Solutions to PHY2049 Exam 1 (Oct. 3, 2017)

Problem 1: *Three charges form an equilateral triangle of side length d. The top charge is $q_3 = -Q$ and is located on the y-axis, while the bottom two are $q_1 = q_2 = 4Q$ and are located on the x-axis. What is the net force acting on q_3?*

Problem 1 Solution: The magnitude of the force between q_3 and each of the two other charges is

$$F = k\frac{4Q^2}{d^2}$$

Both foces are directed towards q_1 and q_2. In vector notation, these directions are

$$\frac{\vec{F}}{|\vec{F}|} = \pm\sin(30°)\hat{\imath} - \cos(30°)\hat{\jmath}$$

So the sum is:

$$\vec{F}_{net} = -2k\frac{4Q^2}{d^2}\cos(30°)\hat{\jmath} = -2k\frac{4Q^2}{d^2}\sqrt{\frac{3}{4}}\hat{\jmath} = -\frac{4kQ^2}{d^2}\sqrt{3}\hat{\jmath}$$

Problem 2: *Two protons are separated by a distance of $d = 10^{-10}$m. What is the magnitude of the acceleration (in m/s) of each proton immediately after both particles are released from rest?*

Problem 2 Solution: The Coulomb force is pushing the protons apart and the acceleration of the proton can be calculated from

$$F = k\frac{e^2}{d^2} = ma \qquad a = 1.38 \times 10^{19}\mathrm{m/s}^2$$

Problem 3: *Two identical conducting spheres A and B carry identical charges. They are separated by a distance much larger than their diameters. The magnitude of the electrostatic force between the spheres is F. A* *third, identical conducting sphere is first touched to earth ground and then touched to sphere A and moved away. The magnitude of the electrostatic force between spheres A and B is now:*

Problem 3 Solution: The force is proportional to the product of the charges on the two spheres. Touching sphere A with a discharged sphere C will reduce the charge on sphere A and therefore the force by a factor 2. So F/2 is the correct answer.

Problem 4: *charge Q is placed on the x-axis at (a,0) from the origin. A second charge -Q is placed on the y-axis at (0,a) from the origin. A third charge of unknown quantity is now placed on an unknown position in the x-y plane and the resulting electric field at the origin vanishes. At which of the following locations could this third charge be?*

Problem 4 Solution: The best way to solve this is to draw the electric fields of the first two charges into a coordinate system. The first charge creates a field which points into the negative x-direction. The second charge creates a field of identical magnitude which points into the positive y-direction. The vector sum of these two points along the diagonal forming an angle of 135deg with the x-axis. The only way to compensate this field with a single point charge is to place the third charge on this diagonal. The only coordinate pair in the potential solutions which are on that diaginal was (a,-a).

Problem 5: *A charge $q_1 = 1\mu C$ is located at (a,0), a charge $q_2 = -2\mu C$ is located at (-2a,0), and a charge $q_3 = 1.5\mu C$ is located at (0,-a). An electron is released at the origin. What is its instantaneous acceleration in m/s^2? ($a = 1cm$)*

Problem 5 Solution: The individual forces are

$$\vec{F}_1 = k\frac{e}{a^2}q_1\hat{i} \qquad \vec{F}_2 = k\frac{e}{a^2}\frac{q_2}{4}\hat{i} \qquad \vec{F}_3 = -k\frac{e}{a^2}q_3\hat{j}$$

So the final net force is:

$$k\frac{e}{a^2}\left(1.5\mu C\hat{i} - 1.5\mu C\hat{j}\right) = 1.5\mu C\frac{ke}{a^2}(\hat{i}+\hat{j}) \qquad |F| = 3.05\times 10^{-11}\text{N}$$

and the resulting acceleration

$$a = F/m_e = 3.35\times 10^{19}\text{m/s}^2$$

Problem 6: *An electric dipole with dipole moment* $\vec{p} = \left(3\hat{\imath} + 4\hat{\jmath}\right) 1.24 \times 10^{-30} Cm$ *is placed into an electric field* $\vec{E} = 4000 N/C\hat{\imath}$. *What is the potential energy of the electric dipole in J?*

Problem 6 Solution: The potential energy is the dot product, so we only need to look at the projection of the dipole moment on the E-field:

$$U = \vec{p} \cdot \vec{E} = 3 \times 1.24 \times 10^{-30}\text{Cm} \cdot 4000\text{N/C} = 1.49 \times 10^{-26}\text{Nm}$$

Problem 7: *In the figure two curved plastic rods, one of charge +3q and the other of charge -q, form a circle of radius R in an xy plane. Note that the positively charged rod is 3 times longer! The charge is distributed uniformly on both rods. What is the magnitude of the electric field produced at P, the center of the circle?*

Problem 7 Solution: The problem was supposed to ask about just the x-component of the electric field, so we gave everyone credit for this problem. The solution of what we wanted to ask is this. The two quarters opposite each other with the same charge compensate each other while the two other quarter circles generate identical fields at P. So we just have to integrate over the q_- part and multiply by two. The generic equation for a curved rod is:

$$E_x = k\frac{\lambda}{R}\int_{\theta_1}^{\theta_2} \cos\theta d\theta = k\frac{\lambda}{R}\left(\sin\theta_2 - \sin\theta_1\right)$$

Here the linear charge density λ is charge over length which is here a third of the positive charge distributed over a quarter circle:

$$\lambda_+ = \frac{q}{\frac{1}{2}\pi R} = -\lambda_-$$

and the angles for the positively charged part can be chosen to be $\theta_1 = 0-$ and $\theta_2 = 90°$ such that $(\sin\theta_2 - \sin\theta_1) = 1$. The angles for the negatively charged part across the origin are then $\theta_1 = 180°$ and $\theta_2 = 270°$ such that $(\sin\theta_2 - \sin\theta_1) = -1$. So the E-field is:

$$E_x = 2k\frac{\lambda}{R} = 2k\frac{q}{\frac{1}{2}\pi R^2} = 4k\frac{q}{\pi R^2}$$

Problem 8: *The figure shows a Gaussian surface in the shape of a cube with edge length 1.4 m. What is the net flux* Φ *(in* $\mathrm{N\ m^2/C}$*) through the surface if* $\vec{E} = 3y\hat{\jmath}$ $\mathrm{N/C}$*, with* y *in meters?*

Problem 8 Solution: Note that this was based on problem 23.9 from the text. Because $\vec{E}$ points along the y axis, only the surfaces at $y = 0$ and $y = 1.4$ m contribute. Further, because $\vec{E} = 0$ at $y = 0$, the only contribution is from the surface at $y = 1.4$ m,

$$\Phi_E = \Big(4.2\ \mathrm{N/C}\Big)\hat{\jmath} \cdot (1.4\ \mathrm{m})^2\hat{\jmath} = 8.232\ \mathrm{N\,m^2/C}\ .$$

Problem 9: *A charged particle is held at the center of two concentric conducting spherical shells as shown in Figure a. Figure b gives the net flux* Φ *through a Gaussian sphere centered on the particle, as a function of the radius* r *of the sphere. The scale of the vertical axis is set by* $\Phi_s = 5.0 \times 10^5\ \mathrm{N\,m^2/C}$*. What is the net charge (in* $\mu\mathrm{C}$*) of shell* A*?*

Problem 9 Solution: Note that this was based on problem 23.20 from the text. The flux just before shell A is $-\frac{9}{5}\Phi_s$, and the flux just after shell A is $+\frac{4}{5}\Phi_s$. Hence Gauss' law gives the charge on shell A as,

$$\epsilon_0\Big(\frac{4}{5}\Phi_s - (-\frac{9}{5}\Phi_s)\Big) = \frac{13}{5}\Phi_s\epsilon_0 \simeq 11.5\ \mu\mathrm{C}\ .$$

Problem 10: *An electron is shot directly toward the center of a large metal plate that has surface charge density* $-2.0 \times 10^{-6}\ \mathrm{C/m^2}$*. If the initial kinetic energy of the electron is* 1.6×10^{-17} J *and if the electron is to stop (due to electrostatic repulsion from the plate) just as it reaches the plate, how far from the plate must the launch point be?*

Problem 10 Solution: Note that this was based on problem 23.41 from the text. This can be done by energy techniques but the chapter 23 technique was to use Newtonian dynamics. The electron's acceleration has magnitude eE/m_e, where the electric field is $E = \sigma/\epsilon_0$. If the initial velocity is v_0 then the position and velocity of the electron are,

$$x(t) = -d + v_0 t - \frac{e\sigma}{2m\epsilon_0}t^2 \qquad , \qquad v(t) = v_0 - \frac{e\sigma}{m\epsilon_0}t\ .$$

The electron stops at $t_{\rm stop} = mv_0\epsilon_0/e\sigma$. Its position at this time is,

$$x(t_{\rm stop}) = -d + \frac{mv_0^2\epsilon_0}{2e\sigma} \ .$$

For the electron to stop at the plate we must have,

$$d = \frac{K\epsilon_0}{e\sigma} \simeq 4.4\times 10^{-4}\ {\rm m}\ .$$

Problem 11: *The electric field in a region of space has the components $E_y = E_z = 0$ and $E_x = 4x{\rm N/C}$, where x is given in meters. Point A is on the y axis at $y = 3$ m, and point B is on the x axis at $x = 4$ m. What is the potential difference $V_B - V_A$?*

Problem 11 Solution: Note that this was based on problem 24.7 from the text. The potential difference between two points is defined as minus the line integral of the electric field between the two points. The path is irrelevant so we can choose it to be along the y axis first(which gives zero because $\vec{E}\cdot d\vec{s} = 0$), then along the x axis. If we measure x in meters and E in N/C, the result is

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s} = -\int_0^4 4x dx = -2x^2\Big|_0^4 = -32\ .$$

Problem 12: *A plastic disk of radius $R = 64$ cm is charged on one side with a uniform surface charge density $\sigma = 7.73\ {\rm fC/m^2}$, and then three quadrants of the disk are removed. The remaining quadrant is shown in the figure. With $V = 0$ at infinity, what is the potential (in μV) due to the remaining quadrant at point P, which is on the central axis of the original disk at distance $D =$ 25.9 cm from the original center?*

Problem 12 Solution: Note that this was based on problem 24.31 from the text. We just integrate kdq/r over the surface,

$$V_P = \int_0^R \frac{k\sigma\frac{1}{2}\pi r dr}{\sqrt{r^2+D^2}} = \frac{k\sigma\pi}{2}\sqrt{r^2+D^2}\Big|_0^R = \frac{k\sigma\pi}{2}\left[\sqrt{R^2+D^2} - D\right] \simeq 47\ \mu{\rm V}\ .$$

Problem 13: *What is the magnitude of the electric field (in N/C) at the point $(3\hat{i} - 2\hat{j} + 4\hat{k})$ m if the electric potential in the region is given by $V = 2xyz^2$, where V is in volts and coordinates x, y and z are in meters?*

Problem 13 Solution: Note that this was based on problem 24.37 from the text. The components of the electric field are,

$$E_x = -\frac{\partial V}{\partial x} = -2yz^2 \ , \ E_y = -\frac{\partial V}{\partial y} = -2xz^2 \ , \ E_z = -\frac{\partial V}{\partial z} = -4xyz \ .$$

Evaluating these at $x = 3$, $y = -2$ and $z = 4$ gives,

$$\vec{E} = (+64, -96, +96) \quad \Longrightarrow \quad E = \sqrt{E_x^2 + E_y^2 + E_z^2} = 32\sqrt{22} \simeq 150 \ .$$

Problem 14: *The figure shows a rectangle whose sides have lengths 5.0 cm and 15 cm. The two charges are* $q_1 = -5\ \mu\mathrm{C}$ *and* $q_2 = +2\ \mu\mathrm{C}$*. With* $V = 0$*at infinity, how much work must be done to move a charge* $q_3 = +3\ \mu\mathrm{C}$ *from B to A along a diagonal of the rectangle?*

Problem 14 Solution: Note that this was based on problem 24.51 from the text. Let's call the long side $\ell = 15$ cm and the short side $d = 5$ cm. The electrostatic energies of charge q_3 at A and B are,

$$U_A = \frac{kq_1q_3}{\ell} + \frac{kq_2q_3}{d} \qquad , \qquad U_B = \frac{kq_1q_3}{d} + \frac{kq_2q_3}{\ell} \ .$$

The work done to move q_3 from B to A is,

$$W = U_A - U_B = kq_3(q_2 - q_1)\Big(\frac{1}{d} - \frac{1}{\ell}\Big) = 2.52 \text{ J} \ .$$

Problem 15: *In the figure the battery has a potential difference of* $V = 10$ V *and the five capacitors each have a capacitance of* $10\ \mu\mathrm{F}$*. What is the charge (in* $\mu\mathrm{C}$*) on capacitor 1?*

Problem 15 Solution: Note that this was based on problem 25.14 from the text. We can forget about the complicated, right hand side of the circuit and just note that the voltage across C_1 is V, so its charge is $Q_1 = C_1V_1 = 100\ \mu\mathrm{C}$.

Problem 16: *The figure shows a circuit formed by two parallel-plate capacitors connected to a battery. Capacitor 1 has a plate area of* $1.5\ \mathrm{cm}^2$ *and an electric field (between its plates) of magnitude 2000 V/m. Capacitor 2 has a* *plate area of* 0.7 cm^2 *and an electric field of magnitude 1500 V/m. What is the total charge on the two capacitors?*

Problem 16 Solution: Note that this was based on problem 25.25 from the text. Recall that the parallel plate capacitance is $C = \epsilon_0 A/d$ and that the voltage is $V = Ed$. Hence the charge on C_1 is

$$Q_1 = C_1 V = \epsilon_0 A_1 E_1 = 2.655 \text{ pC} .$$

Similarly, the charge on C_2 is

$$Q_2 = C_2 V = \epsilon_0 A_2 E_2 = 0.92925 \text{ pC} .$$

Adding the two charges gives $Q = Q_1 + Q_2 \simeq 3.7$ pC.

Problem 17: *In the figure a potential difference of* $V = 100$ V *is applied across a capacitor arrangement with capacitances* $C_1 = 10\ \mu\text{F}$, $C_2 = 5\ \mu\text{F}$ *and* $C_3 = 15\ \mu\text{F}$. *What is the energy* U_1 *stored in capacitor 1?*

Problem 17 Solution: Note that this was based on problem 25.38 from the text. C_1 and C_2 are connected in parallel so their equivalent capacitance is $C_{12} = 15\ \mu\text{F}$. This is the same as C_3, so the voltage is divided between C_3 and C_{12}. Hence the voltage drop across C_1 is 50 V, so its energy is $U_1 = \frac{1}{2} \times 10\ \mu\text{F} \times (50\text{ V})^2 = 12.5$ mJ.

Problem 18: *What is the current in a wire of radius* $R = 3.4$ mm *if the magnitude of the current density is given by* $J = J_0(1 - r/R)$, *in which* r *is the radial distance and* $J_0 = 5.5 \times 10^4\ \text{A/m}^2$*?*

Problem 18 Solution: Note that this was based on problem 26.11 from the text, and was worked in class on Sept. 25. The current is the surface integral of the current density,

$$I = \int_0^R J(r) \times 2\pi r dr = 2\pi J_0 \Big(\frac{1}{2} r^2 - \frac{r^3}{3R}\Big)\Big|_0^R = \frac{1}{3}\pi J_0 R^2 \simeq 0.67 \text{ A} .$$

Problem 19: *A wire with a resistance of* $6\ \Omega$ *is drawn out through a die so that its new length is three times its original length. Find the resistance of* *the longer wire, assuming that the resistivity and density of the material are unchanged.*

Problem 19 Solution: Note that this was based on problem 26.25 from the text. Recall that the resistance is $R = \rho L/A$. When the wire is extended its volume $V = LA$ stays constant, so if its length increases by a factor of 3, its area must decrease by the same factor of 3. Hence the resistance of the new wire is $R_{\text{new}} = \rho \times 3L/(\frac{1}{3}A) = 9R = 54\ \Omega$.

Problem 20: *Wire C and wire D are made from different materials and have length $L_C = L_D = 1$ m. The resistivity and diameter of wire C are $2 \times 10^{-6}\ \Omega \cdot \mathrm{m}$ and 1 mm, and those of wire D are $1 \times 10^{-6}\ \Omega \cdot \mathrm{m}$ and 0.5 mm. The wires are joined as shown in the figure, and a current of 2 A is set up in them. What is the rate at which energy is dissipated between points 2 and 3?*

Problem 20 Solution: Note that this was based on problem 26.51 from the text. The resistance of wire D is,

$$R_D = \frac{\rho_D L_D}{\frac{\pi}{4} D_D^2} = \frac{16}{\pi}\ \Omega\ .$$

Hence the power dissipated in wire D is $P_D = I^2 R_D = \frac{64}{\pi}$ W $\simeq 20$ W.