag{13}$$

3, PHY 2049 Midterm 2 Solutions

(14)



- A side view of a potential configuration for the solenoid, carrying current i_s, and the wire, carrying current i_w, is shown above, along with the directions of magnetic fields they produce.

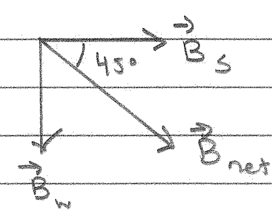
- Using the coordinate system shown, these magnetic fields are given by:

$$\vec{B}_s = \mu_0 i_s n \hat{k} \qquad \vec{B}_w = \frac{\mu_0 i_w}{2\pi x} \hat{j}$$

- The net magnetic field is given by
$$\vec{B}_{net} = \vec{B}_s + \vec{B}_w,$$

and by looking at the figure, this will be at 45° from the axis whenever

$$|\vec{B}_s| = |\vec{B}_w|$$



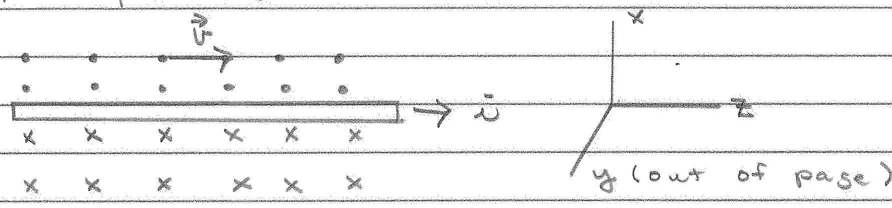
or, when $\mu_0 i_s n = \dfrac{\mu_0 i_w}{2\pi x}$, x being the radial distance. Therefore,

$$x = \frac{i_w}{i_s}\frac{1}{2\pi n} = \frac{(6\ A)}{(20\times10^{-3}\ A)\left(2\pi\ \frac{10\ \text{turns}}{\text{cm}}\right)} = \boxed{4.8\ \text{cm}}$$

(15)

- A particle moving in a magnetic field experiences a force

$$\vec{F} = q\vec{v} \times \vec{B}$$



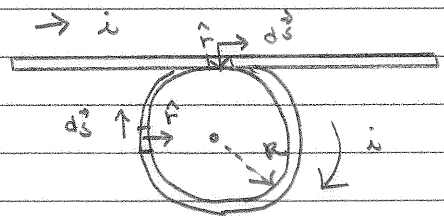
From the figure, $\vec{B}$ of the wire points in the $+\hat{j}$ direction (out of the page), so that

$\vec{v} \times \vec{B} \sim \hat{k} \times \hat{j}$ points in the $-\hat{i}$ direction (downward).

Since the charge q is negative this implies the Force is in the:

$\boxed{+\hat{i} \text{ direction (upward)}}$

(16) • Using the Biot-Savart Law, we know the direction of $\vec{B} \sim d\vec{s} \times \hat{r}$



Looking at the figure, we find **both** the wire and the loop produce magnetic fields **into** the page.

- The net magnetic field will therefore be

$$\vec{B} = (|\vec{B}_{wire}| + |\vec{B}_{loop}|), \text{ into the page}$$

$$= \left(\frac{\mu_0 i}{2\pi R} + \frac{\mu_0 i (\cancel{2}\pi)}{\cancel{4}_{2} \pi R} \right), \text{ into the page}$$

$$= \boxed{\frac{\mu_0 i}{2\pi R}(\pi + 1), \text{ into the page}}$$

(17) Here we have a basic RL circuit operating in two modes:

- 1st, the battery gives rise to a current in the circuit

$$i(t) = \mathcal{E}/R\,(1 - e^{-t/\tau_L}) \quad , \tau_L = L/R$$

- The switch is thrown and the current decays after the battery is disconnected

$$i(t) = i_0\, e^{-t/\tau_L}$$

- The i_0 for the decay will be the maximum current (current after a long time) from just before the switch is thrown:

$$i(t \to \infty) = \mathcal{E}/R \quad (\text{rise})$$
$$= i_0$$

Doubling R will therefore half the initial current,

$$R \to 2R \Rightarrow i_0 \to i_0/2$$

- However, after, doubling both L and R will give rise to the same time constant

$$L \to 2L \, , \, R \to 2R \Rightarrow \tau_L \to \frac{2L}{2R} = \tau_L$$

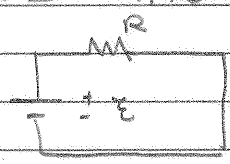
- This means the new curve should also have the same half-life, or reach half its initial value at the same time. Looking at the curves this most closely resembles $\boxed{\text{Curve b}}$

(18)

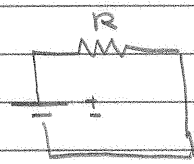
• Recall that for an RL circuit connected to a battery, the current,

$$i(t) = i_{max}(1 - e^{-t/\tau_L})$$ implies L

acts as a short in the circuit after a long time. Therefore, for circuits (1) and (2), (3) after a long time, the circuits will look like

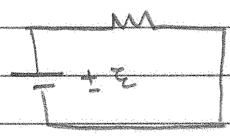


(1)



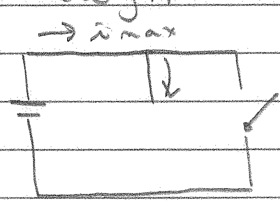
(2)

, and have the same maximum current i_{max}.



(3)

• After the switch is opened, the inductor will instead act as an open switch. This initial current will then flow through the branches containing R:

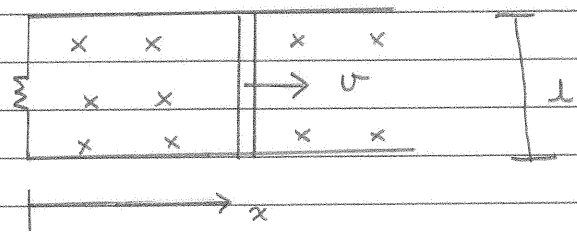


• Since the i_{max} is the same for all 3 circuits, we can see that

$$i_1 = i_2 > i_3,$$

as for circuit 3, the same i_{max} must split between the 2 branches.

(19)



- From Faraday's Law, we can calculate the induced EMF in the loop,

$$|\mathcal{E}| = \left|\frac{d}{dt}\Phi_B\right| = \left|\frac{d}{dt}\int \vec{B}\cdot d\vec{a}\right| = \left|\frac{d}{dt}(BA)\right|$$

$$= \left|\frac{d}{dt}(Blx)\right| = Bl\frac{dx}{dt} = Blv,$$

for the constant $\vec{B}$ field shown.

- The current is then $i = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$

$$= \frac{(0.80\,T)(1.6\,m)(5.0\,m/s)}{96\,\Omega} = \boxed{0.067\,A}$$

(20) For an RL circuit connected to a battery, we know that

$$i_{max} = \mathcal{E}/R$$

- We also know that the energy of an inductor is given by

$$u = \frac{1}{2}Li^2 \Rightarrow u_{max} = \frac{1}{2}L\,i_{max}^2$$

$$u_{max} = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2 \Rightarrow R^2 = \frac{1}{2}\frac{L\mathcal{E}^2}{u_{max}}$$

or $$R = \sqrt{\frac{1}{2}\frac{L\mathcal{E}^2}{u_{max}}} = \sqrt{\frac{1}{2}\frac{(10H)(20V)^2}{80\,J}} = \boxed{5\,\Omega}$$